Astro 501: Radiative Processes
Lecture 35
December 3, 2018

Announcements:

- **Problem Set 11–The Final Frontier** due Friday
  Q1 wordy but not much to calculate!

Last time: began cosmic rays
Q: status as of 1912?

Today: summary of cosmic rays
begin emission from cosmic ray electrons

Cosmic Rays: Vital Statistics

Cosmic rays: population of particles which are

- **electrically charged**
- **energetic** ($\gtrsim 1$ MeV)
- **nonthermal** *Q: meaning?*

Cosmic Ray Sources:

- *solar activity:* ~ 0.1 MeV to ~ 1 GeV, typically few MeV
  `www: Solar Flares`
- but most cosmic rays: *extrasolar*

`www: real-time satellite data`

Cosmic Ray Composition

composition: mostly nuclei (fully stripped of e^-)

- **nuclear ("hadronic") component**
  90% are *protons*
  of remainder, 90% are α
  elements up to Se detected
  `www: proton flux`

- **electron/positron ("leptonic") component**
  mostly e^-, *some* e^+
  at fixed energy, electron flux $\mathcal{I}_E(e) \sim \mathcal{I}_E(p)/100$ of protons

angular distribution:
isotropic over most of energy range

cosmic rays often annoy non-CR astronomers *Q: why?*

Observed Electron Component

Experimental techniques:

- balloons
- space missions
- ground-based (high-energy): atm Čerenkov, air shower arrays

flux at top of atmosphere depends on location *Q: why?*
and on time

anti-correlation between CR flux at Earth and solar activity

$\Rightarrow$ solar "modulation" of CR

- excludes $\lesssim 100$ MeV particles
- reduces $\lesssim 1$ GeV flux

must correct for solar effects ("demodulate") to infer interstellar spectra

Cosmic Ray Spectrum

specific *number* intensity usually expressed in units of energy

$$\mathcal{I}_E = \frac{d\mathcal{N}}{dA\,dt\,d\Omega\,dE} \quad (1)$$

$$= v(E)\frac{d\mathcal{N}}{dV\,d\Omega\,dE} \quad (2)$$

or units of Lorentz $\gamma = E_{\text{tot}}/m_e c^2$

`www: CR electron spectrum`

Q: what is number intensity spectrum $\mathcal{I}_E$? energy intensity spectrum I_E?

`www: CR proton spectrum`

Cosmic-Ray Electron Spectrum

cosmic ray spectrum clearly **nonthermal**
i.e., not a Fermi-Dirac form appropriate for thermal fermions
rather: a succession of *power laws*

• observed CR electrons: *number spectrum* roughly

$$\mathcal{I}_E(e) \propto E^{-3} \quad (3)$$

and thus *usual energy intensity* $I_E(e) = E\ \mathcal{I}_E(e) \propto E^{-2}$

Q: will protons be the same? different?

Cosmic Ray Propagation

consider cosmic ray protons and electrons moving through interstellar space

Q: what interactions will each have?

Q: what will be the effect on interstellar matter?

Q: how will this affect CR propagation ("radiative transfer")?

Q: how could we detect evidence for this?

cosmic rays are highly energetic and penetrating
fill the Galaxy (and other galaxies)

cosmic ray electrons and protons
can and do collide with and scatter off interstellar matter
...and interstellar radiation!

- heating source for interstellar matter
- propagation should include scattering effect

But more important:
Cosmic rays are **charged particles**
$\rightarrow$ couple to Galactic (and intergalactic!) magnetic fields

- cosmic ray trajectories are bent by fields
- *cosmic rays do not point back to their sources*
- *accelerated motion* means that *cosmic rays radiate*
- electrons much more strongly accelerated $\rightarrow$ e synchrotron radiation dominates

∞

`www: radio continuum sky, edge-on spirals, SN remnants`

Synchrotron Radiation

Relativistic Motion in a Uniform B Field

Consider a relativistic classical particle, mass m, charge q
moving in a uniform magnetic field $\vec{B}$
with no electric field $\vec{\mathcal{E}} = 0$

