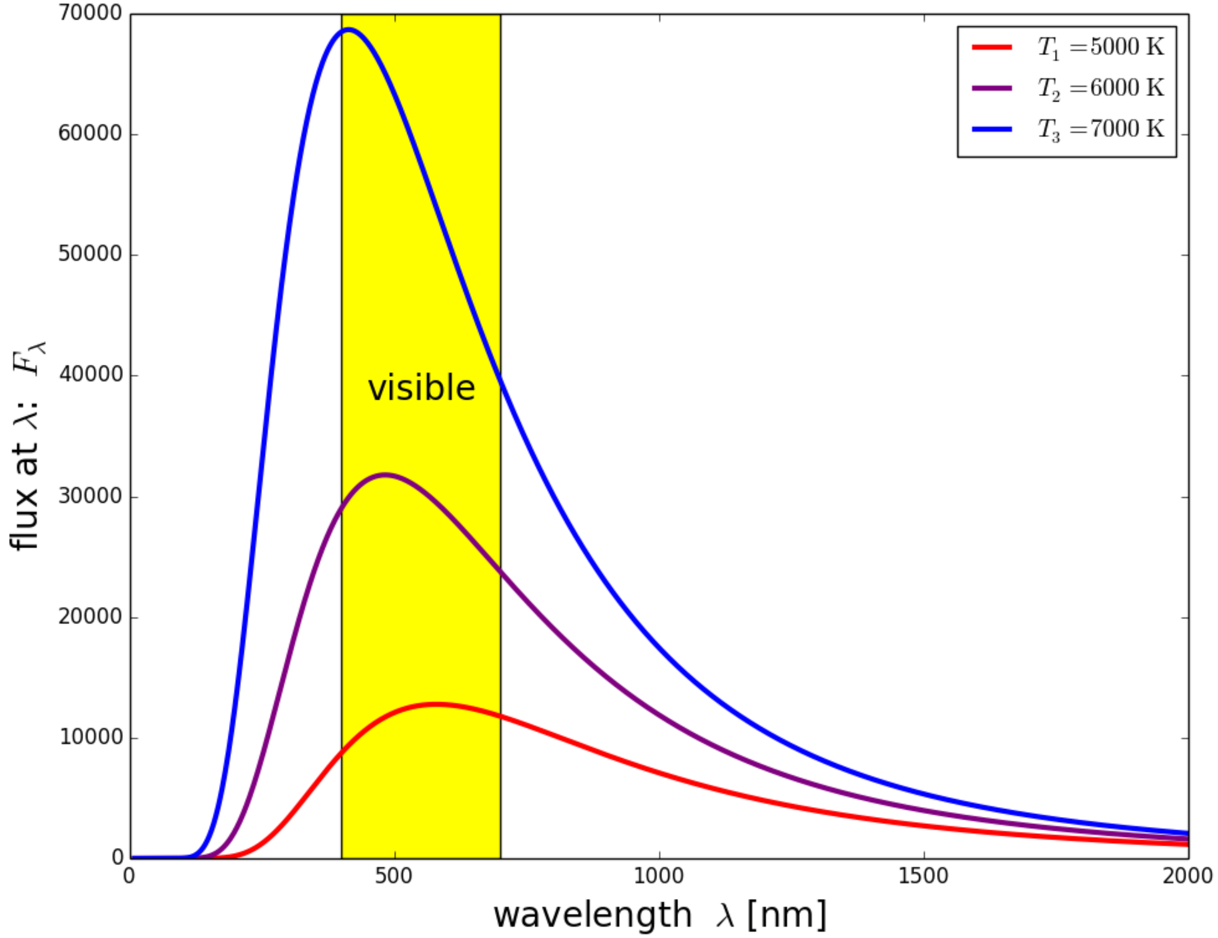
Astro 404
Lecture 5
Sept. 6, 2019

Announcements:
- **Problem Set 1 due on Compass today at 5:00pm**
- **Problem Set 2 posted, due next Friday**
- iClicker registration link now on Compass

Last time:
- star distances
  Q: what's the "gold plated" way to measure distance?
  Q: why can't we use this technique for all cosmic objects?
- stellar spectra and colors: diverse patterns
  but continuum spectra with absorption line features
- light and matter in thermodynamic equilibrium:
  blackbody radiation
  Q: what's a blackbody? what is its radiation pattern?

