Astronomy 501: Radiative Processes

Lecture 5

Sept 7, 2018

Announcements:

- **Problem Set 1** due now
- **Problem Set 2** out, due next Friday

Last time: the mighty equation of radiation transfer

Q: what is it?

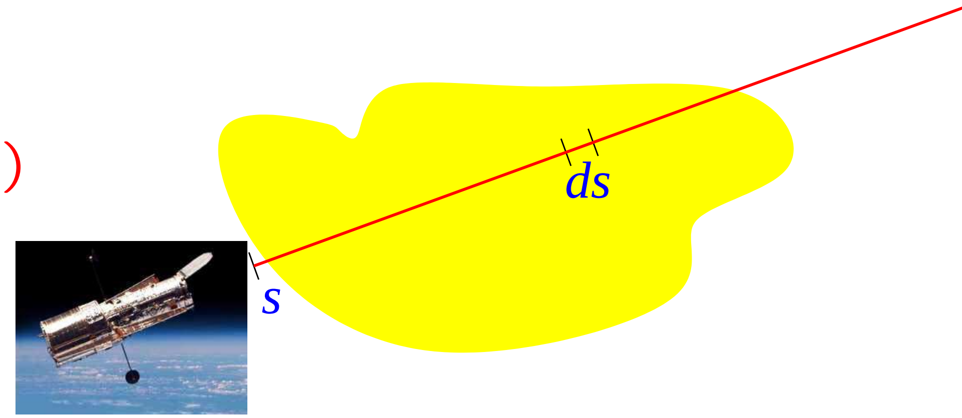
Q: what is optical depth? column density? physical significance?

Q: what is source function? why is it important?

equation of radiation transfer

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu = -\alpha_\nu (I_\nu - S_\nu)$$

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$



where source function set by emission and absorption properties

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = j_\nu \, \ell_{\mathrm{mfp},\nu} \tag{1}$$

and optical depth/thickness $d\tau_\nu = \alpha_\nu \ ds$, so that

$$\tau_\nu = \int_{s_0}^{s} \alpha_\nu \ ds = \int_{s_0}^{s} \frac{ds}{\ell_{\mathrm{mfp},\nu}} = \sigma_\nu \, N_{\mathrm{a}} \tag{2}$$

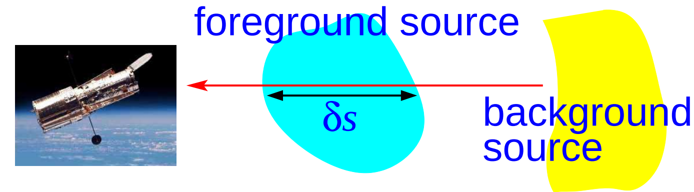
with column density

$$N_{\mathrm{a}} = \int_{s_0}^{s} n_{\mathrm{a}} \ ds \tag{3}$$

Q: optically thin–physical meaning? what see? what learn? everyday examples?

An Optically Thin Source

- $\tau_\nu \ll 1$: **optically thin** = **transparent**
- consider thin foreground source j_ν
  illuminated by background $I_\nu(0)$



$$I_\nu \approx (1 - \tau_\nu)\, I_\nu(0) + j_\nu\, \delta s$$

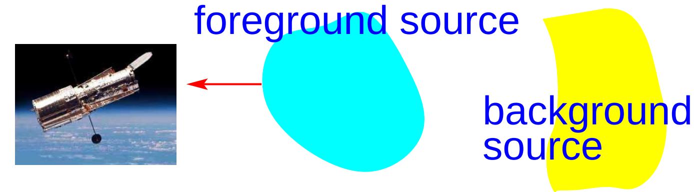
- **physical interpretation:** observed intensity combines
  slightly diminished background emission
  + foreground source along sightline
- **sky view**: foreground object with background shining through
- *all* of foreground source volume is projected on sky!
  useful for probing source interior and global properties

Everyday examples: air, wispy clouds, thin smoke, shallow water

Q: compare/contrast with optically thick case:
physical meaning? what see? what learn?

An Optical Thick Source

- $\tau_\nu \gg 1$: **optically thick** = **opaque**



$$I_\nu \to S_\nu = j_\nu \, \ell_{\text{mfp},\nu}$$

optically thick intensity is source function!

- **sky view**: source **surface**, if solid
  or outermost skin to depth $\ell_{\text{mfp},\nu}$
- measure surface S_ν
- *no information* about interior or background

Everyday examples: most solid objects, deep/muddy water

Note: α_ν and thus τ_ν spectral dependence can mean
the same object can be thin at some ν and thick at others!

Optical Depth and Astrophysical Objects

Q: examples of resolved optically thick astronomical objects?

