Astro 501: Radiative Processes
Lecture 30
November 14, 2018

Announcements:

- **Problem Set 9** due Friday

Last time: began collisional excitation
for two-body inelastic collisions $ac \to a'c'$:
Q: collision rate per volume? per a atom?

for collisions $a + c$, collision rate per volume is

$$\frac{d\mathcal{N}_{\text{collisions}}}{dV \ dt} \equiv \dot{n}_{ac\to a'c'} = \langle \sigma_{ac} v \rangle \ n_a \ n_c \tag{1}$$

where **collision rate coefficient** $\langle \sigma_{ac} v \rangle$
averages over collision *cross section* σ_{ac}
and relative velocity v between a and c

Q: order-of-magnitude estimate for σ_{ac}?
Q: what sets typical v?

collision rate *per a* is

$$\Gamma_{ac\to a'c'} = \frac{\dot{n}_{ac\to a'c'}}{n_a} = \langle \sigma_{ac} v \rangle \ n_c \tag{2}$$

Two-Level Atom: No Radiation

instructive simple case: a *two-level atom*
denote *ground state* 0, *excited state* 1
with atomic number densities n_0 and n_1

consider effect of collisions with partner c
when radiation effects are unimportant:

$$\dot{n}_1 = -\Gamma_{10} n_1 + \Gamma_{01} n_0 = -\langle \sigma_{10} v \rangle n_c n_1 + \langle \sigma_{01} v \rangle n_c n_0 \qquad (3)$$

Q: what is n_1/n_0 ratio in equilibrium ($\dot{n}_1 = 0$)?
Q: what does this imply?

without radiation, in *equilibrium*:

$$\dot{n}_1 = -\langle \sigma_{10} v \rangle \, n_c n_1 + \langle \sigma_{01} v \rangle \, n_c n_0 = 0 \tag{4}$$

which gives $(n_1/n_0)_{\rm eq} = \langle \sigma_{01} v \rangle / \langle \sigma_{10} v \rangle$

but in thermal equilibrium $(n_1/n_0)_{\rm eq} = (g_1/g_0) \; e^{-E_{10}/kT}$

so we have the detailed balance result

$$\langle \sigma_{10} v \rangle = \frac{g_1}{g_0} \; e^{-E_{10}/kT} \; \langle \sigma_{01} v \rangle \tag{5}$$

i.e., excitation is suppressed by Boltzmann factor $e^{-E_{10}/kT}$

Q: how do we add radiation effects?

Two-Level Atom with Radiation

if atoms in excited states exist, they can spontaneously emit
$\longrightarrow$ radiation must be present

volume rate of: *spontaneous emission* is $A_{10} n_1$

volume rate of: *stimulated emission*

$$B_{10} J_\nu n_1 = A_{10} \frac{c^2 J_\nu}{2 h \nu^3} n_1 \equiv A_{10} \ f_\nu \ n_1 \tag{6}$$

where for isotropic radiation $J_\nu = 2 \ \nu^3 / c^2 \ f_\nu$, with
f_ν the *photon distribution function* or occupation number

volume rate of: *true absorption*

$$B_{01} J_\nu n_0 \equiv \frac{g_1}{g_0} A_{10} \ f_\nu \ n_1 \tag{7}$$

putting it all together, the two-level atom
in the presence of collisions and radiation has

$$\dot{n}_1 = \left[\langle \sigma_{01} v \rangle \, n_c + f_\nu \frac{g_1}{g_0} A_{10} \right] n_0 - \left[\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10} \right] n_1 \tag{8}$$

this will seek an equilibrium or *steady state* $\dot{n}_1 = 0$
giving the ratio

$$\left(\frac{n_1}{n_0} \right)_{\rm eq} = \frac{\langle \sigma_{01} v \rangle \, n_c + (g_1/g_0) f_\nu A_{10}}{\langle \sigma_{10} v \rangle \, n_c + (1 + f_\nu) A_{10}} \tag{9}$$

consider the limits of low- and high-density collision partners

$$\left(\frac{n_1}{n_0} \right)_{\rm eq} \rightarrow \begin{cases} (g_1/g_0) \; f_\nu/(1 + f_\nu) \; , & n_c \rightarrow 0 \\ \langle \sigma_{01} v \rangle \, / \, \langle \sigma_{10} v \rangle & n_c \rightarrow \infty \end{cases} \tag{10}$$

