Astro 501: Radiative Processes
Lecture 29
November 12, 2018

Announcements:

- **Problem Set 9** due Friday
- **Iben Lectures: Prof. Wendy Freedman**
  Colloquium tomorrow 3:45pm, NCSA
  Public Lecture Wed 7pm, Lincoln Hall

Last time: 21 cm radiation and astrophysics

Q: Why does the line arise? Why is it special?

Q: what information does it encode?

Q: what information does it omit?

hyperfine transition in **neutral hydrogen atoms:** H i

$u \rightarrow \ell$ is an electron *spin flip* $\uparrow_p \uparrow_e \rightarrow \uparrow_p \downarrow_e$ with

$$A_{u\ell} = 2.8843 \times 10^{-15}\ \mathrm{s}^{-1} = (11.0\ \mathrm{Myr})^{-1} \tag{1}$$

spontaneous emission not measured in lab

but *stimulated emission* exquisitely well-measured in resonating cavities: hydrogen masers

$$\nu_{u\ell} = 1420.405751768(1)\ \mathrm{MHZ} \qquad \lambda_{u\ell} = 21.10611405413\ \mathrm{cm}$$

$\Delta E/k_{\mathrm{B}} = 0.06816\ \mathrm{K} \ll T_{\mathrm{cmb,0}}$ $\rightarrow$ CMB can populate upper level!

$$j_\nu = n_u \frac{A_{u\ell}}{4\pi} h\nu_{u\ell}\ \phi_\nu \approx \frac{3}{16\pi} A_{u\ell}\ h\nu_{u\ell}\ n(\mathrm{H\ I})\ \phi_\nu \tag{2}$$

- *intensity* I_ν measures *atomic hydrogen column* $N(\mathrm{H\ I})$
- *flux/intensity map* $\int I_\nu\ d\Omega$ measures *atomic hydrogen mass*
- *line profile* ϕ_ν usually measures *velocity distribution*

Awesome Example: Cosmic 21 cm Radiation

imagine a Universe which:

- is expanding → photons redshift
- is transparent after recombination $z_{\text{rec}} \sim 1000$
  so CMB is nearly isotropic "backlighting"
  to all lines of sight
- most baryons are hydrogen of some ilk
  some fraction of which will be atomic
- initial conditions are homogenous, then structure forms

consider **thermal atomic H gas** at **redshift** $z < z_{\text{rec}}$

Q: effect if $T_{\text{gas}}(z) = T_{\text{cmb}}(z)$*?*
Q: what if $T_{\text{gas}}(z) \neq T_{\text{cmb}}(z)$*?*
Q: what determines which of these is correct?
Q: resulting 21cm spectrum in each sightline? what does it tell?
Q: resulting 21cm pattern across the sky? what does it tell us?

Awesome Example: Cosmic 21 cm Radiation

radiation transfer along each sightline, at rest-frame 21cm:

$$I = B_\nu(T_{\rm cmb})\ e^{-\tau_\nu} + B_\nu(T_{\rm gas})\ (1 - e^{-\tau_\nu}) \tag{3}$$

in terms of antenna temperature

$$T_b = T_{\rm cmb}\ e^{-\tau_\nu} + T_{\rm gas}(1 - e^{-\tau_\nu}) \tag{4}$$

so: if $T_{\rm gas} = T_{\rm cmb}$ - no signal!

otherwise: emission or absorpbtion
depending on $T_{\rm gas} - T_{\rm cmb}$ sign

$$T_b = T_{\mathrm{cmb}}\ e^{-\tau_\nu} + T_{\mathrm{gas}}(1 - e^{-\tau_\nu}) \tag{5}$$

after gas decoupling, before reionization $z_{\mathrm{reion}} \sim 10 \lesssim z \leq z_{\mathrm{dec}}$
before the first stars and quasars: **cosmic dark ages**
first structure forming, but not yet "lit up"

during dark ages: intergalactic gas has $T_{\mathrm{gas}} < T_{\mathrm{cmb}}$

$$\delta T_b \equiv T_b - T_{\mathrm{cmb}} = (T_{\mathrm{gas}} - T_{\mathrm{cmb}})_z\,(1 - e^{-\tau_\nu})_z \tag{6}$$

we have $\delta T_b < 0$: gas seen in 21 cm *absorption*

Q: what cosmic matter will be seen this way?
Q: what will its structure be in 3-D?
Q: how will this structure be encoded in δT_b?