Blackbody Spectrum

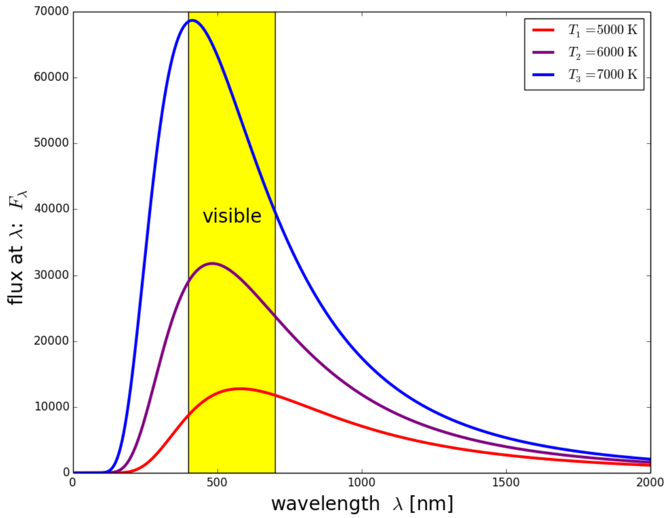


Q: *trends vs* λ?

Q: *trends vs* T?

Q: *hotter* = ???

Thermal Spectrum: Light as Thermometer!



for blackbody at temperature T:
peak λ = color seen:
$\lambda_{\text{peak}} \propto 1/T$
where T is *absolute* temperature
in Kelvin units

Wien's law:

$$\lambda_{\text{peak}} = \frac{0.29 \text{ cm K}}{T} \propto \frac{1}{T} \tag{1}$$

hotter → more blue → shorter λ

⇒ spectrum as **thermometer**
color measures temperature

iClicker Poll: Human Radiation

Humans have temperature $T > 0$

Do humans emit blackbody radiation?

- A no: T_{human} is too low to emit significant radiation
- B no: T_{human} is too high to emit significant radiation
- C yes: human radiation exists, but is invisible
- D yes: human radiation is visible seen all the time! perceived as hair color, eye color, etc.

any object with $T > 0$ emits thermal radiation!
but not always visible to naked eye

Human radiation:
$\lambda_{\text{peak}} = 0.29 \text{ cm K}/300 \text{ K} \approx 10^{-3} \text{ cm} - 10^{-5} \text{ m}$
`www: EM spectrum`
IR!
`www: IR gallery--coffee, people, puppy`

not only good for household objects, but also for galaxies
`www: multiwavelength star clusters, galaxies`

X-ray emission seen from Cassiopeia A
`www: Cas A spectrum`
some of this is thermal emission: how hot is it?

$T \sim 0.29 \text{ cm K}/10^{-7} \text{ cm} = 3 \times 10^{6} \text{ K !}$

Q: what might have made it so hot?

Q: quick-and-dirty way to estimate star temperature?
Q: what part of the star has this temperature?

Stellar Thermometry: Color Temperature

recall – broadband fluxes give "poor person's spectrum"
pro: broad passband filters don't need as much light
so can measure quickly
con: don't get detailed spectra (lines, etc)

also recall: *color* ⇔ *flux ratios*
usually expressed as *color index*
for bands 1 and 2: $m_2 - m_1 = 2.5 \log_{10}(F_1/F_2)$

so if spectrum is well approximated by blackbody
Wien's law: color index estimates **color temperature**
more specifically: the average surface temperature

Q: compare Betelgeuse vs Rigel?
Q: estimate Sun's $T_{\rm color}$?

Color Temperature: Examples

qualitatively:

in Orion, reddish Betelgeuse is cooler than bluish Rigel

in Gemini: red Castor cooler than blue Pollux

quantitatively:

the *Sun*'s color temperature $T_{\text{color},\odot} \approx 5900$ K

check: white sunlight → peaks min-optical $\lambda_{\text{max}} \sim 500$ nm

gives $T_{\text{color}} = 0.3 \text{ mm K}/\lambda_{\text{max}} \sim 6000$ K yay!

Blackbody Flux

hotter objects are glow *brighter* than cooler ones
i.e., blackbody surface flux increases with T

blackbody flux F_{bb}: summed (integrated) over all λ

$$F_{\text{surface}}(T) \overset{\text{blackbody}}{\equiv} F_{bb}(T) = \sigma T^{4} \quad \boxed{\textbf{Stefan-Boltzmann law}} \tag{2}$$

- applies to *surface* of blackbody (solid, liquid, dense gas)
- Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ Watt m^{-2} K^{-4}
- note very strong dependence on (absolute) T!
- note that blackbody flux depends *only* on emitter T
  independent of composition

∞

Q: for blackbody sphere of radius R, sum of flux over surface?

