Astro 501: Radiative Processes
Lecture 16
October 3, 2018

Announcements:

- **Problem Set 5** due Friday
- Problem 2 (a), (b), and (c): no calculations needed
- Problem 3 (c): assume grain composition is silicate, SiO_2 with mean atom weight $A = \langle m \rangle / m_p = 60/3 = 20$ protons

Last time: Thomson scattering

Q: what's that?

Q: what does the scattered power depend on?

Q: what does $d\sigma/d\Omega$ depend on? and not?

Q: what does σ depend on? and not?

Q: lessons?

Thomson Scattering: Electron Dipole Radiation

- Thomson = **scattering by non-relativistic free electrons**
- no change in photon λ, ν: **coherent scattering**

- electron acts as **dipole antenna**

$$\frac{dP}{d\Omega} = \frac{d\sigma}{d\Omega} \; \langle S_{\rm in} \rangle$$

i.e., scattered power $\propto$ incident flux

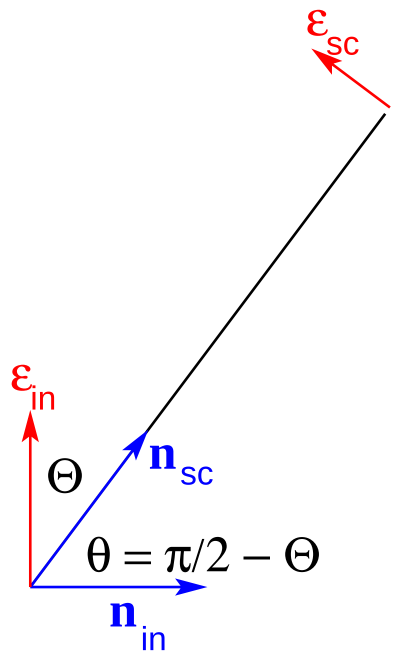
proportionality is **Thomson cross section**

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{e^4}{m_e^2 c^4} \sin^2 \Theta = \frac{3}{2\pi} \sigma_T \cos^2 \theta \\ \sigma_{\rm T} &= \frac{8\pi}{3} \frac{e^4}{m_e^2 c^4} = \frac{8\pi}{3} r_0^2 \end{aligned}$$

maximum at $\hat{\epsilon}_{\rm in} \cdot \hat{n}_{\rm sc} = \cos\Theta = 0$

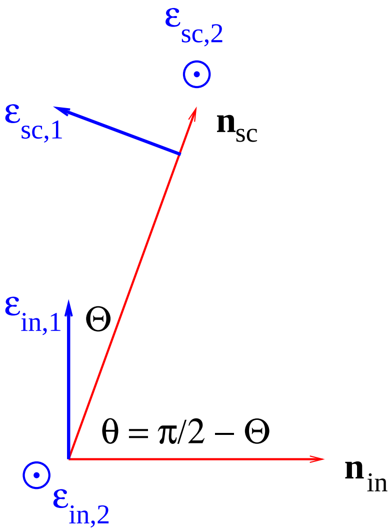
which is also $\hat{n}_{\rm in} \cdot \hat{n}_{\rm sc} = \cos\theta = 1$, with $\theta = \pi/2 - \Theta$

→ *forward and backward scattering*



Thomson Scattering of Unpolarized Radiation

Using result for linear polarization
we can construct result for unpolarized radiation
by *averaging results for two orthogonal linear polarizations*

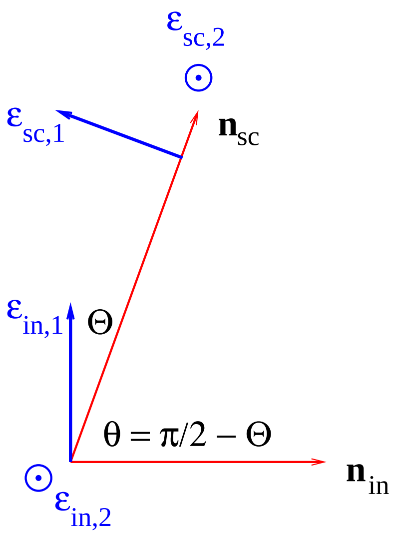


Geometry:
$\hat{n}_{\mathrm{in}}$ direction of incident radiation
$\hat{n}_{\mathrm{sc}}$ direction of scattered radiation
initial polarizations are both $\perp \hat{n}_{\mathrm{in}}$
choose one polarization $\hat{\epsilon}_{\mathrm{in},1}$ in $\hat{n}_{\mathrm{in}} - \hat{n}_{\mathrm{sc}}$ plane
and the other $\hat{\epsilon}_{\mathrm{in},2}$ orthogonal
  to this plane and to $\hat{n}_{\mathrm{in}}$

