

Astro 501: Radiative Processes
Lecture 33
November 28, 2018

Announcements:

- **Problem Set 10** due Friday

Last time: deciphering dust

We are stardust
Billion year old carbon
We are golden
Caught in the devil's bargain
And we've got to get ourselves
Back to the garden
– J. Mitchell (1970)

⌈
Q: *what are grain sizes? how do we know?*
Q: *grain composition? how do we know?*

depletion correlated with condensation temperature

suggests **physical picture:**

- *dust formed out of high-temperature material*
e.g., ejecta from dying stars
note: AGB stars have dusty shells
- *as this vapor cools, refractory elements form dust first*
- small depletion for $T_{\text{cond}} \lesssim 700 - 800 \text{ K}$
either gas never gets this cool,
or more likely, density becomes too low to form dust
by collisional processes

Warning!

when using *interstellar abundances from lines*, never forget:
these include elements in the *gas phase only!*

Dust and Starlight

We have seen: *interstellar dust absorbs starlight*

For local sightlines in the Milky Way plane:

photon mean free path against extinction

$$\frac{1}{\ell_V} = \sigma_V n_{\text{dust}} = \frac{d\tau_V}{ds} \sim \frac{dA_V}{ds} \sim 2^{\text{mag}} \text{ kpc}^{-1} \quad (1)$$

Q: *implications for visible MW? shorter wavelengths? longer?*

www: the multiwavelength sky

Q: *what happens to dust as it absorbs starlight?*

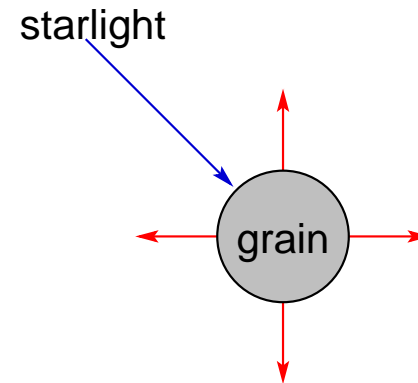
Q: *what dominates the stellar luminosity in a disk galaxy?*

ω Q: *so what does $L_{\text{far-IR}}$ probe?*

Dust Temperature

interstellar dust: radiative equilibrium

- dust grains absorb starlight particularly in optical/UV
- photon energy *thermalized* in grains
- which *radiate* roughly as black bodies



dust temperature set by absorption/emission equilibrium

at $T_{\text{dust}} \approx T_{\text{eff,ISM}} \sim 10 - 100 \text{ K}$, depending on environment
observable in the **infrared** www: mid-far IR sky

Lesson: *dust reprocesses starlight into thermal IR!*

Implication:

starlight dominated by luminous (and short-lived) massive stars

→ far-IR emission traces extincted UV/optical from these stars

→ $L_{\text{far-IR}} \propto dM_{\star}/dt$ *probes star formation rate!*

Shifting to High Gear

Thus far we have only considered radiative properties of **non-relativistic matter**, which means

- speeds $v \ll c$
- kinetic energies $E \ll mc^2$
- temperatures $kT \ll mc^2$

Q: what astrophysical sources satisfy these criteria?

Q: what sources do not?

The Relativistic Cosmos

most of cosmic matter is non-relativistic

stars, planets, interstellar gas all have $T \ll m_p c^2 / k \sim 10^{10}$ K
and bulk motions of stars, planets, ISM have $v \ll c$

but: cosmic accelerators exist, create relativistic motions!

- bulk **jets** launched by: **gamma-ray bursts**
supermassive black holes in **active galaxies** www: M87 jet
- **high-energy particles: cosmic rays**

to describe these objects and their radiation
we need Special Relativity

Special Relativity for the Impatient

Spacetime

see S. Carroll, *Spacetime and Geometry*; R. Geroch, *General Relativity from A to B*

evolving view of space, time, and motion:

Aristotle → Galileo → Einstein

Key basic concept: **event**

occurrence localized in space and time

e.g., firecracker, finger snap

idealized → no spatial extent, no duration in time

a goal (*the* goal?) of physics:

describe relationships among events

∞

Q: consider collection of all possible events—what's included?