Q: implications of limits if $T_{\rm rad} \neq T_{\rm gas}$?

in steady state:

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} = \frac{\langle\sigma_{01}v\rangle\, n_c + f_\nu (g_1/g_0) A_{10}}{\langle\sigma_{10}v\rangle\, n_c + (1+f_\nu) A_{10}} \tag{11}$$

at *low density* of collision partners: $n_c \to 0$ and thus

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{g_1}{g_0}\,\frac{f_\nu}{1+f_\nu} \overset{\rm therm}{=} \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm rad}} \tag{12}$$

$\to$ *level population set by radiation temperature* $T_{\rm rad}$

at *high density* of collision partners: $n_c \to \infty$, and

$$\left(\frac{n_1}{n_0}\right)_{\rm eq} \to \frac{\langle\sigma_{01}v\rangle}{\langle\sigma_{10}v\rangle} = \frac{g_1}{g_0} e^{-E_{10}/kT_{\rm gas}} \tag{13}$$

$\to$ *level population set by gas temperature* $T_{\rm gas}$

Q: characteristic density scale?

Critical Density

for each collision partner c, excited state de-excitation by emission and by collisions are equal when

$$\langle \sigma_{10} v \rangle \, n_c = (1 + f_\nu) A_{10} \tag{14}$$

this defines a **critical density**

$$n_{c,\text{crit}} = \frac{(1 + f_\nu) A_{10}}{\langle \sigma_{10} v \rangle} \tag{15}$$

- if $f_\nu \ll 1$: stimulated emission not important (PS10) $n_{c,\text{crit}} \to A_{10} / \langle \sigma_{10} v \rangle$ depends only on T and atomic properties
- but if f_ν not small, critical density depends on local radiation field

so when partner density $n_c \gg n_{c,\text{crit}}$:
  state population set by $T \to T_{\text{gas}}$

and when $n_c \ll n_{c,\text{crit}}$:
  state population set by $T \to T_{\text{rad}}$

Electron-Atom Collisions

consider inelastic collisions of *atoms* with
thermal *electrons* at T

Q: geometric cross section of electron?

Q: quantum mechanical lengthscale for non-relativistic e*?*

Q: collision cross section, reaction rate estimate for e *at* T*?*

electrons are quantum particles

with de Broglie wavelength $\lambda_{\rm deB} = h/p_e = h/m_e v$

so thermal electrons have a *thermal de Broglie wavelength*

$$\lambda_{{\rm deB},e} \sim \frac{h}{m_e v_T} = \frac{h}{\sqrt{m_e\, kT}} = 52\ \text{Å} \ \left(\frac{1000\ {\rm K}}{T}\right)^{1/2} \tag{16}$$

so for T of interest, $\lambda_{{\rm deB},e} \gg r_{\rm atom} \sim a_0$

so to order-of-magnitude, atom-electron cross section is

$$\sigma_{ae} \sim \pi \lambda_{{\rm deB},e}^2 = \pi \frac{h^2}{m_e\, kT} \tag{17}$$

and thermal collision rate coefficient is

$$\langle \sigma_{ae} v \rangle \sim v_T \ \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e kT}} \propto \frac{1}{\sqrt{T}} \tag{18}$$

to order of magnitude,

$$\langle \sigma_{ae} v \rangle \sim v_T \ \sigma_{ae} \sim \frac{h^2}{m_e \sqrt{m_e k T}} \propto \frac{1}{\sqrt{T}} \tag{19}$$

useful to define dimensionless **collision strength** $\Omega_{u\ell}$
for electron-atom transition $u \rightarrow \ell$:

$$\langle \sigma_{ae} v \rangle \equiv \frac{h^2}{(2\pi m_e)^{3/2} (kT)^{1/2}} \frac{\Omega_{u\ell}}{g_u} \tag{20}$$

in principle, $\Omega_{u\ell}(T)$ depends on T
but in practice, nearly constant with T,
and values are in range $\Omega_{u\ell} \sim 1 - 10$

