Astronomy 596/496 APA, Fall 2009
Homework #4

Due on Compass: Thursday, Sept. 24

1. *Order-of-Magnitude Astrophysics: Supernova Explosions.* Next week's Colloquium Speaker, Laura Lopez, studies supernova remnants. The aftermath of a supernova explosion is not only a driving force in galaxy evolution, but also a canonical example of dimensional analysis.

   As usual, this is wordy but should not be difficult.

   (a) A star explodes, ejecting a mass M and releasing an energy E that is much larger than the star's gravitational binding energy. The star is embedded in a cold gaseous insterstellar medium (ISM) of uniform mass density ρ.

   The blast expands to radius r, sweeping up the ISM gas it encounters. Trivially estimate an expression for the mass of the swept-up medium.

   Then estimate an expression for the radius $r_{\rm sw}$ at which the swept-up mass is equal to the ejected mass M.

   Finally, evaluate $r_{\rm sw}$ in parsecs. Use a typical supernova value $E = 10^{51}$ erg, and estimate M to be typical for a high-mass star. And put $\rho = m_p n$ with m_p the proton mass and a gas particle number density $n = 1 \text{ atom/cm}^3$ typical of the interstellar medium.

   How would you expect the ejecta velocity to change early after the explosion, when $r \ll r_{\rm sw}$?

   (b) For $r \gg r_{\rm sw}$, the supernova remnant dynamics no longer depend on the ejected mass, but only on the explosion energy E and medium density ρ.

   Find a dimensionless combination of these variables, and the remnant size r and age t. Call this dimensionless number θ.

   Is θ unique?

   (c) Guess a physical value that θ might have for a typical supernova remnant. Using this, find an expression that estimates the remnant age t as a function of E, ρ, and r.

   (d) Now consider an explosion in a medium that is not cold but has temperature T, and thus nonzero pressure. Find an expression that estimates the supernova remnant radius $r_{\rm eq}$ at which the blast pressure is equal to that of the surrounding medium.

   Evaluate this for the E and ρ values above, and $T = 10^4$ K.

   Comment on the physical significance of $r_{\rm eq}$. How might you use observations of supernova remnants to look for signatures of this lengthscale?

   (e) Using your expression from parts 1c and 1d, estimate the age in years of a supernova remnant of size $r_{\rm eq}$.

   Our Galaxy has ~ 3 supernova explosions per century. How many supernova remnants would you estimate our Galaxy has right now? How would you test this prediction?