Astro 501: Radiative Processes
Lecture 37
December 7, 2018

Announcements:

- **Problem Set 11–The Final Frontier** due now
  rest for the weary at last!

last time: synchrotron radiation from cosmic-ray electrons

- *Q: why do cosmic-ray electrons radiate?*
- *Q: characteristic synch frequency for electron with γ?*
- *Q: what about proton synchrotron? frequency, intensity?*
- *Q: synchrotron spectrum for power-law e energy distribution?*

Synchrotron Radiation

isotropic electrons with single γ, β:

- synchrotron power

$$P_e = \left|\frac{dE_e}{dt}\right| = \left(\frac{2}{3}\right)^2 r_0^2 \; c \; \gamma^2 \beta B^2 = \frac{4}{3} \sigma_{\mathrm{T}} \; c \; \beta^2 \gamma^2 \; u_B \tag{1}$$

where $\sigma_{\mathrm{T}} = 8\pi r_0^2/3$ and $u_B = B^2/8\pi$

- synchrotron spectrum peaks at **critical frequency**

$$\nu_{\mathrm{c}} \sim \gamma^2 \nu_{\mathrm{cyc}} = \gamma^2 \frac{m_e c}{qB} \tag{2}$$

for electron distribution $dN/d\gamma = C\gamma^{-p}$

$$4\pi j_{\mathrm{tot}}(\omega) = \frac{\sqrt{3} q^3 C B \sin\alpha}{2(p+1)\pi m c^2} \, \Gamma\left(\frac{p}{4} + \frac{9}{12}\right) \, \Gamma\left(\frac{p}{4} - \frac{1}{12}\right) \, \left(\frac{mc\omega}{3qB\sin\alpha}\right)^{-(p-1)/2} \tag{3}$$

Synchrotron Radiation: the Big Picture

for relativistic electrons with power-law energy distribution

emission coefficient

$$j_\nu \propto \nu^{-(p-1)/2} \tag{4}$$

absorption coefficient

$$\alpha_\nu \propto \nu^{-(p+4)/2} \tag{5}$$

source function (note nonthermal character!)

$$S_\nu \propto \nu^{5/2} \tag{6}$$

Q: optical depth vs ν? implications?
Q: spectrum of a synchrotron emitter?

`www: awesome example: pulsar wind nebulae`
young pulsars are spinning down
much of rotational energy goes into relativistic wind
which collides with the supernova ejecta an emits synchrotron

Build Your Toolbox–Synchrotron Radiation

emission physics: matter-radiation interactions

Q: physical conditions for synchrotron emission? absorption?

Q: physical nature of sources?

Q: spectrum characteristics?

Q: frequency range?

real/expected astrophysical sources of synchrotron radiation

Q: what do we expect to emit synchrotron? absorb?

Q: relevant temperatures? EM bands?

Toolbox: Synchrotorn Radiation

emission physics

- physical conditions: relativistic charged particles in magnetic field
- physical sources: relativistic electrons dominate
- spectrum: for electron energy distribution $dN_e/dE_e \propto E_e^{-p}$
  synchrotron emission is continuum
  with *power law* $j_\nu \sim \nu^{-(p-1)/2}$ spectrum
  and source function $S_\nu \sim \nu^{5/2}$

astrophysical sources of synchrotron

- emitters: relativistic electrons: cosmic rays in galaxies or in jets
- temperatures: trick question! sources are nonthermal!
- EM bands: max synch energy depends on max γ and magnetic field, can go from radio to X-ray!

Astrophysical Context: Blazars

we met radio galaxies in the context of synchrotron radiation
but there are many beasts in the active galaxy zoo

Blazars

- seen as luminous nuclear region
  at center of giant elliptical galaxies
  `www: optical blazar images (`R`-band)`
- but *do not* show the elongated jets seen in radio galaxies
- flux shows rapid and large-amplitude time variability
- subclasses: BL Lacertae objects–weak radio emission
  optically violent variables (OVV)–strong radio emission
- demographics: many fewer blazars than other AGN
  e.g., Seyfert galaxies
  `www: AGN demographics plot (INTEGRAL)`
- blazar emission spans radio to TeV gamma rays

Q: what does this suggest about the nature of blazars?