The "21 cm Forest"

what will absorb at 21 cm?
any neutral hydrogen in the universe!
but after recomb., most H is neutral, and most baryons are H
so absorbers are *most of the baryons in the universe*

thus absorber spatial distribution is *3D distribution of baryons*
i.e., intergalactic baryons as well as seeds of galaxies and stars!
baryons fall into potentials of dark matter halos, form galaxies
so *cosmic 21 cm traces formation of structure and galaxies!*

gas at redshift z absorbs at $\lambda(z) = (1+z)\lambda_{\ell u}$
and o responsible for decrement $\delta T_b[\lambda(z)]$
$\rightarrow$ thus $\delta T_b(\lambda)$ *encodes redshift history* of absorbers
a sort of "21 cm forest"

Q: what about sky pattern of $\delta T_b(\lambda)$ at fixed λ?

and at fixed λ, sky map of $\delta T_b(\lambda)$
gives baryon distribution in "shell" at $1 + z = \lambda/\lambda_0$
$\rightarrow$ a radial "slice" of the baryonic Universe!

so by scanning through λ, and at each
making sky maps of $\delta T_b(\lambda)$
$\rightarrow$ we build in "slices" a *3-D map of cosmic structure evolution!*
"cosmic tomography"! a cosmological gold mine!
encodes huge amounts of information

sounds amazing! and it is! but there is a catch!

Q: why is this measurement very difficult to do?
Hint: it hasn't yet been done

21 cm Cosmology: The Challenge

The 21 cm cosmic goldmine lies at redshifts $z \sim 6$ to 150 corresponding to:

- $\lambda_{\text{obs}} \sim 1.5 - 30$ m
  enormous wavelengths! `www: LOFAR`
- $\nu_{\text{obs}} \sim 200 - 10$ MHz
  but ionosphere opaque $> \nu_{\text{plasma}} \sim 20$ MHz
  for highest z (most interesting!) have to go to space! in fact, have to go to far side of the Moon *Q: why?*
  `www: proposed lunar observatories`

But wait! It's worse!
at these wavelengths, dominant emission is *Galactic synchrotron* with brightness $T_{\text{B,synch}} \sim 200 - 2000 \text{ K} \gg T_{\text{cmb}} \gg T_{\text{B,21 cm}}$

∞ `www: radio continuum sky`

Q: implications? how to get around this?

sky intensity $T_{\mathrm{B,synch}} \sim 200 - 2000\ \mathrm{K} \gg T_{\mathrm{cmb}}$
$\longrightarrow$ Galactic synchrotron foreground dominates cosmic 21 cm
curse you, cosmic rays!

But there remains hope!
recall: cosmic-ray electron energy spectrum is a power law
so their *synchrotron spectrum is a power law*
i.e., $I_{\nu,\mathrm{synch}}$ is *smooth function of* ν

compare 21 cm at high-z: a "forest" of absorption lines
not smooth! full of spectral *lines & features*
$\longrightarrow$ can hope to measure with very good spectral coverage
and foreground subtraction

also: can use spatial (i.e., angular) distribution
e.g., consider effect of first stars (likely massive) *Q: namely?*

first stars: likely massive $\rightarrow$ hot $\rightarrow$ large UV sources
ionizing photons carve out "bubble" neutral H
$\rightarrow$ corresponding to a *void* in 21 cm
$\rightarrow$ sharp bubble edges may be detectable
$\rightarrow$ 21 cm can probe *epoch of reionization*

hot, ongoing research area!

stay tuned!

