Astro 406

Lecture 17

Oct. 4, 2013

Announcements:

- **PS 5 due now**
- **PS 6 due next Friday**
- ASTR 401: draft section due Monday

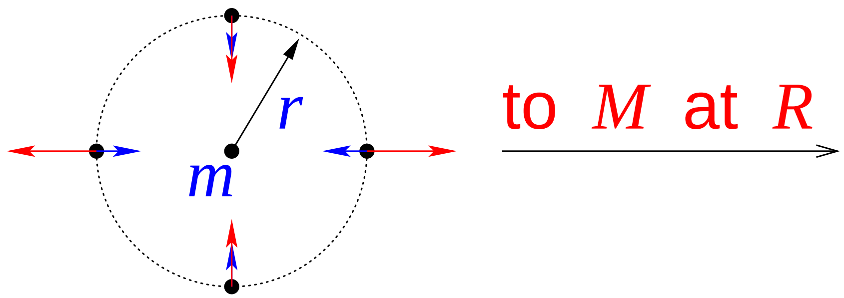
Last time: globular star clusters

Q: What's a globular cluster?

Q: what's special/important about them?

Globular Cluster Vital Statistics

- dense, bound spherical systems of stars
- typical mass: $10^5 - 10^6 M_\odot$
- not rotationally supported, no much/any dark matter
- heavy element content ("metallicity") low
  typically cluster stars have $(\mathrm{Fe/H})_\star \sim 10^{-2}(\mathrm{Fe/H})_\odot$
- HR diagram: clusters are very old
- found within galaxies, not in isolation
  $\sim$ 150 clusters in spherical distribution around MW
  and other galaxies `www: M31, M87 globular clusters`
- often a $\approx$ const density core at $r_c \sim 1.5$
- outer radius $r_t \sim 50$ pc *Q: what sets this?*



Tidal stripping: external tides (from M, R) overcome self-gravity (of m, r)?

$$\frac{2GMr}{R^3} = \frac{Gm}{r^2} \tag{1}$$

→ stripped at distance

$$r_{\rm tide} = \left(\frac{m}{2M}\right)^{1/3} R \tag{2}$$

For globular clusters in Galaxy:

$$r_{\rm tide} = \left(\frac{m_{\rm gc}}{2M_{\rm MW}}\right)^{1/3} R_{\rm MW} \sim \left(\frac{10^6 \ M_\odot}{10^{12} M_\odot}\right)^{1/3} 10 \text{ kpc} \sim 100 \text{ pc} \tag{3}$$

agrees with observed GC values!
→ any stars outside this radius are already gone!

Note: unclustered "field" stars in stellar halo (spheroid)
low metallicity, large ages
and similar spherical spatial distribution to GCs

Q: implication?

The Stellar Halo and Globular Cluster Stripping

when globular cluster stars stripped
they remain in spherical distribution
$\rightarrow$ become field stars in stellar halo

many if not all halo stars
likely were once in young & much more massive globulars

tidal disruption still going!
`www: SDSS image of GC Palomar 5, not tidal tails`
`www: Magellanic stream`
`www: colliding galaxies`

Globular Cluster Dynamics: Orbits

How do stars move within a globular cluster?
$\rightarrow$ orbits controlled by cluster gravity

globular clusters are (almost all) spherical

motions of stars in spherical systems an important problem
also applies to stellar bulges in spiral galaxies
and to nearly-spherical elliptical galaxies

Q: what does sphericity imply for the density $\rho(r)$*?*
Q: for the gravitational acceleration $g(r)$*?*

for spherical globular cluster, symmetry implies:

- *mass density* $\rho = rho(r)$
- *gravity* $\vec{g}(r) = -GM_{\rm enc}(r)/r^2\ \hat{r}$

  i.e., gravity acceleration entirely *radial*

  and thus gravity force $\vec{F} = m\vec{g}$ also radial

angular momentum $\vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$

with $\vec{v} = \dot{\vec{r}} = d\vec{r}/dt$

time change:

$$\dot{\vec{L}} = \dot{\vec{r}} \times \vec{p} + \vec{r} \times \dot{\vec{p}} \quad (4)$$

$$= m\vec{v} \times \vec{v} + \vec{r} \times \vec{F} = \vec{r} \times \vec{F} \quad (5)$$