ω scattering angle of pol 1 has $\cos\Theta_1 = \hat{\epsilon}_1 \cdot \hat{n}_{\mathrm{sc}}$ *Q: which means?*
scattering angle of pol 2 has $\cos\Theta_2 = \hat{\epsilon}_2 \cdot \hat{n}_{\mathrm{sc}}$ *Q: which means?*

thus scatter polarization 1 by angle $\Theta = \pi/2 - \theta$
and polarization 2 by angle $\pi/2$, and so

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_1 + \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_2$$
$$= \frac{r_0^2}{2}\left(\sin^2\Theta + 1\right)$$
$$= \frac{r_0^2}{2}\left(1 + \cos^2\theta\right)$$



which only depends on angle θ
between incident $\hat{n}_{\text{in}}$ and scattered $\hat{n}_{\text{sc}}$ radiation directions

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{unpol}} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \tag{1}$$

- **forward-backward symmetry**: $\theta \to -\theta$ invariance

- angular pattern: $\cos^2\theta \propto \cos 2\theta$ term

→ scattered radiation has 180^0 periodicity

→ 4 extrema = "poles": **quadrupole** pattern!

- total cross section $\sigma_{\mathrm{unpol}} = \sigma_{\mathrm{pol}} = \sigma_{\mathrm{T}}$

→ electron at rest has no preferred direction

- Polarization degree of scattered radiation

$$\Pi = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \tag{2}$$

Q: what does this mean?

Thomson Scattering Creates Polarization

Thomson scattering of *initially unpolarized* radiation has

$$\Pi = \frac{1 - \cos^2\theta}{1 + \cos^2\theta} \tag{3}$$

i.e., degree of polarization $\Pi \neq 0$!

Thomson-scattered radiation is linearly polarized!

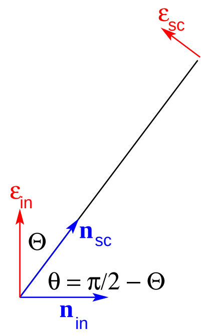
quadrupole pattern in angle θ between $\hat{n}_{\rm init}$ and $\hat{n}_{\rm scattered}$

- 100% polarized at $\theta = \pi/2$
- 0% polarized at $\theta = 0, \pi$

classical picture: e^- as dipole antenna

incident linearly polarized wave accelerates e^-

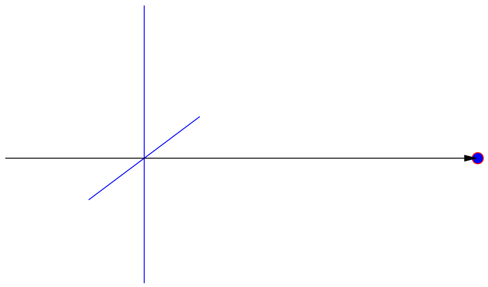
→ $\sin^2\theta$ pattern, peaks at $\theta = 0, \pi$



Thompson Scattering: A Gut Feeling

Discussion swiped from Wayne Hu's website

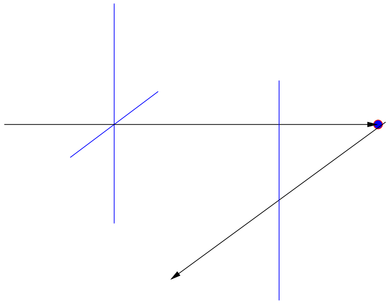
Consider a beam of unpolarized radiation
propagating in plane of sky, incident on an electron
think of as superposition of linear polarizations
one along sightline, one in sky



Q: why is scattered radiation polarized? in which direction?

