Astronomy 596/496 APA

Lecture 5

Sept. 22, 2016

Today's Agenda

⋆ Order of Magnitude

Buckingham Pi theorem

HW recap

Next week: Skype conversation

Stephen Licata, U of I Alumnus and NASA/JPL Scientist

Homework Recap: Gravitational Timescale

recall: given density ρ, and Newton's G
we can form a unique timescale: $\tau = 1/\sqrt{G\rho}$

Q: value when $\rho = \rho_\oplus$? implications?

Q: value for $\rho_0 = \rho_{\text{universe}}$? implications?

Q: implications for collapse of cloud with outwardly decreasing density?

Restated: given ρ and τ, plus G
there is a single unique **dimensionless combination**

$$\theta = G\rho\tau^2 \tag{1}$$

Homework Implications, Continued

Compare:

- Earth gravitational collaps timescale
- treetop satellite orbit period
- period for object dropped into center-Earth borehole

Compare:

- Universe gravitational collapse timescale
- Universe expansion rate

Buckingham Pi Theorem

Simpleminded Example Becomes More Complex

last time:
- uniform (Newtonian) gravity field g
- particle released *from rest*
- falls height h in time t

Q: unique dimensionless grouping?

now:
- particle released with initial speed v_0
  $\rightarrow$ new dimensionful parameter
  Q: any new fundamental units?

Notice: two constants g and v_0
- can construct characteristic fundamental scale(s)
  Q: namely? physical significance?
- dimensionless group(s) *Q: namely?*
  we want $h = f(t, g, v_0)$: *Q: how to proceed?*

Multiple Dimensionless Groups: Physics to the Rescue

New characteristic scales:

- time $t_{\text{acc}} = v_0/g$: timescale for acceleration to v_0
- length $h_{\text{acc}} = v_0 t_{\text{acc}} = v_0^2/g$: lengthscale for acceleration to v_0

Before: found dimensionless group $\theta_1 = gt^2/h$

physics only depends on this: some $F(\theta_1) = const$

in fact, good results with $F(\theta) = \theta$

Now: one new group possible, e.g., $\theta_2 = v_0 t/h$

must find $F(\theta_1, \theta_2) = const$

$\rightarrow$ need physical insight to proceed

use characteristic scales:

9 *Q: expected behavior for $t/t_{\text{acc}} \ll 1$? $t/t_{\text{acc}} \gg 1$?*

Q: implications for F? e.g., why not $F \sim \theta_1\theta_2$?

Short timescales $t/t_{\rm acc} \ll 1$:

- acceleration hasn't changed speed much
- problem reduces to h, t, v_0
  $\Rightarrow$ $F = F(\theta_2)$ only $\rightarrow$ $h \sim v_0 t$

Long timescales: $t/t_{\rm acc} \gg 1$:

- acceleration overwhelms v_0
  problem reduces to h, t, g
  $\Rightarrow$ $F = F(\theta_1)$ only $\rightarrow$ $h \sim g t^2$

Full $F(\theta_1, \theta_2)$ must include these limits
$F \sim \theta_1 \theta_2 \rightarrow 0$ if $g = 0$ or $v_0 = 0$! illegal!
$\Rightarrow$ simplest legal choice: linear combo $F \sim \theta_1 + \theta_2$
$h \sim v_0 t + g t^2$!

Counting Dimensionless Groups

The Buckingham Pi Theorem (Buckingham 1914, 1915)
In your problem identify:

- the number of physical **variables**
- the number of independent fundamental **units** $\in [M, L, T]$

Theorem:

$$\#\text{ of dimensionless groups} = \#\text{ of variables} - \#\text{ of units}$$

Example:

- h, t, g problem: variables = 3, units = 2
  $\rightarrow$ groups = 1 *check!*
- h, t, g, v_0 problem: variables = 4, units = 2
  $\rightarrow$ groups = 2 *check!*

Q: What if you get # groups $\leq$ 0?