$\rightarrow$ angular momentum changes due to **torque** $\vec{r} \times \vec{F}$

Q: what is torque on a globular cluster star?

for any *radial* force: $\vec{F}(r) = F(r)\ \hat{r}$, and so

$$\vec{r} \times \vec{F} = F(r)\ \vec{r} \times \frac{\vec{r}}{r} = 0 \tag{6}$$

torque is zero! $\rightarrow \dot{\vec{L}} = 0$
angular momentum is conserved!
each star orbit maintains magnitude & direction of $\vec{L}$

every star keeps its $\vec{L} = \vec{r} \times \vec{p}$ constant
to do this, *each orbit must lie in a plane*

orbit dynamics obey Newton II: for star of mass m

$$\dot{\vec{p}} = m\dot{\vec{v}} = \vec{F} = m\ \vec{g} \tag{7}$$

$$\dot{\vec{v}} = \vec{g}(r) = -\nabla\Phi(r) \tag{8}$$

with **gravitational potential** Φ

- for single point mass M: $\Phi = -GM/r$
- for mass density ρ, satisfies $\nabla^2\Phi = -4\pi G\rho$

Gravity and Energy

consider a test particle of mass m

with velocity $\vec{v}$

and living in a gravitational potential Φ write the test particles' **energy** as

$$E = \frac{1}{2} m \vec{v} \cdot \vec{v} + m\Phi = \frac{1}{2} m v^2 + m\Phi \tag{9}$$

How does this change with time?

$$\frac{dE}{dt} = m\vec{v} \cdot \dot{\vec{v}} + m \frac{d\phi}{dt} \tag{10}$$

use chain rule

$$\frac{d}{dt}\phi(x,y,z,t) = \frac{\partial \phi}{\partial x}\frac{dx}{dt} + \frac{\partial \phi}{\partial y}\frac{dy}{dt} + \frac{\partial \phi}{\partial z}\frac{dz}{dt} + \frac{\partial \phi}{\partial t} = \vec{v} \cdot \nabla \phi + \frac{\partial \phi}{\partial t} \tag{11}$$

physically: first term – change due to movement

second term is change due to actual time variation so we have

$$\frac{dE}{dt} = \vec{v} \cdot \left(m\dot{\vec{v}} + m\nabla\phi \right) + m\frac{\partial \phi}{\partial t} = m\frac{\partial \phi}{\partial t} \qquad (12)$$

Q: why do the terms cancel?

Q: under what conditions is E conserved? not conserved?

Bottom line: in gravity potential ϕ
test particle energy changes as

$$\frac{dE}{dt} = m\frac{\partial \phi}{\partial t} \tag{13}$$

so **if** $\partial\phi/\partial t = 0$: static potential
then $dE/dt = 0 \to$ particle $E = const$: conserved!
but in a time-changing potential $\partial\phi/\partial t \neq 0$
single particle energies are *not conserved!*

Q: what's a system with static ϕ?
Q: what's a system with time-varying ϕ?
`www: examples`
Q: how would this lead to particle energy non-conservation?
Q: so why all the hype about energy conservation?

Globular Cluster Orbits

for unchanging globular cluster gravitational potential
each star's energy is conserved

orbit is in a plane:

- use polar coordinates (r, θ)
- constant angular momentum magnitude $L = mr^2\dot{\theta}$
  per unit mass: $L/m = \ell\ r^2\dot{\theta}$
- constant energy per unit mass ("specific energy") $\varepsilon = E/m$ is:

$$\varepsilon = \frac{1}{2}\vec{v}^2 + \Phi = \frac{1}{2}\dot{r}^2 + \frac{1}{2}r^2\dot{\theta}^2 + \Phi \quad (14)$$

$$= \frac{1}{2}\dot{r}^2 + \frac{\ell^2}{2r^2} + \Phi(r) \quad (15)$$

Q: what happens physically when $\dot{r} = 0$?

Q: how to find the r where this happens?

Q: how many r values will have $\dot{r} = 0$?