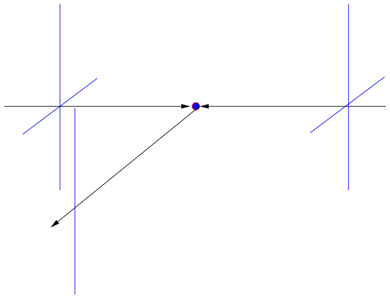
Q: now what if unpolarized beams from opposite directions?

scattering of one unpolarized beam:



→ see radiation from e motion in sky plane
→ linear polarization!

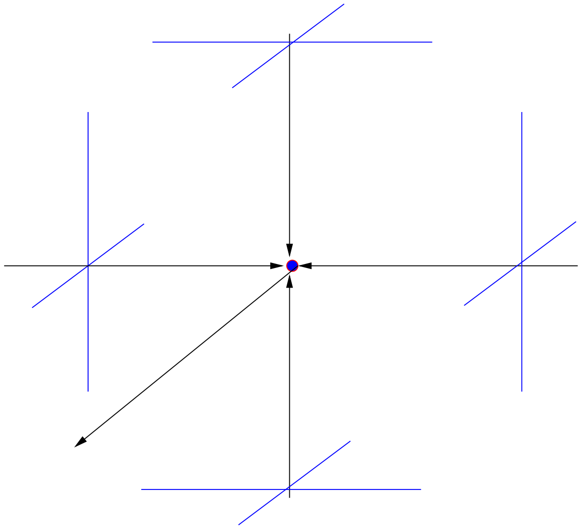
scattering of two unpolarized beams in opposite directions:



→ the other side only adds to e motion in sky plane
→ also linear polarization!

Q: what if isotropic initial radiation field?

isotropic initial radiation field:

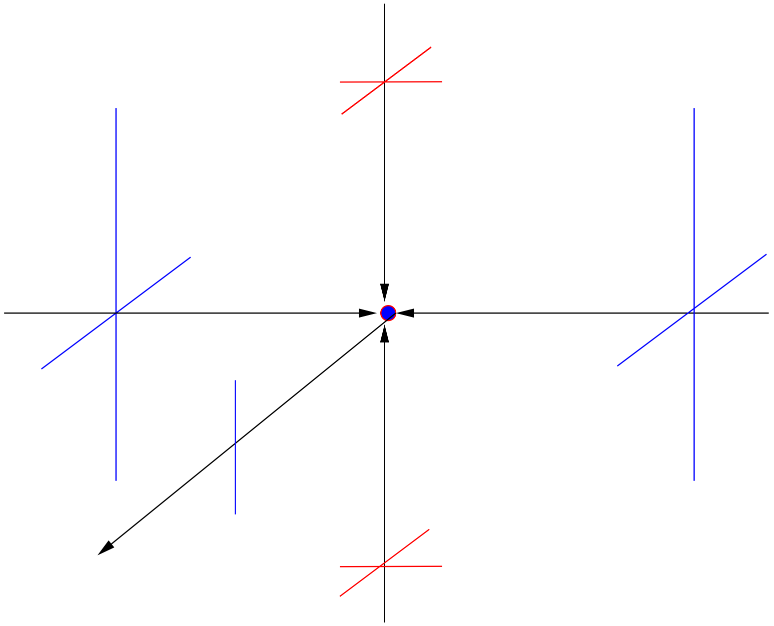


e motions in x and y sky directions cancel
$\longrightarrow$ no net polarization

Q: what incident radiation fields do create polarization?

Q: lesson?

if initial radiation field has quadrupole intensity pattern



linear polarization!

lesson: polarization arises from Thomson scattering
when electrons “see” quadrupole anisotropies in radiation field

Q: If Thomson scattering is the only process acting
what is the appropriate transfer equation?

Thomson Scattering in Radiation Transfer

recall: in *coherent scattering*

- photon number and energy preserved
- but directions changed

$$\frac{dI_\nu(\hat{n})}{ds} = -n_e \sigma_{\mathrm{T}} \; [I_\nu(\hat{n}) - S_\nu(\hat{n})]$$

for scattering of unpolarized radiation, source is not isotropic!

$$S_\nu(\hat{n}) = \frac{1}{\sigma_T} \int I_\nu(\hat{n}') \, \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \, \frac{d\Omega'}{4\pi} = \frac{3}{16\pi} \int I_\nu(\hat{n}') \, \left[1 + (\hat{n} \cdot \hat{n}')^2\right] \, d\Omega'$$

where the *redistribution function*

$$\mathcal{R}(\hat{n}, \hat{n'}) = \frac{1}{4\pi\sigma_{\mathrm{tot}}} \frac{d\sigma}{d\Omega}(\hat{n}, \hat{n}') \overset{\mathrm{Thom}}{=} \frac{3}{16\pi} \left[1 + (\hat{n} \cdot \hat{n}')^2\right]$$

encodes the scattering directionality

Q: what if scattering is isotropic?