Nebular Diagnostics

Collisional Excitation

so far we have considered atomic line transitions
due to emission or absorption of radiation
but atom *collisions* can also drive transitions

⋆ collisions can place atoms in excited states
de-excite radiatively (line emission) → cooling source

⋆ collisions populate atomic levels
observing line emission can diagnose density, temperature, radiation field

key physical input: *collision rates*
consider inelastic collisions $a + c \rightarrow a' + c'$
of species a with "collision partner" c
Q: what is collision rate per volume? per a *atom?*

for collisions $a + c$, collision rate per volume is

$$\frac{d\mathcal{N}_{\text{collisions}}}{dV\ dt} \equiv \dot{n}_{ac\rightarrow a'c'} = \langle \sigma_{ac} v \rangle\ \ n_a\ n_c \tag{7}$$

where **collision rate coefficient** $\langle \sigma_{ac} v \rangle$
averages over collision *cross section* σ_{ac}
and relative velocity v between a and c

Q: order-of-magnitude estimate for σ_{ac}?
Q: what sets typical v?

collision rate *per a* is

$$\Gamma_{ac\rightarrow a'c'} = \frac{\dot{n}_{ac\rightarrow a'c'}}{n_a} = \langle \sigma_{ac} v \rangle\ \ n_c \tag{8}$$

Two-Level Atom: No Radiation

instructive simple case: a *two-level atom*
denote *ground state* 0, *excited state* 1
with atomic number densities n_0 and n_1

consider effect of collisions with partner c
when radiation effects are unimportant:

$$\dot{n}_1 = -\Gamma_{10} n_1 + \Gamma_{01} n_0 = -\langle \sigma_{10} v \rangle n_c n_1 + \langle \sigma_{01} v \rangle n_c n_0 \qquad (9)$$

Q: what is n_1/n_0 *ratio in equilibrium* ($\dot{n}_1 = 0$)*?*
Q: what does this imply?

without radiation, in *equilibrium*:

$$\dot{n}_1 == -\langle\sigma_{10}v\rangle\, n_c n_1 + \langle\sigma_{01}v\rangle\, n_c n_0 = 0 \tag{10}$$

which gives $(n_1/n_0)_{\rm eq} = \langle\sigma_{01}v\rangle \,/\, \langle\sigma_{10}v\rangle$

but in thermal equilibrium $(n_1/n_0)_{\rm eq} = (g_1/g_0)\ e^{-E_{10}/kT}$

so we have the detailed balance result

$$\langle\sigma_{10}v\rangle = \frac{g_1}{g_0}\ e^{-E_{10}/kT}\ \ \langle\sigma_{01}v\rangle \tag{11}$$

i.e., excitation is suppressed by Boltzmann factor $e^{-E_{10}/kT}$

Q: how do we add radiation effects?

Two-Level Atom with Radiation

if atoms in excited states exist, they can spontaneously emit
$\longrightarrow$ radiation must be present

volume rate of: *spontaneous emission* is $A_{10}n_1$

volume rate of: *stimulated emission*

$$B_{10}J_\nu n_1 = A_{10}\frac{c^2 J_\nu}{2h\nu^3}n_1 \equiv A_{10}\ f_\nu\ n_1 \tag{12}$$

where for isotropic radiation $J_\nu = 2\ \nu^3/c^2\ f_\nu$, with
f_ν the *photon distribution function* or occupation number

volume rate of: *true absorption*

$$B_{01}J_\nu n_0 = \equiv \frac{g_1}{g_0}A_{10}\ f_\nu\ n_1 \tag{13}$$

putting it all together, the two-level atom in the presence of collisions and radiation has

$$\dot{n}_1 = \left[\langle \sigma_{01} v \rangle \, n_c + f_\nu \frac{g_1}{g_0} A_{10} \right] n_0 - \left[\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10} \right] n_1 \tag{14}$$

this will seek an equilibrium or *steady state* $\dot{n}_1 = 0$ giving the ratio

$$\left(\frac{n_1}{n_0} \right)_{\rm eq} = \frac{\langle \sigma_{01} v \rangle \, n_c + (g_1/g_0) f_\nu A_{10}}{\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10}} \tag{15}$$

consider the limits of low- and high-density collision partners

$$\left(\frac{n_1}{n_0} \right)_{\rm eq} \rightarrow \left\{ \begin{array}{ll} (g_1/g_0) \; f_\nu/(1 + f_\nu) \; , & n_c \rightarrow 0 \\ \langle \sigma_{01} v \rangle \, / \, \langle \sigma_{10} v \rangle & n_c \rightarrow \infty \end{array} \right. \tag{16}$$