Spacetime Coordinates

Each event specifies a unique point in space and time
collection of all possible events = **spacetime**

lay down coordinate system: 3 space coords, one time
4-dimensional, but as yet time & space unrelated

e.g., time t , Cartesian x, y, z : event $\rightarrow (t, x, y, z)$
physics asks (and answers) what is the relationship
between two events, e.g., (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2)

Note: more on spacetime in Director's Cut Extras to today's notes

Galilean Relativity

consider *two laboratories*

(same apparatus, funding, required courses, vending machines)

move at constant velocity with respect to each other

Galileo:

no experiment done in either lab (without looking outside)

can answer the question “which lab is moving”

→ *no absolute motion*, only relative velocity

Newton: laws of mechanics invariant

for observers moving at const \vec{v}

“*inertial observers*”

Implications for spacetime

no absolute motion → *no absolute space*

(but still no reason to abandon absolute time)

Galilean Frames

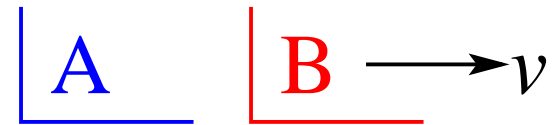
each inertial obs has own personal frame:

obs (“Angelina”) at rest in own frame: (x, y, z) same for all t

but to another obs (“Brad”) in relative motion $\vec{v} = v\hat{x}$

B sees A’s frame as time-dependent:

$$x_{\text{A as seen by B}} = x' = x - vt$$



but still absolute time: $t' = t$

Newton’s laws (and Newtonian Gravity) hold in both frames

$$\text{can show: } d^2\vec{x}/dt^2 = \vec{F}(\vec{x}) \Rightarrow d^2\vec{x}'/dt'^2 = \vec{F}(\vec{x}')$$

“Galilean invariance”

Trouble for Galileo

Maxwell: equations govern light
very successful, but:

- predicts unique (constant) *light speed* c —relative to whom?
- Maxwell eqs **not** Galilean invariant

Lorentz: Maxwell eqs invariant when

$$t' = \gamma(t - vx/c^2) \quad (2)$$

$$x' = \gamma(x - vt) \quad (3)$$

$$y' = y \quad (4)$$

$$z' = z \quad (5)$$

with Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$

Einstein:

Lorentz transformation not just a trick
but correct relationship between inertial frames!
 \Rightarrow this is the way the world is

Einstein: Special Relativity

consider two *nearby events*

(t, x, y, z) and $(t + \Delta t, x + \Delta x, y + \Delta y, z + \Delta z)$

different inertial obs *disagree* about Δt and $\Delta \vec{x}$
but all *agree* on the value of the **interval**

$$\Delta s^2 \equiv (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad (6)$$

$$= (c\Delta t)^2 - (\Delta \ell)^2 \quad (7)$$

Note: interval can have $\Delta s^2 > 0, < 0, = 0$

quantities agreed upon by all observers: *Lorentz invariants*

Light pulse: $\Delta \ell = c\Delta t$

→ $\Delta s_{\text{light}} = 0$

→ light moves at c in all frames!

Motion and time:

Consider two events, at rest in one frame:

$\Delta \vec{x}_{\text{rest}} = 0$ in rest frame, so

$\Delta s = c\Delta t_{\text{rest}}$: $c \times$ elapsed time in rest frame

In another inertial frame, relative speed v :

events separated in space by $\Delta x' = v\Delta t'$

$$\Delta s = \sqrt{c^2 \Delta t'^2 - \Delta x'^2} = \sqrt{c^2 - v^2} \Delta t' = \frac{1}{\gamma} c \Delta t' \quad (8)$$

since Δs same: infer $\Delta t' = \gamma \Delta t_{\text{rest}} > \Delta t_{\text{rest}}$

\Rightarrow moving clocks appear to run slow

(special) relativistic **time dilation**

\Rightarrow no absolute time (and no absolute space)

H. Minkowski:

“Henceforth, space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.”