Turning Points

when $\dot{r} = 0$, instantaneously no radial motion
$\rightarrow$ radial position r is at extremum!
$\rightarrow$ *turning point* in orbit

stars orbit lies between
maximum radius: *apocenter* $r_{\rm ap}$
minimum: *pericenter* $r_{\rm peri}$

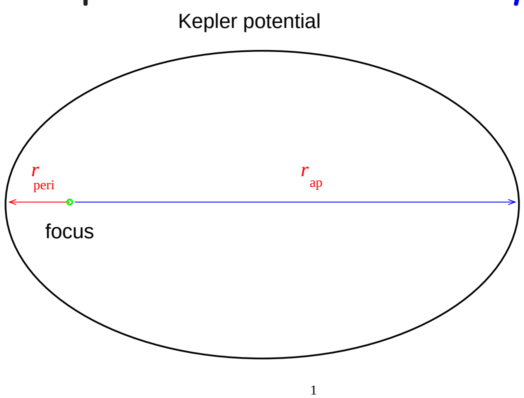
for a given potential Φ, can calculate

- time $\Delta T_{\rm r}$ from max to max (or min to min)
- time ΔT_{θ} to go around $\Delta\theta = 2\pi$

$\rightarrow$ *these times need not be the same*
$\rightarrow$ values depend on the potential

13 consider a point (Kepler/Newton) potential $\Phi = -GM/r$
Q: how are $\Delta T_{\rm r}$ *and* ΔT_{θ} *related?*

Kepler motion: *ellipse*, source M at one *focus*



orbit period $P = \Delta T_\theta$ obeys $P^2 = (4\pi^2/GM)a^3$

the same as max-max time: $\Delta T_\mathsf{r} = \Delta T_\theta$

but is this always true?

consider *uniform density sphere* $\rho(r) = \rho_0$

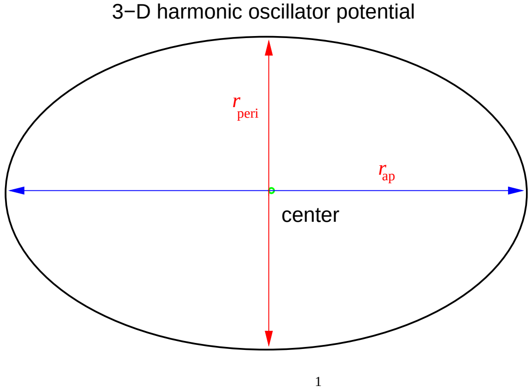
PS6: can show potential has *harmonic oscillator form*

"3D harmonic oscillator"

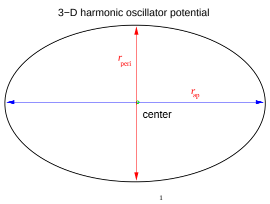
and can show:

- oscillations in x and y have same period $P = \Delta T_\theta$
- orbit is *ellipse*

  centered on cluster center



iClicker Poll: Uniform Sphere Orbits



uniform sphere orbits are ellipses centered on cluster center

How is period $P = \Delta T_\theta$ related to $\Delta T_{\rm r}$?

A $\Delta T_\theta < \Delta T_{\rm r}$

B $\Delta T_\theta = \Delta T_{\rm r}$

C $\Delta T_\theta = 2\Delta T_{\rm r}$

D $\Delta T_\theta > 2\Delta T_{\rm r}$

Q: implication?

for point mass potential: $\Delta T_\theta = \Delta T_{\mathrm{r}}$

for uniform density potential: $\Delta T_\theta = 2\Delta T_{\mathrm{r}}$

in both cases: $\Delta T_\theta / \Delta T_{\mathrm{r}}$ is a *rational number*
$\rightarrow$ this means that the *orbits close*
i.e., keep same shape, orientation in plane *Q: why*

but real globular clusters (and elliptical galaxies)
are neither point masses nor uniform density

Q: expectations for $\Delta T_\theta / \Delta T_{\mathrm{r}}$*?*
Q: what does this mean for stellar orbits?

Globular Cluster Orbits

realistic spherical stellar systems like globular clusters are *centrally concentrated*:
$\rho(r)$ largest at center, decreases outwards

density *more* concentrated than uniform sphere *less* concentrated than point mass

intuitive (and correct) expectation: $1 < \Delta T_\theta / \Delta T_\mathrm{r} < 2$
and generally *not a rational number*
$\rightarrow$ *orbits do not close*
make "rosette" pattern

`www: orbit simulation animations`

Q: implications for cluster structure?

Q: what dynamical effects have we ignored so far?