Astro 501: Radiative Processes
Lecture 38
December 10, 2018

Announcements:

- **Final Exam – Friday Dec 14.**
- take home. Questions assigned 2pm, due by 10pm.
- designed to take 3 hours
- open book, open notes. No internet, no collaboration.

last time:

Compton scattering *Q: what's that?*

Q: differences with Thompson scattering?

Inverse Compton scattering *Q: what's that?*

Q: astrophysical sources of inverse Compton?

Electron/Photon Scattering: $e\gamma \rightarrow e\gamma$

Thomson

Classical treatment works

photon frequency unchanged: $\epsilon_{\rm f} = \epsilon_{\rm i}$ *coherent scattering*

$\lambda \gg \lambda_{\rm C} = h/m_e c$ and $\epsilon \ll m_e c^2$

cross section $\sigma = \sigma_{\rm T}$ is energy independent

Compton

photon *recoil* important: scattering not coherent/elastic

cross section drops for $\epsilon \gtrsim m_e c^2$

Inverse Compton

"inverse" kinematics: *high-energy electron upscatters photon*

Inverse Compton Scattering

the usual Compton scattering expressions
assume the electron is initially *at rest*
and the *photon loses energy* in scattering
$\longrightarrow$ "ordinary kinematics"
but this is not the case we are interested in!

in a frame where the electron is relativistic

- then there can be a momentum and energy transfer and the photon *gains energy*
- "upscattered" to higher frequencies

$\longrightarrow$ "inverse kinematics" – **inverse Compton scattering**

New Dance Craze: Inverse Compton Style

Step 0: plant your feet = consider *lab/observer frame*:
- relativistic electrons with $E = \gamma m_e c^2$
- isotropic photon distribution, energies ϵ

Step 1: jump (boost) to *electron rest frame*
Ask ourselves: what does the electron "see"?
Q: incident photon angular distribution? typical energy ϵ'?

for simplicity: let $\gamma\epsilon \ll m_e c^2$
$\rightarrow$ Thompson approximation good in e frame K'
Q: then what is angular distribution of scattered photons in K'?
Q: scattered photon energy lab frame, roughly?

Step 2: jump (boost) again, return to *lab frame*
Q: what is angular distribution of scattered photons?
Q: scattered photon energy in e rest frame, roughly?

Inverse Compton and Beaming

Recall: a photon distribution isotropic in frame K
is *beamed* into angle $\theta \sim 1/\gamma$ in highly boosted frame K'

so in *electron rest frame* K'
most lab-frame photons "seen" in head-on beam
with energy $\epsilon' \sim \gamma\epsilon$

if rest-frame energies in Thompson regime:
- scattered photon directions $\propto d\sigma/d\Omega \propto 1 + \cos^2\theta$
  $\rightarrow$ isotropic + quadrupole piece
- scattered energy $\epsilon'_1 \sim \epsilon' \sim \gamma\epsilon$

back in lab frame
- boost $\rightarrow$ scattered photons beamed forward
- scattered photon energy *boosted* to $\epsilon_1 \sim \gamma\epsilon'_1 \sim \gamma^2\epsilon$

Q: implications for blazar spectra?

Inverse Compton Power for Single-Electron Scattering

Consider a relativistic electron (γ, β)
incident on an isotropic distribution of ambient photons

Order of magnitude estimate of *power* into inverse Compton
- if typical ambient photon energy is ϵ
  then typical *upscattered energy* is $\epsilon_1 \sim \gamma^2 \epsilon$
- if ambient photon number density is $n_{\rm ph}$
  then *scattering rate per electron* is $\Gamma = n_{\rm ph} \sigma_{\rm T} c$ *Q: why?*

thus expect power = rate of energy into inverse Compton

$$\frac{dE_{1,\rm upscatter}}{dt} \sim \Gamma \ \epsilon_1 \sim \gamma^2 \epsilon n_{\rm ph} \sigma_{\rm T} c \sim \gamma^2 \sigma_{\rm T} c u_{\rm ph} \tag{1}$$

where $u_{\rm ph} = \langle \epsilon \rangle \, n_{\rm ph}$ is the ambient photon
energy density in the lab (observer) frame

