Astronomy 596/496 APA, Fall 2009
Homework #2
Due in class: Thursday, Sept. 10

1. *Order-of-Magnitude Astrophysics: Gravity and Dimensional Analysis.* This problem is wordy but straightforward, and hopefully interesting.

   (a) Consider a test particle at a characteristic lengthscale a from a central point mass M. Associated with this system is a characteristic time, the "gravitational timescale" or "dynamical time" $\tau_{\rm grav}$.

   Show that there is one and only one way to form a timescale from the variables M, a, and the gravitational constant G. Use this to find an expression that estimates $\tau_{\rm grav}$.

   (b) Consider a test particle in a circular orbit (with constant speed) around a mass M at a distance a. If the (Newtonian) gravity of M provides the centripetal acceleration, find the exact expression for the orbit period P.

   Compare and contrast your result to the gravitational timescale estimate from part (1a). What does and doesn't the dimensional analysis give us?

   (c) For a system with a characteristic mass density ρ, find a gravitational timescale $\tau_{\rm grav}$.

   (d) Now consider a spherical matter distribution of characteristic size R and characteristic density ρ. Assume this object feels no internal or external forces other than gravity. In the absence of opposing forces, the object will collapse under its own gravity.

      i. Estimate $\tau_{\rm grav}$ for this system.

      ii. To see that this is a reasonable result, make a different estimate for the collapse timescale by asking: how long it would take a particle at the surface of the sphere to fall to the center? For simplicity you may assume it always feels a constant gravitational acceleration g, which you may take to be the pre-collapse acceleration at R. How does your result compare with $\tau_{\rm grav}$?

      iii. Finally, evaluate the gravitational timescale for the Earth, the Sun, and a neutron star, in seconds. *Hint:* this is most simply done using mean densities. Comment on your results. If you find that the timescale is shorter than the known ages of these objects, explain the discrepancy.

   (e) Now consider a sphere of *non*-uniform density, also undergoing gravitational collapse. If, as in most stars, the density *decreases* with increasing radius, from a maximum at the center to a minimum at the surface, how would you expect the collapse to proceed? Don't do any calculation here, but use the form of $\tau_{\rm grav}$ to guide your reasoning. Also, what if the density were to *increase* with radius from a minimum at the center to a maximum at the surface?

   (f) The mean mass density of the Universe today is about $\rho_0 \sim 3 \times 10^{-30}\ {\rm g\,cm^{-3}}$. Estimate the associated gravitational timescale, in seconds and in billions of years (10^9 yr = 1 Gyr). What is the physical significance of this timescale?