Astro 210
Lecture 6
Sept. 9, 2013

Announcements:

- **PS 2 due Friday**
- ASTR 401: abstracts due next Monday

PS 1: many questions & funny looks, about *dimensional analysis*
a "quick and dirty" way to get rough, approximate answers
→ a way to *estimate* results

But: You've taken many courses and spent a lot of time
learning complex and powerful tools for precise calculation
Isn't estimation/dimensional analysis a step backwards?
Maybe: *Those who can't calculate, they approximate!*

After all you've done to do things precisely
Q: why ever make rough, imprecise approximations?

Approximation is Real Science

the real world is subtle and rich ($\equiv$ complicated)
physics/astro phenomena elaborately detailed, but
not all details equally important

no real-world system *ever* simple enough
to calculate without *any* approximation
and even if you could, complicated result hides insight

faced with a new problem: simplify!
...but keep the essentials

approximations and estimates help you
• to see what is relevant
• to see what is irrelevant
• to test ideas/hunches quickly
★ identify which detailed calculation(s) are worth doing

Dimensional Analysis: The Estimator's Workhorse

physical quantities have dimensions (units)
all units can ultimately be expressed in terms of
three *fundamental dimensions (units)*
[length] $\equiv [L]$, [time] $\equiv [T]$, and [mass] $\equiv [M]$

example: universal gravitation
force definition $F = G m_1 m_2 / r^2 \Rightarrow [G] = [L^3 \ M^{-1} \ T^{-2}]$
so: given *only one* mass scale m and lengthscale ℓ
the *unique* timescale satisfies $\tau^2 \sim \ell^3 / GM \sim 1/G\rho$
$\rightarrow$ applies to *any* problem with gravity *only*

Keplerian motion around Sun:
mass scale is $m = M_\odot$, lengthscale is $\ell = a$
$\rightarrow$ estimate gravitational timescale of *period* $\tau^2 = P^2 \sim a^3 / GM_\odot$
compare to honest calculation: period $P^2 = 4\pi^2 a^3 / GM_\odot$
found guts of a law of Nature–Kepler III! but not $4\pi^2$ factor...
more details below in Director's Cut Extras

Last time:

stellar evolution: main sequence

Q: what is the origin of the main sequence on the HR diagram?

Q: how does stellar lifetime depend on mass?

Q: what is the power source for main sequence stars?

Q: how do we know for sure?

Journey to the Center of the Sun: Solar Neutrinos

the Sun now is on the *main sequence = hydrogen burning*
a variety of nuke reactions occur in solar core
net effect: 4p $\rightarrow$ ^{4}He $+$ energy

First link: $p + p \rightarrow d = \boxed{\text{np}} + e^+ + \nu$
d deuterium: *Q: which is what kind of atom?*
e^+ positron: **antimatter** partner of e^-
opposite charge, same mass
ν neutrino: no charge, tiny mass ($m_\nu \ll m_e$)
very weakly interacting, only created in nuke transformations

neutrinos come directly from solar core $\rightarrow$ detect on earth
`www: SNO detector`
`www: Super-K image of Sun`
$\Rightarrow$ proof Sun powered by fusion!

Q: what happens when core of star is all He?

Post-MS Evolution: Death and Dying

depends on mass

low-mass: $m \lesssim 0.8 M_\odot$
$\tau(m) > t_0$ age of the universe
Q: what does this mean for these stars?

intermediate mass: $0.8 M_\odot \lesssim m \lesssim 8 M_\odot$
He core contracts, heats
H shell ignites, energy balance lost
outer layers expand, cool
red giant
He core ignites, burns $3\,^4\text{He} \rightarrow {}^{12}\text{C}$, also oxygen *diagram: He core, H shell, env*

for $M \lesssim 1.5 M_\odot$, most energy release in giant phase
$\rightarrow$ in old $\star$ systems L dominated by giants

`www: MW near-IR`

`www: elliptical galaxy`

pulsations $\rightarrow$ outer layers ejected
planetary nebula

when core $\rightarrow$ C+O, can't burn
$\rightarrow$ white dwarf

Intermediate Mass Stars: Element Production

`www: chemist's periodic table`

nucleosynthesis: production and cycling of elements
heavy elements = all but H and He = "metals" = Z
(e.g., famous "metals" C, N, O)
astronomer's periodic table: | H | | He | | Z |

all heavy elements are created in stars

`www: circle of life`
intermediate mass stars: ^{4}He, C

∞

High Masses: James Dean of Stars

high-mass: $m \gtrsim$ 8 to$10 M_\odot$

after MS $\rightarrow$ supergiant
`www: Betelgeuse`
cycles: core ash contracts $\rightarrow$ heats $\rightarrow$ ignites
ash $\rightarrow$ fuel
"onion-skin" structure

when core $\rightarrow$ iron
can't burn more (Fe fusion takes away E)
core collapse $\rightarrow$ bounce
Demo: astro blaster!
$\Rightarrow$ supernova explosion
Q: where is star's material after explosion?

ejected material:
hot ($\gtrsim 10^6$ K), fast 10,000 km/s nucleosynthesis products: almost all metals
lots of O, Mg, Si, S, Fe

leftover ultradense core:
neutron star or black hole

Connection with galactic environment and evolution:
Q: do you expect SN from massive stars in elliptical galaxies?
Q: how about spirals?
`www: SN in galaxy`

stars in ellipticals have higher metal content than in spirals
Q: what does this say about the past history?