Q: but what about scattering "removal" of incident photons?

upscattering "removes" some photons from ambient distribution

removal rate is scattering rate per electron: $\Gamma = n_{\text{ph}} \sigma_{\text{T}} c$

and thus rate of energy "removal" per electron is

$$\frac{dE_{1,\text{init}}}{dt} = -\Gamma \langle \epsilon \rangle = -\sigma_{\text{T}}\ c\ \langle \epsilon \rangle\, n_{\text{ph}} = -\sigma_{\text{T}}\ c\ u_{\text{ph}} \tag{2}$$

because $\langle \epsilon \rangle \equiv u_{\text{ph}}/n_{\text{ph}}$

Note that

$$\frac{dE_{1,\text{upscatter}}}{dt} \simeq \gamma^2 \left| \frac{dE_{1,\text{init}}}{dt} \right| \tag{3}$$

$\longrightarrow$ for $\gamma \gg 1$, large net energy gain!

net inverse Compton power per electron, when done carefully:

$$P_{\text{Compt}} = \frac{4}{3} \sigma_{\text{T}}\ c\ \gamma^2\ \beta^2\ u_{\text{ph}} \tag{4}$$

Q: note any family resemblances?

Synchrotron vs Compton Power

We found the single-electron inverse Compton power to be

$$P_{\text{Compt}} = \frac{4}{3}\sigma_{\text{T}}\ c\ \gamma^2\ \beta^2\ u_{\text{ph}} \tag{5}$$

but recall *synchrotron power*

$$P_{\text{synch}} = \frac{4}{3}\sigma_{\text{T}}\ c\ \gamma^2\ \beta^2\ u_B \tag{6}$$

formally identical! and note that

$$\frac{P_{\text{synch}}}{P_{\text{Compt}}} = \frac{u_B}{u_{\text{ph}}} \tag{7}$$

for *any* electron velocity as long as $\gamma\epsilon \ll m_e c^2$

∞ we turn next to spectra: good time to ask
Q: what is conserved in Compton scattering? implications?

Inverse Compton Spectra: Monoenergetic Case

in Compton scattering, the *number of photons is conserved*
i.e., ambient photons given new energies, momenta
but neither created nor destroyed

thus: the photon *number* emission coefficient $\mathcal{J}(\epsilon_1)$
must have $4\pi \int \mathcal{J}(\epsilon_1)\ d\epsilon_1 =$ number of scatterings per unit volume
and $4\pi \int (\epsilon_1 - \epsilon)\ \mathcal{J}(\epsilon_1)\ d\epsilon_1 =$ net Compton power

detailed derivation appears in RL: answer is

$$j(\epsilon_1; \epsilon, \gamma) = \frac{3}{4} N(\gamma)\ \sigma_{\rm T}\ c\ \frac{du_{\rm ph}}{d\epsilon}(\epsilon)\ x\ f(x) \qquad (8)$$

with $N(\gamma) = dN/d\gamma$ the electron flux at γ
and $du_{\rm ph}/d\epsilon$ the ambient photon energy density at ϵ
and $f(x) = 2x \ln x + 1 + x - 2x^2$, with $x = \epsilon_1/(4\gamma^2\epsilon)$

Inverse Compton Scattering: Power-Law Electrons

as usual, assume power-law electron spectrum $N(\gamma) = C\ \gamma^{-p}$

still for a single ambient photon energy
integrate emission coefficient over all electron energies

$$
\begin{aligned}
j(\epsilon_1; \epsilon) &= \int j(\epsilon_1; \epsilon, \gamma) d\gamma && (9) \\
&= \frac{3}{4} \sigma_{\mathrm{T}}\ c\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \int\ x\ f(x)\ N(\gamma) d\gamma && (10)
\end{aligned}
$$

with $x = \epsilon_1/(4\gamma^2\epsilon)$
and where $x\ f(x)$ is peaked, with max at $x = 0.611$

Q: notice a family resemblance?