Blazars: Staring Down the Jet

AGN "Unification Model" `www: unification cartoon`
idea: all active galaxies have similar physical conditions

- a supermassive black hole (SMBH)
  possibly actively accreting matter
- a surrounding accretion disk, and dusty torus
- a relativistic jet, if SMBH is actively accreting

in unification picture: *blazar = jet pointing directly at us!*
"looking down the barrel of the gun"
emission from small region of jet "tip" → highly variable

blazar spectra `www: example`
over full EM range, two large features

- power-law rise from radio, peaks near optical
- falls to X-rays, then peak and power-law fall at gamma-ray

Q: what could be going on?

Blazar Spectra

Power-law rise from radio to ~optical

- nonthermal
- similarity with radio galaxies suggests *synchrotron origin* from relativistic electrons in jet

Peak and power-law fall in gamma rays

- in non-flare ("quiescent") state, gamm-ray energy content similar to synchrotron
- suggests similar origin
  → perhaps a *reprocessing* of the synchrotron photos
- reprocessed how? *by the relativistic electrons themselves!*

Thus: we want to understand how relativistic electrons interact with photons *Q: the name for which is...?*

Note: blazar neutrinos seen! → imply proton emission $pp \to \pi^0 \to \gamma\gamma$!

Compton Scattering

Thomson Scattering

We already discussed the scattering of light by electrons in the context of *Thomson scattering*

Thomson highlights:
energies of incident photon ϵ and scattered photon ϵ_1 related by

$$\epsilon_1 = \epsilon \tag{7}$$

differential cross section, with $\hat{k} \cdot \hat{k}_1 = \cos\theta$

$$\frac{d\sigma_{\mathrm{T}}}{d\Omega} = \frac{1}{2} r_0^2 \left(1 + \cos^2\theta\right) \tag{8}$$

total cross section, with $r_0 = e^2/m_e c^2$

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3} r_0^2 \tag{9}$$

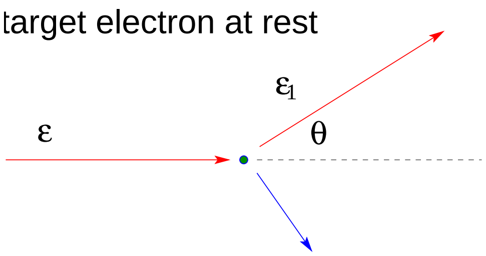
Q: what assumptions went into this? When will they fail?

Enter the Quantum: Compton Scattering

Thomson scattering derived for classical EM wave
- $\nu_1 = \nu$ classically
- carrying this to photon picture: $h\nu_1 = h\nu$
  → coherent or elastic scattering

but really: photon quanta carry momentum and energy
→ and electron will *recoil* and carry away energy
→ expect scattered photon to have less energy,
  and to move in a different direction

Compton: treat light as massless particle



for photon incident on electron *at rest*
conservation of energy and momentum implies

$$\epsilon_1 = \frac{\epsilon}{1 + (\epsilon/m_e c^2)(1 - \cos\theta)} \tag{10}$$

scattered photon energy is lower, and direction different

so the wavelength shifts by

$$\lambda_1 - \lambda = \lambda_C(1 - \cos\theta) \tag{11}$$

where the electron *Compton wavelength* $\lambda_C = h/m_e c = 0.02426$ Å

Q: what energy does a photon with λ_C have?
Q: What region of the spectrum is this?

Q: when is the wavelength shift important? negligible?

Cross Section for Compton Scattering

Compton wavelength shift is $\Delta\lambda \sim \lambda_{\mathrm{C}}$

- *small* if $\lambda \gg \lambda_{\mathrm{C}}$ i.e., $h\nu \ll m_e c^2$
  i.e., radio through soft X-rays
- *large* if $h\nu \gg m_e c^2$: hard X-rays, gamma rays

differential cross section: **Klein-Nishina** formula

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2}\,\frac{\epsilon_1^2}{\epsilon^2}\,\left(\frac{\epsilon}{\epsilon_1} + \frac{\epsilon_1}{\epsilon} - \sin^2\theta\right) \tag{12}$$