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Q: Observe and interpret:

`www:  supernova remnants in optical`

`www:  Orion nebula in optical`

`www:  multiwavelength dark cloud Barnard 68`

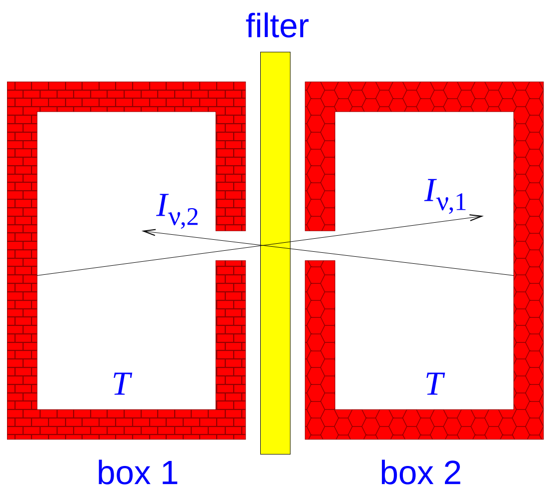
`www:  galaxies`

`www:  all sky:  optical, microwave, near infrared`

Gossip Break: Chandra Story

Blackbody Radiation

Universality of Blackbody Radiation



From last time, experiment: two boxes
each in thermodynamic equilibrium at T

separate boxes by *filter passing only frequency* ν
radiation from each box incident on screen

if both boxes at *same* $T \Rightarrow$ *no net energy transfer*
but this requires $I_{\nu,1} = I_{\nu,2}$ and so the radiation is:

- independent of the composition of the box
- a universal function of T
- **blackbody radiation** with intensity $I_\nu^{\text{blackbody}} \equiv B_\nu(T)$

∞ Lesson: radiation has energy, exchanges it with environment
$\rightarrow$ *radiation can be treated thermodynamically*

Thermodynamics Recap

First Law of Thermodynamics: heat is work!
adding *heat energy* dQ to system changes
system *energy* U and/or *pressure* P:

$$dQ = dU + pdV \tag{4}$$

Second Law of Thermodynamics: heat is entropy!

$$T\ dS = dQ \tag{5}$$

together

$$T\ dS = dU + P\,dV \tag{6}$$

and thus entropy $S = S(T, V)$ obeys

$$dS = \frac{dU}{T} + \frac{P}{T}dV \tag{7}$$

entropy $S = S(T, V)$ obeys

$$dS = \frac{dU}{T} + \frac{P}{T} dV \tag{8}$$

and thus we have

$$\partial_T S = \frac{\partial_T U}{T} \tag{9}$$

$$\partial_V S = \frac{\partial_V U + P}{T} \tag{10}$$

which means

$$\partial_V \partial_T S = \frac{\partial_V \partial_T U}{T} \tag{11}$$

$$\partial_T \partial_V S = \frac{\partial_T \partial_V U}{T} - \frac{\partial_V U}{T^2} + \partial_T \left(\frac{P}{T}\right) \tag{12}$$

but mix partial derivatives equal, e.g., $\partial_V \partial_T S = \partial_T \partial_V S$,
and note that $\partial_V U|_T = u$ energy density, so

$$u = T^2 \, \partial_T \left(\frac{P}{T}\right) \tag{13}$$

Radiation Thermodynamics

general thermodynamic considerations give:

$$u = T^2 \; \partial_T \left(\frac{P}{T} \right) \tag{14}$$

now specialize to *radiation:* $P = P(T) = u(T)/3$

$$T \frac{d}{dT} \left(\frac{u}{T} \right) = 3 \frac{u}{T} \tag{15}$$

which gives

$$\frac{d(u/T)}{u/T} \; = \; 3 \; \frac{dT}{T} \tag{16}$$

$$\ln \left(\frac{u}{T} \right) \; = \; 3 \ln(T) + \ln(a) \tag{17}$$

$$u(T) \; = \; a \; T^4 \tag{18}$$

This is Huge! *Q: why?*

radiation energy density gotten just from theromdynamic considerations

$$u(T) = a\, T^4 \tag{19}$$

- $u(T) \propto T^4$: strong T dependence!
- also get total intensity and flux!

$$B(T) = \frac{ac}{4\pi}\, T^4 \tag{20}$$

$$F(T) = \pi B(T) = \frac{ac}{4}\, T^4 \tag{21}$$

- where a is the "radiation constant"
  value not determined by thermodynamics alone

Note: *blackbody quantities fixed entirely by* T
no adjustable parameters!

Radiation Entropy

Using $U = aT^4 V$ and $P = u/3$, can solve for **radiation entropy**

$$S_{\rm rad} = \frac{4}{3} a T^3 \ V \qquad (22)$$

and thus *entropy density* $s_{\rm rad}(T) = S/V = 4/3 \ aT^3$

if entropy $S_{\rm rad}$ constant in a parcel of radiation
$\longrightarrow$ *adiabatic* process:

$$T_{\rm adiabat} \propto V^{-1/3} \qquad (23)$$

$$P_{\rm adiabat} \propto T^4_{\rm adiabat} \propto V^{-4/3} \qquad (24)$$

writing $P \propto V^{-\gamma}$, we have
an *adiabatic index* $\gamma_{\rm rad} = 4/3$

Q: but how do we get the radiation constant a?

The Quantum Mechanics of Blackbody Radiation

to have deeper understanding of radiation thermodynamics
and to find radiation constant a
need to study radiation in more detail
$\longrightarrow$ need physical picture of radiation

can try classical description: radiation as EM waves
different frequencies (“modes”) all thermally excited
$\longrightarrow$ gives somewhat wrong answers, e.g., $u(T) = 8\pi\, kT/c^3 \int_0^\infty \nu^2\, d\nu \to \infty$
“ultraviolet catastrophe”

Historically, this disaster drove Planck & Einstein to a new
microscopic picture of quanta: **photons**
$\longrightarrow$ of course this gives correct blackbody description
in a *statistical mechanics* description of photons