Stellar Thermometry II: Effective Temperature

for a *blackbody sphere* of radius R
sum (really, integration) of flux over surface gives **luminosity**!

$$4\pi R^2\, B = 4\pi R^2\ \sigma T^4 = \text{flux} \times \text{area} = \text{power} = L \tag{3}$$

for a real star, if R known, can compute **effective temperature**

$$T_{\text{eff}} = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4} \tag{4}$$

Q: What is T_{eff} for a perfect blackbody?
Q: What if $T_{\text{eff}} \neq$ color temperature? How could that be?

Temperatures of Real Stars

if star were perfect blackbody:
color temperature $= T_{\rm eff} =$ true thermodynamic temperature T

but real star spectra are not perfect blackbodies
so in general, none of these "temperatures" agree!

in practice: color temp vs $T_{\rm eff}$ tests blackbody approximation
$T_{{\rm eff},\odot} = 5780$ K, close to but not same as $T_{{\rm color},\odot}$
blackbody approximation not too bad!

better: make detailed model of stellar atmosphere
  compute spectrum in presence of lines
  and changing temperature with depth
use this to infer temperature structure

A Census of Stars

We now have the technology to take a census of stars!

For large sample of stars, measure L and T for each
plot each star's (T, L) point on diagram of L vs T

Some possible trends:
- random scatter
- all stars fall onto same point
- tight clump of points
- a line or curve

Q: what would each of these imply?

iClicker Poll: Star Temperature and Luminosity

Vote your conscience!

For large sample of stars, measure L and T for each
plot points on diagram of L vs T
What will the data show?

A random scatter: stars have large range of L, and of T, and in any combination

B tight clump of points: stars are nearly identical, all with very similar L and T

C a clear trend: stars have large range of L and of T but the two vary together (correlated)

D none of the above

A Stellar Census: Hertzsprung-Russell Diagram

Hertzsprung-Russell: plot L vs$T$ for lotsa stars
really, abs mag M_V vs spectra type
but these are equivalent to L and T

`www: Gaia H-R diagrams for 3700 / 30,000 / 2 million nearest stars`

Q: what patterns do you notice?
Q: where are most stars?
Q: where is the Sun?
Q: how does the Sun compare to other stars?

Hertzsprung-Russell Diagram

for a “fair sample” of stars
(i.e., not a specially picked cluster)
trends emerge

`www: Gaia HRD for 4+ million stars`

most stars ($\sim 90\%$) fall on curve: “main sequence”
  (including the Sun!); “dwarfs”
most of the rest: cooler but more luminous: “giants”
  Q: how do we know they are giant?
a rare few: hot but luminous: “supergiants”
not rare but dim and hard to find:
  very hot but very low-L objects: “white dwarfs”
  Q: how do we know they are teeny?

note huge range in luminosity – more than $10^{-4} L_\odot < L < 10^4 L_\odot$
and in temperature: $3000 \text{ K} < T < 30,000 \text{ K}$

imagine some main sequence stars are binaries (pairs)
Q: if unresolved (appear as one star), effect on HR diagram?

Hertzsprung-Russell Diagram and Binaries

if a single Gaia point source is an unresolved binary
flux sums light both stars

if stars are identical:

- color same for both–*unchanged* in unresolved binary
- binary's luminosity is 2× single-star luminosity
  PS1: absolute magnitude brighter by ≈ 0.75 mag
- net result: HRD vertical shift by ≈ 0.75 mag

`www: Gaia HRD with` 0.75 `mag shift shown`
Q: lesson?

Q: what does the HR diagram tell us about the Sun?

H-R and the Sun

The Sun on H-R diagram:
- found on the main sequence
- position is in the middle of the curve

but the main sequence is where most stars are found!

thus: *the Sun is a typical star!*
- lies in heart of main sequence L vs T trend
- neither most nor least luminous, not hottest or coolest

Other questions arise:
- *why* do stars lie on the main sequence?
- what controls their position on the diagram?
- what's up with the giants, supergiants, and white dwarfs?

