Astro 406
Lecture 28
Nov. 1, 2013

Announcements:

- **PS 8 due now**
- **PS 9 due next Friday**

Last time: expansion of the universe

- cosmic scale factor $a(t)$
  Q: what is it? present value? past, future values?
  Q: connection with Hubble "constant"?
- expansion → Hubble's law *Q: how?*

Expansion: Einstein → Hubble

transparency demo: photocopy universe

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances

$$\vec{r}(t) = \vec{r}_0 a(t) \tag{1}$$

so velocity is

$$\begin{aligned} \vec{v}(t) &= \dot{\vec{r}} = \vec{r}_0 \dot{a} = \frac{\dot{a}}{a}\, a\vec{r}_0 \qquad (2) \\ &= H(t)\ \vec{r}(t) \qquad (3) \end{aligned}$$

⇒ this is *Hubble's law!* holds at all times!

we now can interpret the "Hubble parameter"
as the **cosmic expansion rate** $\boxed{H(t) = \dot{a}/a}$

Expansion and Cosmology

All of cosmology is nothing more or less
than the evolution of a system that is

- homogeneous
- isotropic
- expanding

⋆ much of cosmology amounts to imagining a box

  - filled homogeneously with galaxies (today)
    or atoms/particles (in the early Universe)
  - with other identical expanding boxes on all sides

and asking questions, like:
how do the contents respond as the box expands?
how does the universe respond to changes in its contents?

⋆ to do this don't need to know if U. has finite or infinite volume!
question is interesting but can distract and confuse

Expansion and the Hubble ’Constant”

Rate (“speed”) of expansion controlled by Hubble parameter

$$H(t) = \frac{da/dt}{a} = \frac{\dot{a}(t)}{a(t)} \tag{4}$$

ex: time δt for a to *double* roughly given by

$2a(t) = a(t + \delta t) \approx a(t) + \delta t\, \dot{a}$

$\rightarrow \delta t \approx a/\dot{a} = 1/H \Rightarrow H$ *is roughly cosmic doubling rate*

present-day value: $H_{\mathrm{today}} = H(t_0) \equiv H_0$

well-measured, thanks largely to Hubble Space Telescope:

$$H_0 = 71 \pm 2.5 \ \mathrm{km\ s^{-1}\ Mpc^{-1}} = \frac{1}{13.8 \pm 0.7 \ \mathrm{Gyr}} \tag{5}$$

but generally need not be (and isn’t!) constant!

Confusing vestige #1: H often still called “Hubble constant”

Confusing vestige #2: “little h”

when dinosaurs roamed the earth (20 yrs ago): H_0 uncertain
$H_0 = 50 - 100$ km s^{-1} Mpc^{-1}
bad news: many cosmo results hinge on this value
e.g., for $h = (0.5, 1)$, a $z = 0.1$ galaxy is at $D = v/H_0 = cz/H_0 = (600, 300)$ Mpc

so wrote $H_0 = 100h$ km s^{-1} Mpc^{-1}
put uncertainty in “little h” factor $h = 0.5 - 1.0$
and thus a $z = 0.1$ galaxy is at 300 h^{-1} Mpc

In modern times: we know H_0 to a few percent
and so we now know that $h = 0.71 \pm 0.025$
but h still fills the literature...sorry!

Cosmic Scale Factor Revisited

for two “particles” (possibly Galaxies!)
distance evolves according to

$$\vec{\ell}(t) = \underset{\substack{\text{scale factor} \\ \textit{time varying}}}{a(t)} \quad \underset{\substack{\text{present distance} \\ \textit{fixed once and for all}}}{\vec{\ell}_0} \tag{6}$$

and thus

$$\vec{v} = H\vec{\ell} \tag{7}$$

with $H = \dot{a}/a$

Note: a depends on t, but
not spatial position

Q: why not?

Q: implications–present, past, future values for a*?*

present: at t_0, $a(t) = 1$
Universe is expanding, so

past: $a(t) < 1$
future: $a(t) > 1$

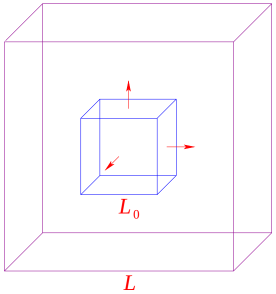
e.g., at some time in past $a = 1/2$
“galaxies twice as close”

Q: how do cosmic volumes depend on a?
e.g., *Q: when $a = 1/2$?*

Q: plot possible behavior(s) of $a(t)$?