- classical Thomson expression recovered when $\epsilon \sim \epsilon_1$
- main effect: *smaller* cross section at high energy
- total cross section, with $x = h\nu/m_e c^2$ in e rest frame

$$\sigma \to \begin{cases} \sigma_{\mathrm{T}}\ (1 - 2x + \cdots) & x \ll 1 \\ 3\sigma_{\mathrm{T}}/8\ x^{-1}(\ln 2x + \cdots) & x \gg 1 \end{cases} \tag{13}$$

recall: to understand blazars, we are interested in

- high-energy electrons interacting with ambient photons

Q: why can't *we just use the Compton scattering formulae?*

Q: how can *we use the formulae?*

Inverse Compton Scattering

the usual Compton scattering expressions
assume the electron is initially *at rest*
and the *photon loses energy* in scattering
→ "ordinary kinematics"
but this is not the case we are interested in!

in a frame where the electron is relativistic

- then there can be a momentum and energy transfer and the photon *gains energy*
- "upscattered" to higher frequencies

→ "inverse kinematics" – **inverse Compton scattering**

strategy: use Lorentz transformations *twice*

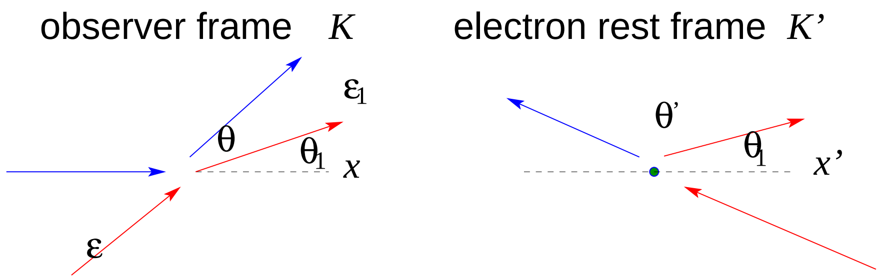
0. start in "lab frame" were e is relativistic
1. boost to e rest frame, find scattered photon energy/momentum
2. boost back to lab frame to find scattered photon energy

Lab/observer frame K:

- initial electron *relativistic*
- Lorentz factor γ, speed $\beta = v/c$

electron rest frame K':

- ordinary kinematics



Doppler expression: find photon energies in rest frame

$$\epsilon' = \gamma\, \epsilon\, (1 - \beta \cos\theta) \tag{14}$$

$$\epsilon'_1 = \gamma\, \epsilon_1\, (1 - \beta \cos\theta_1) \tag{15}$$

Compton: with $\hat{k}' \cdot \hat{k}'_1 = \cos\Theta$, and assuming $\epsilon' \ll m_e c^2$

$$\epsilon'_1 \approx \epsilon \left[1 - \frac{\epsilon'}{m_e c^2} (1 - \cos\Theta) \right] \tag{16}$$

if initial lab-frame photon energy is ϵ

Q: initial photon energy in e rest frame, roughly?

Q: scattered photon energy in e rest frame, roughly?

Q: scattered photon energy lab frame, roughly?

Inverse Compton: Order of Magnitude

if initial lab-frame photon energy is ϵ
and lab-frame electron with $\gamma \gg 1$:
in e rest frame: photon energy *boosted*
$\rightarrow$ initial energy $\epsilon' \sim \gamma\epsilon$

still in e rest frame:
if $\epsilon' \ll m_e c^2 \rightarrow$ small photon energy change (Thomson)
scattered energy $\epsilon'_1 \sim \epsilon' \sim \gamma\epsilon$

back to lab frame:
scattered photon energy *boosted* to $\epsilon_1 \sim \gamma\epsilon'_1 \sim \gamma^2\epsilon_1$

Bottom line: upscattered photon energy $\epsilon_1 \sim \gamma^2\epsilon$
Q: implications for blazars?

Inverse Compton Power for Single-Electron Scattering

Consider a relativistic electron (γ, β)
incident on an isotropic distribution of ambient photons
$\longrightarrow$ find power going into inverse Compton

Order of magnitude estimate:

- if typical ambient photon energy is ϵ
  then typical *upscattered energy* is $\epsilon_1 \sim \gamma^2 \epsilon$
- if ambient photon number density is n_γ
  then *scattering rate per electron* is $\Gamma \sim n_\gamma \sigma_\top c$ *Q: why?*

thus expect power = rate of energy into inverse Compton

$$\frac{dE_1}{dt} \sim \Gamma\ \epsilon_1 \sim \gamma^2 \epsilon n_\gamma \sigma_\top c \sim \gamma^2 \sigma_\top c u_\gamma \tag{17}$$

where $u_\gamma = \langle \epsilon \rangle\, n_\gamma$ is the ambient photon energy density
Q: looks familiar?