Lorentz Transformations

consider two coordinate systems K, K'
moving with *relative* speed $\vec{v} = v\hat{x}$

$$t' = \gamma(t - vx/c^2)$$

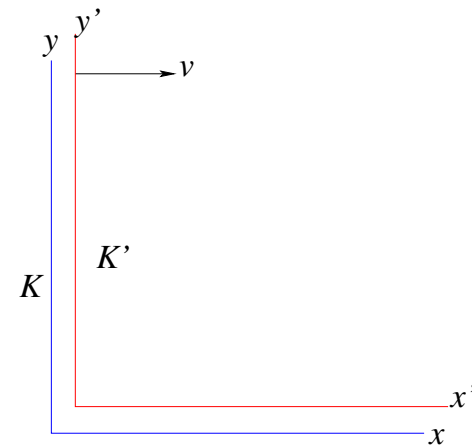
$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

- **boost** from one frame to another
 - truly mix space and time \rightarrow *spacetime*
 - look like rotations, but 4-dimensional
- \rightarrow should express laws in terms of 4-D vectors:

“4-vectors,” t, x components transform via Lorentz



Length Contraction

consider a *standard ruler*: measure length at factory

- ruler is at rest wrt observer
- measure both ends *at same time* $\delta t = t_2 - t_1 = 0$
- ends are $x_1 = 0$, $x_2 = L \rightarrow$ length $L = \delta x = x_2 - x_1$

observer flying by a speed $\vec{v} = v\hat{x}$, makes measurement

$$\delta x' = \gamma(\delta x - v\delta t) = \gamma(L - v\delta t/c^2) \quad (9)$$

$$\delta t' = \gamma(\delta t - v\delta x/c^2) = \gamma(\delta t - vL/c^2) \quad (10)$$

but length measurement is done at same time

$\rightarrow \delta t' = 0 \rightarrow \delta t = vL/c^2$ Q: implications?

and thus length found is $L' = \delta x' = \gamma(1 - v^2/c^2)L$

17 $\Rightarrow L' = L/\gamma$ **length contraction**

Q: what if the observer were moving in \hat{y} ?

Addition of Velocities

consider an object moving wrt to frame K'
as seen by K' : in time interval dt , moves distance dx → has
velocity $u' = dx'/dt'$

What is speed in frame K (speed v wrt K)?

$$dt = \gamma(dt' + v dx'/c^2) \quad (11)$$

$$dx = \gamma(dx' + v dt') \quad (12)$$

$$dy = dy' \quad dz = dz' \quad (13)$$

and thus

$$u_x = dx/dt = \frac{u'_x + v}{1 + u'_x v/c^2} \quad (14)$$

$$u_y = dy/dt = \frac{u'_y}{\gamma(1 + u'_x v/c^2)} \quad (15)$$

$$u_z = dz/dt = \frac{u'_z}{\gamma(1 + u'_x v/c^2)} \quad (16)$$

If object is moving with arbitrary velocity \vec{u}' in K'
 decompose $\vec{u}' = \vec{u}'_{\parallel} + \vec{u}'_{\perp}$ where \parallel is along $K - K'$ motion:

$$u_{\parallel} = \frac{u'_{\parallel} + v}{1 + u'_{\parallel}v/c^2} \quad (17)$$

$$u_{\perp} = \frac{u'_{\perp}}{\gamma(1 + u'_{\parallel}v/c^2)} \quad (18)$$

boost causes **change in velocity direction**

$$\tan \theta = \frac{u_{\perp}}{u_{\parallel}} = \frac{u' \sin \theta'}{\gamma(u' \cos \theta' + v)} \quad (19)$$

and consider **the case where $u' = c$**

$$\tan \theta = \frac{\sin \theta'}{\gamma(\cos \theta' + \beta)} \quad (20)$$

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'} \quad (21)$$

angular shift is the **aberration of light**

a light signal emitted in K' at angle θ'
is seen in K at angle θ

$$\cos \theta = \frac{\cos \theta' + v/c}{1 + v/c \cos \theta'} \quad (22)$$

Q: what if $\theta' = 0? \pi?$

Q: how can we understand this physically?

