Astronomy 596/496 APA

Lecture 4

Sept. 15, 2016

Today's Agenda

★ Guest Lecture & Colloquium Recap

★ Order of Magnitude

Buckingham Pi theorem

★ Colloquium Preview

No HW was due today! But there is some next week!

Looking ahead:

- Sept. 22: Special Lecture on NASA/Kelper
- Sept. 29: Guest speaker from JPL

this past Tuesday: Xuening Bai
"The microphysics of astrophysics: adventures in computational magnetohydrodynamics"

Q: What was the talk about?

Q: Key/memorable results?

Q: What did you like about the presentation?

Q: Lingering questions?

Q: Other comments?

Dimensional Analysis: The Estimator's Workhorse

physical quantities have dimensions (units)

all units can ultimately be expressed in terms of
three *fundamental dimensions (units)*

- [length] $\equiv [L]$
- [time] $\equiv [T]$, and
- [mass] $\equiv [M]$

of course, some measurable physical quantities are dimensionless
Q: example?

Profound but seemingly innocent observation I:

the behavior of a physical system is independent of the units used to describe it

Profound but seemingly innocent observation II:

in any expression (equation) describing a physical system each term must have the same units

i.e., physical equations must be dimensionally **homogeneous**

Dimensional Analysis Illustrated

Consider
- a Newtonian particle in a uniform gravity field g
- released from rest, then after time t
- falls some height $h \leftarrow$ *want to find this*

You know the exact result, but imagine you don't

If we have fully characterized the problem
then it should be possible to write

$$h = f(g, t) \tag{1}$$

where f is an arbitrary (for now) function

to solve the problem: specify f
- could use Newtonian mechanics, honest calculation takes work (integration), gives exact result
- but we can get far just by looking at dimensions

Q: what does dimensional homogeneity imply for $h = f(g, t)$?

what does dimensional homogeneity mean
for our relation $h = f(g, t)$?

- since $[h] = [L]$
  then we must have $[f] = [L]$
- but also: if h is measured in meters, then f must be as well
- so if we change to h' in yards, then
  $h' = \lambda h$, and in yards $f' = \lambda f$,
  where both expressions have the *same* conversion rescaling λ

so we have: $h = f(g, t)$ dimensionally homogeneous
rewrite: $h/f(g, t) = const = 1$
$\Rightarrow$ holds regardless of the units used

we see $h/f(g,t)$ forms a dimensionless constant
but our variables have:

- $[g] = [LT^{-2}]$
- $[t] = [T]$

given these dimensions, *only one grouping*
of variables h, t, and g is dimensionless

Q: find this grouping!
Q: use this to find the most general form of $f(g,t)$!

we have $[f(g,t)] = [L]$
but the only way to form a length from g and t
is the unique combination: $\boxed{gt^2}$

so the most general dimensionally legal expression is

$$f(g,t) = Cgt^2 \tag{2}$$

with C a dimensionless *constant*
Q: what's wrong with $Cgt^2+\Lambda$, *or* $C(gt^2)^2/\Lambda$, *with* Λ *a constant?*

and thus our dimensionless ratio can only be

$$\frac{h}{f(g,t)} = \frac{1}{C}\frac{h}{gt^2} = const = 1 \tag{3}$$

and so we can now solve

$$\boxed{h = Cgt^2} \tag{4}$$

Without calculus, but only considering dimensions, we find

$$h = Cgt^2 \tag{5}$$

with C an undetermined dimensionless constant that is independent of units used for h, g, t

Q: what does this equation teach us?
Q: what does this not *give us?*
Q: how could you test this equation without knowing C*?*
Q: if you didn't know C*, what's a reasonable order-of-magnitude guess?*
Q: how could you find C *if you didn't know calculus?*
Q: what is the actual value of C*?*

Dimensional Analysis: Lessons

what has

$$h = Cgt^2 \tag{6}$$

done for us?

- *scaling* relations $h \propto g$ and $h \propto t^2$
- don't know C: constant, so "invisible" to dim. analysis
- can test $h \propto t^2$ without knowing C
  measure fall time for different h, see if quadratic
- if you had to guess, would try $C \sim 1$
- without calculus, could get this *experimentally*:
  measure h vs t, find $C = h/gt^2$
- of course, freshman physics says $C = 1/2$
  order-of-magnitude guess off by factor 2: not bad!

Dimensional Analysis: T-Shirt Version

$\boxed{\text{What else could it be?}}$

E.g.: the only length arising from g and t is gt^2
so we must have $h \sim gt^2$: what else could it be?

Lessons:

- gather *all relevant* variables
- find dimensionless grouping(s)
- use to solve for the result of interest
- shortcut: find combinations of variables with dimensions of the answer you want

But: what if variables allow
> 1 independent dimensionless grouping?
i.e., more than one possible dimensionful answer?
...wait till next time!

Colloquium Preview

Next week, Sept. 23

- Robert Scherrer, Vanderbilt
- "Parameterizing Dark Energy"

Q: what is dark energy? why has it been proposed?

Q: how is dark energy similar to or different from dark matter?

Q: What is the simplest form of dark energy?

Q: What is w*? Why would it be a Big Deal if* $w = -0.95 \pm 0.01$*?*

Q: What if there isn't dark energy?