GALAXIES: SWEET HOME MILKY WAY

iClicker Poll: Our Milky Way Galaxy

Milky Way to naked eye: irregular band of light

`www: MW mosaic`

Vote your conscience!

What is the dominant naked-eye Milky Way light source?

A predominantly gas

B predominantly stars

C roughly equal mix

Milky Way: Overview and History

Galileo (1610): first telescope for astronomy
revolutionized our view of the universe, e.g.

- Venus phases ruled out Earth-centered (geocentric) cosmology
- away from Milky Way discovered stars too faint for naked eye
  philosophical problem: what's the use of stars we can't see?

observing Milky Way's light:
  Galileo saw it is *made of stars*

- huge numbers of stars
- very crowded on sky
- individually very faint

eye can't see MW stars individually, light blends together

MW band on 2-D sky is a great circle
Q: what's that?
Q: what does this mean for MW in 3-D space?

Director's Cut Extras

Dimensional Analysis and Estimation

Profound but seemingly innocent observation I:

the behavior of a physical system is independent of the units used to describe it

Profound but seemingly innocent observation II:

in any expression (equation) describing a physical system each term must have the same units

i.e., physical equations must be dimensionally **homogeneous**

Dimensional Analysis Illustrated

Consider
- a Newtonian particle in a uniform gravity field g
- released from rest, then after time t
- falls some height $h \leftarrow$ *want to find this*

You know the exact result, but imagine you don't

If we have fully characterized the problem
then it should be possible to write

$$h = f(g, t) \tag{1}$$

where f is an arbitrary (for now) function

to solve the problem: specify f
- could use Newtonian mechanics, honest calculation takes work (integration), gives exact result
- but we can get far just by looking at dimensions

Q: what does dimensional homogeneity imply for $h = f(g, t)$?

what does dimensional homogeneity mean
for our relation $h = f(g, t)$?

- since $[h] = [L]$
  then we must have $[f] = [L]$
- but also: if h is measured in meters, then f must be as well
- so if we change to h' in yards, then
  $h' = \lambda h$, and in yards $f' = \lambda f$,
  where both expressions have the *same* conversion rescaling λ

so we have: $h = f(g, t)$ dimensionally homogeneous
rewrite: $h/f(g, t) = const = 1$
$\Rightarrow$ holds regardless of the units used

we see $h/f(g,t)$ forms a dimensionless constant
but our variables have:

- $[g] = [LT^{-2}]$
- $[t] = [T]$

given these dimensions, *only one grouping* of variables h, t, and g is dimensionless

Q: find this grouping!
Q: use this to find the most general form of $f(g,t)$!

we have $[f(g,t)] = [L]$
but the only way to form a length from g and t
is the unique combination: $\boxed{gt^2}$

so the most general dimensionally legal expression is

$$f(g,t) = Cgt^2 \tag{2}$$

with C a dimensionless *constant*
Q: what's wrong with $Cgt^2+\Lambda$*, or* $C(gt^2)^2/\Lambda$*, with* Λ *a constant?*

and thus our dimensionless ratio can only be

$$\frac{h}{f(g,t)} = \frac{1}{C}\frac{h}{gt^2} = const = 1 \tag{3}$$

and so we can now solve

$$\boxed{h = Cgt^2} \tag{4}$$

Without calculus, but only considering dimensions, we find

$$h = Cgt^2 \tag{5}$$

with C an undetermined dimensionless constant that is independent of units used for h, g, t

Q: what does this equation teach us?
Q: what does this not *give us?*
Q: how could you test this equation without knowing C?
Q: if you didn't know C, what's a reasonable order-of-magnitude guess?
Q: how could you find C if you didn't know calculus?
Q: what is the actual value of C?

Dimensional Analysis: Lessons

what has

$$h = Cgt^2 \tag{6}$$

done for us?

- *scaling* relations $h \propto g$ and $h \propto t^2$
- don't know C: constant, so "invisible" to dim. analysis
- can test $h \propto t^2$ without knowing C
  measure fall time for different h, see if quadratic
- if you had to guess, would try $C \sim 1$
- without calculus, could get this *experimentally*:
  measure h vs t, find $C = h/gt^2$
- of course, freshman physics says $C = 1/2$
  order-of-magnitude guess off by factor 2: not bad!

Dimensional Analysis: Twitter Version

What else could it be?

E.g.: the only length arising from g and t is gt^2
so we must have $h \sim gt^2$: what else could it be?

Lessons:

- gather *all relevant* variables
- find dimensionless grouping(s)
- use to solve for the result of interest
- shortcut: find combinations of variables
  with dimensions of the answer you want