if we *approximate Thomson as isotropic*, then

$$\frac{d\sigma}{d\Omega} \xrightarrow{\text{iso}} \sigma_{\mathsf{T}}/4\pi$$

and we recover our old result

$$S_\nu \xrightarrow{\text{iso}} J_\nu = \frac{1}{4\pi}\int I_\nu \; d\Omega \tag{4}$$

for which the redistribution function is just

$$\mathcal{R}(\hat{n}, \widehat{n'}) = \frac{1}{4\pi} \tag{5}$$

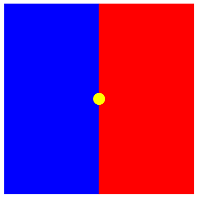
Awesomest Example of Thompson Polarization: the CMB

The CMB is nearly isotropic radiation field
arises from $\tau_S \sim 1$ "surface of last scattering" at $z \approx 1000$
when free e and protons "re" combined $e + p \to H + \gamma$

- *before recombination:*
  Thomson scattering of CMB photons, Universe opaque
- after recombination: no free e, Universe transparent

the CMB is the cosmic photosphere!

electrons during last scattering see anisotropic radiation field

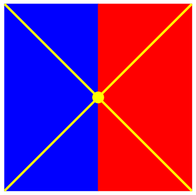


consider point at hot/cold "wall"
locally sees *dipole* T anisotropy
net polarization towards us: zero! *Q: why?*
Q: what about edge of circular hot spot? cold spot?

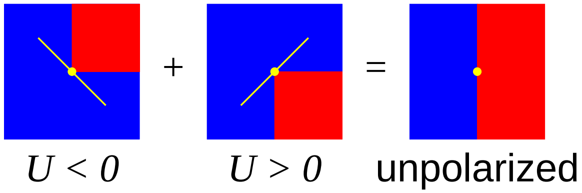
at wall: see local dipole

hot side horizontal and vertical contributions are equal!

→ no net polarization!



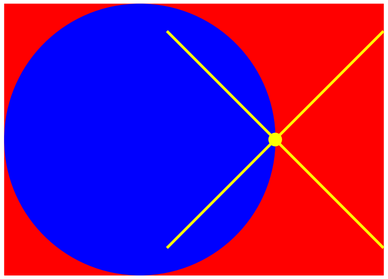
also follows from this superposition



polarization tangential (ring) around hot spots

radial (spokes) around cold spots

(superpose to “+” = zero net pol)



www: WMAP polarization observations of hot and cold spots

Note: polarization & T anisotropies *linked*

→ consistency test for CMB theory and hence hot big bang

Polarization Observed

First detection: pre-WMAP!
★ DASI (2002) ground-based interferometer
at level predicted based on T anisotropies! Woo hoo!

WMAP (2003): first polarization-T correlation function

Planck (March 2013): much more sensitive to polarization

Build Your Toolbox: Thomson Scattering

microphysics: matter-radiation interactions
Q: physical origin of Thomson scattering?
Q: physical nature of sources?
Q: spectrum characteristics?
Q: frequency range?

real/expected astrophysical sources of Thomson scattering
Q: where do we expect this to be important?
Q: relevant EM bands? temperatures?

Toolbox: Thomson Scattering

emission physics

- physical origin: scattering by *non-relativistic free electrons*
- physical sources: need free e^- → ionized gas
  scattering → *photons conserved, need incident radiation*
  scattering induces *polarization* even for unpolarized sources
- spectrum: Thomson coherent *scattered energy unchanged*
  σ_T indept of ν: *spectral shape preserved in scattered radiation*

astrophysical sources of Thomson scattering

- sites are illuminated and highly ionized gas: stellar interiors, stellar coronae, hot nebulae (HII regions), early Universe
- EM bands *radio to X-ray*
  for γ-rays relativistic effects are important → Compton
- temperatures *up to* $\sim 10^6$ K
  above this, relativistic effects are important → Compton