Q: what if $\theta' = \pi/2?$

Q: how can we understand this physically?

consider photons emitted *isotropically* in K'
with v/c not small

20 Q: what is angular pattern in K ? implications?

Relativistic Beaming

for light emitted in K' at $\theta' = \pi/2$
observed angle after boosting is

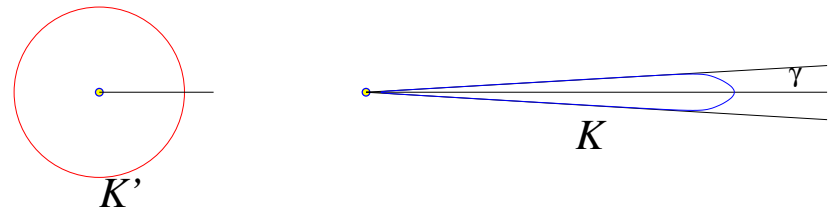
$$\tan \theta = \frac{1}{\gamma v/c} \quad (23)$$

and thus

$$\sin \theta = \frac{1}{\gamma} \quad (24)$$

if emitted K' is highly relativistic,
then $\gamma \gg 1$, and

$$\theta \rightarrow \frac{1}{\gamma}$$



21 i.e., a small forward angle!
a highly relativistic emitter gives a **beamed radiation pattern**
strongly concentrated ahead of emitter direction

Director's Cut Extras

Pre-Relativity: Aristotle

x, y, z Cartesian (Euclidean geometry)

spatial distance ℓ between events is:

$$\ell^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 \quad (25)$$

and is independent of time

elapsed time between events is: $t_2 - t_1$

and is independent of space

“absolute space” and “absolute time”

Is a particle at rest? \Leftrightarrow do (x, y, z) change?

\rightarrow “absolute rest, absolute motion”

\approx *Diagram: Aristotelian spacetime*

unique “stacking” of “time slices”

Life According to Aristotle

Note: even in absolute space

event location indep of coordinate description

e.g., two observers choose coordinates different by a rotation:

(x, y) and $(x', y') = (x \cos \theta - y \sin \theta, y \cos \theta + x \sin \theta)$

preserves distance from origin: $x^2 + y^2 = (x')^2 + (y')^2$

objects (observers) at rest:

same x, y, z always, t ticks forward

geometrically, a line in spacetime: **“world line”**

if at rest: world line vertical

constant speed: $x = vt$: diagonal line light: moves at “speed of light” c

→ well-defined, since motion absolute

in spacetime: light pulse at origin $(t, x, y, z) = (0, 0, 0, 0)$

moves so that distance $\ell = \sqrt{x^2 + y^2 + z^2} = ct$

geometrically: **light cone**

Galilean Transformations: Spacetime Picture

Geometrically:

different inertial frames \rightarrow transformation of spacetime

slide the “space slices” at each time

(picture “shear,” or beveling a deck of cards)

Spacetime and Relativity

Pre-Relativity: space and time separate and independent but *rotations* mix *space* coords, e.g.,

$$x' = x \cos \theta - y \sin \theta \quad ; \quad y' = y \cos \theta + x \sin \theta \quad (26)$$

without changing underlying vector (rotation of coords only)

transform rule holds not only for \vec{x}

but all other physical directed quantities: e.g., $\vec{v}, \vec{a}, \vec{p}, \vec{g}, \vec{E}$

all transform under rotations following same rule, e.g.,

$$E'_x = E_x \cos \theta - E_y \sin \theta \quad ; \quad E'_y = E_y \cos \theta + E_x \sin \theta \quad (27)$$

Lesson: express & guarantee underlying rotational invariance

by writing physical law in vector form

26 e.g., $\vec{F} = m\vec{a}$ gives same physics for any coord rotation