Expansion and Areas, Volumes

consider a cube, galaxies at corners
present side length L_0
at any time: length $L(t) = a(t)\, L_0$



cube is “comoving” w/ expansion

- *volume* $V = L^3 = L_0^3 a^3 = V_0\ a^3$, thus $V \propto a^3$
- area of a side: $A = L^2 = A_0\ a^2$, thus $A \propto a^2$

`www: raisin cake analogy`
`www: balloon analogy`
Q: what is tricky, imperfect about each analogy?

Redshifts

wavelengths are lengths! it's right there in the name!

expansion stretches photon λ

$\lambda(t) \propto a(t)$

if emit at $t_{\rm em}$, then

$$\lambda(t) = \lambda_{\rm emit} \frac{a(t)}{a(t_{\rm em})} \tag{8}$$

Q: why is this right?

if observe later: $\lambda_{\rm obs} = \lambda_{\rm em}\ a_{\rm obs}/a_{\rm em}$

so measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \quad \Rightarrow \quad a_{\rm em} = \frac{1}{1+z}$$

Q: notice what the redshift isn't?

Note: redshift is really due to cosmic stretching
not a Doppler shift! Doesn't measure galaxy speed
neither at emission nor at observation

Also note: Scale factor ↔ redshift

$$a = \frac{1}{1+z} \tag{9}$$

$$z = \frac{1}{a} - 1 \tag{10}$$

both are ways to label an instant in the cosmic past

Example: most distant QSO has $z \approx 7$
→ scale factor was $a = 1/(1+z) = 0.125$
interparticle (intergalactic) distances 12.5% of today!
→ galaxies $1+z = 8$ times closer
squeezed into volumes $8^3 = 512$ times smaller!

Photon Energies and Thermal/Blackbody Radiation

since $E_\gamma = hc/\lambda$
$E \propto 1/a \to \gamma$: photon energy redshifts

consider photon radiation with thermal spectrum
Q: what does this mean? examples?
Q: $T \leftrightarrow \lambda$ *connection?*
Q: expansion effect on T*?*

Wien's law: $T \propto 1/\lambda_{\mathrm{max}}$

but since $\lambda \propto a$
then *cosmic temperature* obeys

$$T \propto \frac{1}{a} \tag{11}$$

temperature "redshifts" with time
⇒ T decreases → *the Universe cools as it expands!*

today: cosmic temperature $T_0 = 2.725 \pm 0.001$ K
distant but still "garden variety" QSO: $z = 3$
"feels" $T = 8$ K (effect observed!)

Cosmic Time Dilation

GR: gravitational redshifting goes hand-in-hand with gravitational time dilation

emitted, waves observed (today) have

$$\lambda_{\text{obs}} = \lambda_{\text{em}} \frac{a_{\text{obs}}}{a_{\text{em}}} = \lambda_{\text{em}} \frac{1}{a_{\text{em}}} = (1+z)\lambda_{\text{em}} \tag{12}$$

but wave crests are like clock!
emitted periodically: $\Delta t = 1/f = \lambda/c$
$\rightarrow \lambda = c\Delta t$: wavelength is light travel time between period
and thus we have

$$\Delta t_{\text{obs}} = \Delta t_{\text{em}} \frac{1}{a_{\text{em}}} = (1+z)\Delta t_{\text{em}} \tag{13}$$

Q: what does this mean physically?
Q: how could we test for it observationally?

→ i.e., redshifted objects also *appear to have* slow clocks

cosmic time dilation

Cosmic time dilation observed! And only recently!

Challenge: need "standard clock" in order to know that it's running slow

Tool: exploding stars (supernovae) – known timing of brightness observe high-z supernovae, see lengthening of duration in explosion and aftermath!

Woo hoo!

Cosmic Density Evolution: Matter

matter for cosmo defined as *non-relativistic particles*
$\rightarrow \langle v \rangle \ll c$, $kT \ll mc^2$

if a non-relativistic species is not produced/destroyed
Q: when would this assumption break down?

- number density $n_{\rm matter} \propto a^{-3}$ (volume effect)
- mass density $\rho_{\rm matter} = m n_{\rm matter} \propto a^{-3}$

$$\boxed{\rho_{\rm matter} = \rho_{\rm matter,0}\ a^{-3}}$$

example: most distant QSO has $z \approx 7 \rightarrow a = 1/8$
when cosmic matter density was higher by factor $a^{-3} \approx 512$

Q: by analogy, what is "radiation" to a cosmologist?