Q: strategies for doing integral?

Q: anticipated result?

for both IC and synchrotron: spectrum is integral of form

$$j(\epsilon_1;\epsilon) \propto \int G\left(\frac{\epsilon_1}{\gamma^2\epsilon_0}\right)\ \gamma^{-p}\ d\gamma \tag{11}$$

strategy is to change variables to $x = \epsilon_1/(\gamma^2\epsilon_0)$

result factorizes into product of

- dimensionless integral, times
- power law $j \propto (\epsilon_1/\epsilon)^{-(p-1)/2}$

so once again:
peaked emission spectrum for single-energy electron
smoothed to power-law emission spectrum, index $s = (p-1)/2$
for power-law electron energy distribution

full result in RL, guts are (up to numerical factors)

$$4\pi\ j(\epsilon_1;\epsilon) \sim \sigma_{\mathrm{T}}\ c\ C\ \epsilon_1^{-(p-1)/2}\ \epsilon^{(p-1)/2}\ \frac{du_{\mathrm{ph}}}{d\epsilon}(\epsilon) \tag{12}$$

Q: interesting choice of ambient photon distribution?

emission coefficient is

$$4\pi \ j(\epsilon_1;\epsilon) \sim \sigma_{\mathsf{T}} \ c \ C \ \epsilon_1^{-(p-1)/2} \ \epsilon^{(p-1)/2} \ \frac{du_{\mathsf{ph}}}{d\epsilon}(\epsilon) \tag{13}$$

depends on background photon distribution via $du_{\mathsf{ph}}/d\epsilon$

for a thermal (Planck) photon distribution:
- $du/d\epsilon \sim T^4/T \sim T^3$, and
- $\epsilon^{(p-1)/2} \sim T^{(p-1)/2}$ and so

expect temperature scaling $j \sim T^{3+(p-1)/2} = T^{(p+5)/2}$

in fact:

$$4\pi \ j(\epsilon_1) = 4\pi \int j(\epsilon_1;\epsilon) \sim \frac{\sigma_{\mathsf{T}} \ C}{h^3 c^2} (kT)^{(p+5)/2} \ \epsilon_1^{-(p-1)/2} \tag{14}$$

Awesome Examples

`www:  Fermi sky movie:  mystery object`

Q: what strikes you?

Q: how does the mystery object radiate > 100 MeV *photons?*

`www:  WMAP Haze`

Q: what strikes you?

haze spectrum: $\propto \nu^{-0.5}$, flatter than usual synchrotron

Q: what electron index would this imply?

Q: if electrons continue to high E*, what should we see?*

`www:  Fermi search for that feature`

Inverse Compton: Non-Relativistic Electrons

if electrons are nonrelativistic
but still on average more energetic than the photons
we have $\beta = v/c \ll 1$
and $\gamma \approx 1 + \beta^2/2 + \cdots$, so that

$$P_{\rm Compt} = \frac{4}{3}\sigma_{\rm T}\ c\ \gamma^2\ \beta^2\ u_{\rm ph} \approx \frac{4}{3}\sigma_{\rm T}\ c\ \beta^2\ u_{\rm ph}\ +\ \vartheta(\beta^4) \tag{15}$$

if electrons has a **thermal velocity distribution** at T_e
then velocities have Maxwell-Boltzmann distribution $e^{-v^2/2v_T^2} v^2\ dv$
with $v_T^2 = kT_e/m_e$, and so averaging, we get

$$\left\langle v^2 \right\rangle = 3v_T^2 = 3\frac{kT_e}{m_e} \tag{16}$$

and thus

$$\left\langle P_{\rm Compt} \right\rangle = 4\sigma_{\rm T}\ c\ \frac{kT_e}{m_e c^2}\ u_{\rm ph} \tag{17}$$