Equations of motion: total relativistic *energy* $E = \gamma mc^2$

$$\frac{dE}{dt} = mc^2 \frac{d\gamma}{dt} = q\ \vec{v} \cdot \vec{\mathcal{E}} = 0 \tag{4}$$

and so γ is *constant* and hence $|\vec{v}|$ is too

Equations of motion: *momentum*

$$\frac{d\vec{p}}{dt} = m\frac{d}{dt}\gamma\vec{v} = \frac{q}{c}\vec{v} \times \vec{B} \tag{5}$$

$$\frac{d}{dt}\gamma\vec{v} = \frac{q}{mc}\vec{v} \times \vec{B} \tag{6}$$

but γ and $|\vec{v}|$ are constant, so

$$\frac{d}{dt}\vec{v} = \frac{q}{\gamma mc}\vec{v} \times \vec{B} \tag{7}$$

take dot product with $\vec{B}$

$$\vec{B} \cdot \frac{d}{dt}\vec{v} = B\frac{d}{dt}v_{\parallel} = 0 \tag{8}$$

$\rightarrow$ *velocity component $v_{\parallel}$ parallel to $\vec{B}$ is constant*

decompose velocity into $\vec{v} = \vec{v}_{\parallel} + \vec{v}_{\perp}$

$$\frac{d}{dt}\vec{v}_{\perp} = \frac{q}{\gamma mc}\vec{v}_{\perp} \times \vec{B} \tag{9}$$

Q: resulting motion orthogonal to field?

$$\frac{d}{dt}\vec{v}_\perp = \frac{q}{\gamma mc}\vec{v}_\perp \times \vec{B} = \vec{v}_\perp \times \vec{\omega}_B \tag{10}$$

perpendicular velocity *precesses* around $\vec{B}$
with gyrofrequency

$$\vec{\omega}_B = \frac{q}{\gamma mc}\vec{B} \tag{11}$$

note: nonrelativistic gyrofrequency $\omega_{B,\mathrm{nr}} = qB/mc$
is independent of v
but in relativistic case has factor $1/\gamma$

Q: full motion of charge?

orthogonal to $\vec{B}$, particle with speed $v_\perp$
moves in circle with **gyroradius**

$$r_{\rm g} = \frac{v_\perp}{\omega_B} = \frac{mc\gamma v_\perp}{qB} = \frac{cp_\perp}{qB} \quad (12)$$

thus the general motion is a combination of

- *constant velocity $v_\parallel$ along $\vec{B}$*
- *uniform circular motion in plane orthogonal $\vec{B}$*

net result: **spiral** around $\vec{B}$

numerically: gyroradius

$$r_{\rm g} = 3.3 \times 10^{12} \ {\rm cm} \ \left(\frac{cp_\perp}{1 \ {\rm GeV}}\right) \ \left(\frac{1 \ \mu{\rm Gauss}}{B}\right) \quad (13)$$

Q: why these choices for $p_\perp$ and B? implications?
Q: what if p very very large?

typical "blue collar" cosmic ray energy $E \sim cp \sim 1\text{GeV}$
and typical interstellar magnetic field $B \sim 1\ \mu\text{Gauss}$

thus typical cosmic ray gyroradius is

$$r_{\rm g} \sim 0.02\ \text{AU} = 10^{-6}\ \text{pc} \tag{14}$$

so $r_{\rm g} \ll$ solar system, interstellar scales
cosmic rays definitely do not move in straight lines

possible exception: $r_{\rm g} \gtrsim R_{\rm MW} \sim 10$ kpc
for $p \gtrsim 10^{10}$ GeV $= 10^{19}$ eV
$\rightarrow$ "ultra-high-energy cosmic rays"
www: arrival directions for UHECR

returning to typical cosmic rays: gyrofrequency

$$\nu_{\rm g} = \frac{\omega_{\rm g}}{2\pi} = \frac{eB}{2\pi\gamma mc} = 2.8\text{Hz}\ \gamma^{-1} \left(\frac{B}{1\ \mu\text{Gauss}}\right) \left(\frac{m_e}{m}\right) \tag{15}$$

Q: implications for electrons? protons?

gyrofrequency for mildly relativistic electrons:
cyclotron frequency $\nu_{\rm g} \sim few$ Hz
$\rightarrow$ very slow! huge wavelengths
if radiation is only at this frequency
would seem undetectable

but we will see: for relativistic electrons
radiation is at much higher frequencies!
synchrotron radiation

even so, low gyrofrequency hints that *radio* frequencies
likely to be important for synchrotron emission
`www: Kepler supernova remnant at 6 cm (VLA)`
Q: implications of intensity pattern?

Q: how to evaluate emitted synchrotron power from CR electrons?