Q: implications of limits if $T_{\rm rad} \neq T_{\rm gas}$?

in steady state:

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} = \frac{\langle \sigma_{01} v \rangle\, n_c + f_\nu (g_1/g_0) A_{10}}{\langle \sigma_{10} v \rangle\, n_c + (1+f_\nu) A_{10}} \tag{17}$$

at *low density* of collision partners: $n_c \to 0$ and thus

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{g_1}{g_0} \frac{f_\nu}{1+f_\nu} \stackrel{\rm therm}{=} \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm rad}} \tag{18}$$

→ *level population set by radiation temperature* $T_{\rm rad}$

at *high density* of collision partners: $n_c \to \infty$, and

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{\langle \sigma_{01} v \rangle}{\langle \sigma_{10} v \rangle} = \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}} \tag{19}$$

→ *level population set by gas temperature* $T_{\rm gas}$

Q: characteristic density scale?

Critical Density

for each collision partner c, excited state de-excitation by emission and by collisions are equal when

$$\langle \sigma_{10} v \rangle \, n_c = (1 + f_\nu) A_{10} \tag{20}$$

this defines a **critical density**

$$n_{c,\mathrm{crit}} = \frac{(1 + f_\nu) A_{10}}{\langle \sigma_{10} v \rangle} \tag{21}$$

- if $f_\nu \ll 1$: stimulated emission not important (PS11)
  $n_{c,\mathrm{crit}} \rightarrow A_{10} / \langle \sigma_{10} v \rangle$ depends only on T and atomic properties
- but if f_ν not small, critical density depends on local radiation field

so when partner density $n_c \gg n_{c,\mathrm{crit}}$:
  state population set by $T \rightarrow T_{\mathrm{gas}}$
and when$n_c \ll n_{c,\mathrm{crit}}$:
  state population set by $T \rightarrow T_{\mathrm{rad}}$

Director’s Cut Extras

Awesome Example: Cosmic 21 cm Radiation

CMB today, redshift $z = 0$, has $T_{\rm cmb}(0) = 2.725\ \mathrm{K} \gg \mathrm{T}_{\rm ex,21\,cm}$
but what happens over cosmic time?

fun & fundamental cosmological result:
(relativistic) *momentum redshifts*: $p \propto 1/a(t)$, which means

$$p(z) = (1+z)\ p(0) \tag{22}$$

where $p(0)$ is observed momentum today ($z = 0$)

why? photon or de Broglie wavelength λ is a *length*, so

$$\lambda(t) = a(t)\ \lambda_{\rm emit} = \frac{\lambda_0}{(1+z)} \tag{23}$$

and quantum relation $p = h/\lambda$ implies $p \propto (1+z)$

Q: implications for gas vs radiation after recombination?

Thermal History of Cosmic Gas and Radiation

until recombination (CMB formation) $z \geq z_{\rm rec} \sim 1000$
(mostly) hydrogen gas is ionized, tightly coupled to CMB
via Thomson scattering: $T_{\rm cmb} = T_{\rm gas}$

after recombination, before gas decoupling $z_{\rm dec} \sim 150 \lesssim z \leq z_{\rm rec}$

- most gas in the Universe is *neutral*
  but a small "residual" fraction $x_e \sim 10^{-5}$ of e^- remain ionized

- Thompson scattering off residual free e^- $(x_e \sim 10^{-5})$
  still couples gas to CMB $\rightarrow$ $T_{\rm cmb} = T_{\rm gas}$ maintained
- until about $z_{\rm dec} \sim 150$, when Thomson scattering ineffective,
gas *decoupled*

Q: after decoupling, net effect of 21 cm transition?