Sunyaev-Zel'dovich Effect

The CMB Reprocessed: Hot Intracluster Gas

CMB is cosmic photosphere: "as far as the eye can see"
CMB created long ago, comes from far away

- all other observable cosmic objects are in *foreground*
- CMB passes through all of the observable universe

Sunyaev & Zel'dovich:
what happens when CMB passes through hot gas *Q: examples?*

consider gas of electrons at temperature $T_e \gg T_{\text{cmb}}$
but where $kT_e \ll m_e c^2$ *Q: how good an approximation is this?*

Q: what's probability for scattering of CMB photon with ν*?*

CMB Scattering by Intracluster Gas

mean free path is that for Thompson scattering:
$\ell_\nu^{-1} = \alpha_\nu = n_e \sigma_{\mathrm{T}}$ independent of frequency
and thus optical depth is integral over cloud sightline

$$\tau_\nu = \int \alpha_\nu \; ds = \sigma_{\mathrm{T}} \int n_e \; ds \tag{18}$$

thus transmission probability is $e^{-\tau_\nu}$, and so
absorption probability is $1 - e^{-\tau_\nu}$

but for galaxy clusters: $\tau < 10^{-3} \ll 1$,
and so *absorption probability* is just τ
Q: implications?
Q: effect of scattering if electrons cold, scattering is elastic?
Q: what if electrons are hot?

if electrons are hot, they transfer energy to CMB photons
change temperature pattern, in frequency-dependent way

What is net change in energy?
initial photon energy density is $u_0 = u_{\rm cmb} = 4\pi B(T_{\rm cmb})/c$
power transfer per electron is $P_{\rm Compt} = 4(kT_e/m_e c^2)\sigma_{\rm T} c\ u_0$, so

$$\frac{\partial u}{\partial t} = P_{\rm Compt}\ n_e = 4\frac{kT_e}{m_e c^2}\sigma_{\rm T} c\ u_0\ n_e \tag{19}$$

and thus net energy density change

$$\Delta u = 4\sigma_{\rm T}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \tag{20}$$

Q: implications?

CMB energy density change through cluster

$$\Delta u = 4\sigma_{\mathrm{T}}\ u_0 \int \frac{n_e\ kT_e}{m_e c^2} ds = 4\frac{kT_e}{m_e c^2}\tau\ u_0 \equiv 4y\ u_0 \qquad (21)$$

- dimensionless **Compton-y parameter**

$$y \equiv \sigma_{\mathrm{T}} \int \frac{n_e\ kT_e}{m_e c^2} ds \simeq \tau \frac{kT_e}{m_e c^2} \simeq 3\tau\beta^2 \qquad (22)$$

- note $n_e kT_e = P_e$ electron pressure
  $\rightarrow y$ set by line-of-sight pressure

fractional change in (integrated) energy density $\Delta u / u_0 = 4y$
- positive change $\rightarrow$ (small) net heating of CMB photons
- since $u \propto I$, this also means

$$\frac{\Delta I_{\mathrm{cmb}}}{I_{\mathrm{cmb}}} = 4y \qquad (23)$$

cluster generated net CMB "hotspot"

Q: expected frequency dependence?

SZ Effect: Frequency Dependence

on average, we expect photons to gain energy
adding intensity at high ν, at the expense of low ν

but note that in isotropic electron population
- some scatterings will reduce energy
- while others will increase it

detailed derivation is involved:
- allow for ordinary and stimulated emission
- include effects of electron energy distribution
- allow for Compton shift in energy
- use Thomson (Klein-Nishina) angular distribution

full equation (Kompaneets and generalization)
describes *"diffusion" in energy (frequency) space*
but key aspect comes from basic Compton property *Q: namely?*