Awesome Example: C^+ 158μm

singly ionized carbon: C^+ or C ii
ground state hyperfine splitting $J = 3/2, 1/2$

$$\Delta E/k = 91.21 \text{ K} \quad (21)$$
$$\lambda = 158 \ \mu\text{m} \quad (22)$$
$$A_{10} = 2.4 \times 10^{-6} \text{ s}^{-1} = (4.8 \text{ days})^{-1} \quad (23)$$

Q: waveband? appropriate telescopes?

critical densities

$$n_{\text{crit}}(\text{H}) \sim 3000 \text{ cm}^{-3} \quad (24)$$
$$n_{\text{crit}}(e^-) \sim 50 \sqrt{T/10^4 \text{K}} \text{ cm}^{-3} \quad (25)$$

consider a low density, optically thin region with C^+
Q: what are the level populations?
Q: if upper level collisionally excited, what happens?
Q: where is this likely to occur?

C^+ Hyperfine Emission as a Star-Formation Coolant

low density parts of star-forming regions

- contain C ii,
- but are below critical densities
- and optically thin: not radiatively excited

so: *upper level "subthermal" → collisions can excite*

and if collisional excitation occurs

- radiative de-excitation is the most likely
- [C ii] 158 μm photon emitted
- usually optically thin, lost from system: *observable!*
- removes energy: *coolant*

This line is a major tracer of diffuse star-forming regions!

Q: what should an all-sky map of 158 μm look like?

`www: all-sky`, `www: external galaxies`

Nebular Diagnostics

consider a *diffuse nebula*: low-density gas generally irradiated by stars

Q: expected optical spectrum?

`www: example spectra`

Q: how to use spectra to measure T? density?

Nebular Temperature Diagnostic

diffuse nebulae: usually optically thin in visible band
continuum radiation is not blackbody
and reprocesses stellar radiation with $T \sim 3000 - 30,000$ K
spectra dominated by *emission lines*
$\rightarrow$ need to use these to determine T, density

temperature diagnostics: *pairs of lines* that are

- energetically accessible: $E_{u\ell} \lesssim kT$
- widely spaced: $\Delta E \sim kT$

consider an idealized *3-level atom*

- ground state $n = 1$, excited states $n = 2, 3$
- excited states populated by *electron collisions*
  at volume rate $\dot{n}_{13} = \langle \sigma_{e1\rightarrow 3} v \rangle \; n_1 \; n_e \propto \Omega_{13} e^{-E_{13}/kT} n_1 n_e$
- probability for $3 \rightarrow 1$ transition: $A_{31}/(A_{31} + A_{32})$ *Q: why?*

if electron density $n_e \ll n_{e,\rm crit}$
then de-excitation occurs via spontaneous emission
and integrated emissivity from the $3 \rightarrow 1$ transition is

$$j_{31} = E_{31}\dot{n}_{13}\frac{A_{31}}{A_{31}+A_{32}} = E_{31}\left\langle\sigma_{31}v\right\rangle\frac{A_{31}}{A_{31}+A_{32}}n_1 n_e \qquad (26)$$

and from the $3 \rightarrow 1$ transition is

$$j_{21} = E_{21}\left(\left\langle\sigma_{12}v\right\rangle + \left\langle\sigma_{13}v\right\rangle\frac{A_{32}}{A_{31}+A_{32}}\right)n_1 n_e \qquad (27)$$