Power Emitted by a Relativistic Charge

non-relativistic Larmor: $P' = 2q^2/3c^3 \ |\vec{a}'|^2$
want to re-express using 4-acceleration

can show: in instantaneous rest frame, $a^{0\prime} = 0$
and thus $|\vec{a}'|^2 = a \cdot a$
Lorentz-invariant Larmor expression for total radiated power

$$P = \frac{2}{3}\frac{q^2}{c^3} a \cdot a \tag{16}$$

manifestly invariant, can evaluate in any frame

$$P = \frac{2}{3}\frac{q^2}{c^3} a \cdot a \tag{17}$$

in instantaneous rest frame, 4-acceleration transforms as

$$\begin{aligned} a'_{\|} &= \gamma^3 a_{\|} && (18) \\ a'_{\perp} &= \gamma^2 a_{\perp} && (19) \\ & && (20) \end{aligned}$$

and so power emitted is

$$\begin{aligned} P &= \frac{2}{3}\frac{q^2}{c^3} a' \cdot a' = \frac{2}{3}\frac{q^2}{c^3}(a'^2_{\perp} + a'^2_{\|}) && (21) \\ &= \frac{2}{3}\frac{q^2}{c^3}\, \gamma^4\, (a^2_{\perp} + \gamma^2 a^2_{\|}) && (22) \end{aligned}$$

note large boost for relativistic particles ($P \propto \gamma^4$ or γ^6)

Synchrotron Power

Lorentz-invariant power emitted from accelerated charge is

$$P = \frac{2}{3}\frac{q^2}{c^3} a' \cdot a' = \frac{2}{3}\frac{q^2}{c^3}(a'^2_{\parallel} + a'^2_{\perp}) \quad (23)$$

$$= \frac{2}{3}\frac{q^2}{c^3}\,\gamma^4\,(a^2_{\perp} + \gamma^2 a^2_{\parallel}) \quad (24)$$

for our case of circular motion: $a_{\parallel} = 0$,and

$a_{\perp} = \omega_B v_{\perp}$, so

$$P = \frac{2}{3}\frac{q^2}{c^3}\,\gamma^4\,\frac{q^2 B^2}{\gamma^2 m^2 c^2} = \frac{2}{3} r_0^2\; c\; \gamma^2 \beta_{\perp}^2 B^2 \quad (25)$$

but electron distribution is isotropic

so must *average over* distribution of *pitch angle* $\hat{v} \cdot \hat{B} = \cos\alpha$

$$\left\langle \beta_{\perp}^2 \right\rangle = \frac{\beta^2}{4\pi}\int \sin^2\alpha\; d\Omega = \frac{2}{3}\beta^2 \quad (26)$$

total synchrotron power from isotropic electrons

$$P = \left(\frac{2}{3}\right)^2 r_0^2 \; c \; \gamma^2 \beta B^2 = \frac{4}{3} \sigma_\mathsf{T} \; c \; \beta^2 \gamma^2 \; u_B \tag{27}$$

where $\sigma_\mathsf{T} = 8\pi r_0^2/3$ and $u_B = B^2/8\pi$

Q: how to find the spectrum of synchrotron radiation?

Q: why is it non-trivial? hint–think of relativistic circular motion

Director's Cut Extras

Cosmic-Ray Proton Spectrum

- protons w/ 1 GeV $\lesssim E \lesssim$ 300 TeV:

$$I_E(p) \simeq 1.4 \left(\frac{E}{\mathrm{GeV}}\right)^{-s} \text{ protons cm}^{-2}\text{ s}^{-1}\text{ sr}^{-1}\text{GeV}^{-1} \quad (28)$$

  where spectral index ("slope") $s \simeq 2.7$
- beyond "knee' at $E_{\mathrm{knee}} \sim 10^{15}$ eV
  power law index steepens to $s \sim 3$
- then beyond "ankle" at $E_{\mathrm{anlke}} \simeq 10^{18}$ eV, flattens again

Q: Tevatron energy? LHC? implications?
historically: many particles first discovered via CRs

 Q: in which regime are most CR particles? most CR energy?

What's typical?

cosmic-ray number flux

$\Phi(> E) = 4\pi \int I(E)\ dE = 4\pi \int E\ I(E)\ d\ln E$

per log energy interval, number distribution is

$d\Phi/d\ln E \sim E\ I(E) \sim E^{-(s-1)}$

$\rightarrow$ *number peaks at smallest (but still relativistic) energies*

typical proton: $E \sim 1$ GeV

cosmic-ray energy proton flux $F(> E) = 4\pi \int E\ I(E)\ dE$

per log energy interval, $dF/d\ln E \sim E^2\ I(E) \sim E^{-(s-2)}$

$\Rightarrow$ since $s > 2$, *energy also peaks at low energies*

ensemble of cosmic rays acts as *mildly relativistic gas*

spectrum poses questions:

- origin(s) of the power-law behavior?
- what leads to the different regimes?