thus the emissivity ratio and hence line ratio is

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}}\frac{(A_{31}+A_{32})\left\langle\sigma_{31}v\right\rangle}{(A_{31}+A_{32})\left\langle\sigma_{21}v\right\rangle + A_{31}\left\langle\sigma_{31}v\right\rangle} \qquad (28)$$

$$= \frac{A_{31}E_{31}}{A_{32}E_{32}}\frac{(A_{31}+A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31}+A_{32})\Omega_{21} + A_{31}\Omega_{31}e^{-E_{32}/kT}} \qquad (29)$$

excellent! Q: Why?

3-level atom line ratio

$$\frac{j_{31}}{j_{21}} = \frac{A_{31}E_{31}}{A_{32}E_{32}} \frac{(A_{31} + A_{32})\Omega_{31}e^{-E_{32}/kT}}{(A_{31} + A_{32})\Omega_{21} + A_{31}\Omega_{31}e^{-E_{32}/kT}} \tag{30}$$

depends only on T and atomic properties

so: for appropriate systems

- measure line ratio
- look up the atomic properties
- use observed ratio to solve for T!

Interstellar Dust

Strange Things are Afoot at the Circle K

E. E. Barnard (1907, 1910)

noted "vacancy" on the sky, now called "*dark clouds*"

`www: Barnard's images; modern images of dark clouds`

"It almost seems to me that we are here brought face to face with a phenomenon that may not be explained with our present ideas of the general make-up of the heavens." –Barnard 1907

R. J. Trumpler (1930)

compared distance measures to open star clusters

- *luminosity distance* $d_L = \sqrt{L/4\pi F}$
- *angular diameter distance* $d_A = R/\theta$

  Q: but how did he know luminosity L? physical size R?

`www: Trumpler data`

⇒ found that for distant clusters, $d_L > d_A$

also found stellar *colors* increasingly *red* with larger distance

Q: possible explanations? implications?

Cosmic Dust: Evidence

Trumpler 1930: found increasing ratio $d_L/d_A > 1$ with distance, with

$$\frac{d_L}{d_A} \propto \frac{1}{R}\sqrt{\frac{L}{F}} \tag{31}$$

observed d_L/d_A increase requires distant clusters are either:

- progressively more luminous – but why?
- progressively smaller – but why?
- anomalously dimmer, i.e., flux F increasingly *attenuated*

increased reddening with distance $\longrightarrow$ not a geometric effect

$\longrightarrow$ space filled with medium that *absorbs* and *reddens* light

$\Rightarrow$ **interstellar dust**

Interstellar Extinction

consider an object of *known flux density* F_λ^0
Q: candidates?

due to dust absorption, *observed flux* density is $F_\lambda < F_\lambda^0$
quantify this via **extinction** A_λ

$$\frac{F_\lambda}{F_\lambda^0} = 10^{-(2/5)A_\lambda} \tag{32}$$

compare optical depth against dust absorption:
$F_\lambda / F_\lambda^0 = e^{-\tau_\lambda}$, so

$$A_\lambda = \frac{5}{2} \log_{10} e^{\tau_\lambda} = 2.5\ \log_{10}(e)\ \tau_\lambda = 1.086\ \tau_\lambda \text{ mag} \tag{33}$$

extinction measures optical depth

Q: what does reddening imply about A_λ?

Reddening

observed reddening implies A_λ stronger for shorter λ
$\rightarrow$ increases with $1/\lambda$

for source of known F^0_λ, can measure this

`www: extinction curve as a function of wavelength`

observed trend: "*reddening law*"

- general rise in A_λ vs $1/\lambda$
- broad peak near $\lambda \sim 2200$ AA $= 0.2\mu$ m

Q: implications of peak? of reddening at very short λ?

in photometric bands, define *redding*: for: B and V

$$E(B-V) \equiv A_B - A_V \tag{34}$$