...stay tuned

Weighing Stars

We saw that clever measurements give a stars
- luminosity
- surface temperature
- radius

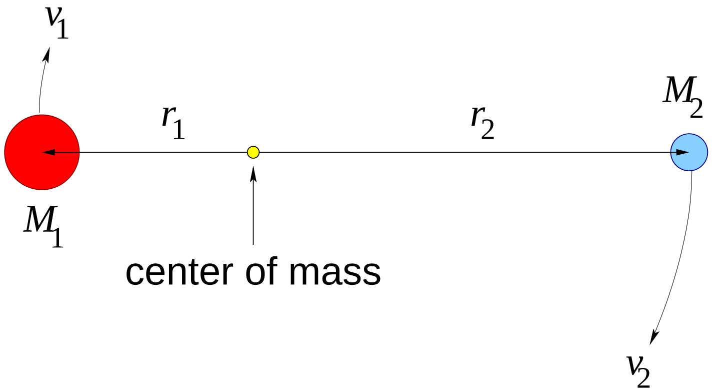
What about mass?

For single stars:
mass determination difficult, very indirect
but we *can* find masses for stars in **binary** systems

Measuring Star Masses: Binary Systems

for single stars without companions: can't accurately find mass

But can find masses for **binary** systems:
two stars orbiting common center of mass



Binary Star Systems

most stars are in gravitationally bound multiple systems
binary pairs are most common, but 3 of more star systems exist

observational classes:
visual binaries both stars resolved
  can track orbit around each other
astrometric binaries only the brighter star resolved
  moves in orbit around unseen partner
spectroscopic binaries appear as single point in scope
  but spectrum shows lines that split into pairs
  due to different Doppler shifts along sightline
eclipsing binary stars orbit plane seen edge-on
  when aligned one blocks the other

binary orbit info + gravity physics → star masses!

Q: what gravity force between point masses?

Universal Gravitation: Point Masses

consider point masses m_1 and m_2 at separation $\vec{r}$
gravitational force of 2 on 1:

$$\vec{F}_1 = -\frac{Gm_1m_2}{r^2}\hat{r} \tag{5}$$

- *inverse square* *Q: why minus sign?*
- $\hat{r} = \vec{r}/r$: unit vector along $\vec{r}$
  force is along line between particle centers: *central force*

Q: motion in center of mass system?
Q: equation of motion?

Motion in Center of Mass System

PS2: for two interacting particles with no external forces

- center of mass feels no net force
- particles stay on opposite sides of center of mass
- relative motion: particle separation $\vec{r}$ set by

$$\mu \frac{d^2\vec{r}}{dt^2} = \vec{F} \tag{6}$$

where

- $\mu = m_1 m_2/(m_1 + m_2)$ is *reduced mass*
- $\vec{F}$ is force between particles

so for 2-body gravitational interaction

$$\mu \frac{d^2\vec{r}}{dt^2} = -\frac{G m_1 m_2}{r^2}\hat{r} \tag{7}$$

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G(m_1 + m_2)}{r^2}\hat{r} \tag{8}$$

Q: conditions for circular motion?

Kepler Motion: Circular Case

for circular motion

- particle separation unchanged: constant radius $r = a$
- all *acceleration* is radial and thus centripetal v^2/r
- circular speed $v = v_{\mathrm{circ}} = 2\pi a/P = a\omega$
  with orbit period P

$$\frac{d^2\vec{r}}{dt^2} = -\frac{G(m_1+m_2)}{r^2}\hat{r} \tag{9}$$

$$\frac{v_{\mathrm{circ}}^2}{a} = \frac{G(m_1+m_2)}{a^2} \tag{10}$$

and motion has

- constant circular speed v_{circ} and angular speed ω
- $4\pi^2 a^3 = GMP^2$

Q: generalize to non-circular bound orbits?

Kepler's Laws

in general, orbits of gravitationally bound point masses:

I. orbit in space:
ellipses, with center of mass at one focus

II. orbit speed: sweeps equal areas in equal time

III. period and size related:

$$4\pi^2 a^3 = G(m_1 + m_2)P^2 \tag{11}$$

Q: how does the circular case fit in?
Q: equal mass particle motions about COM? unequal masses?

Motion About Center of Mass

COM positions: $r_1/r_2 = m_2/m_1$

measure P, and r_1, r_2
→ find mass ratio

problem: must measure r's *Q: how?*

Types of Binary Stars

visual binary

can see both stars!

`www: visual binary orbit`

eclipsing binary

stars pass in front of each other

can see in light curve:

diagram: light curve → get rs from timing of eclipses

spectroscopic binary

periodic Doppler shifts in spectrum

see $\Delta\lambda_1$, $\Delta\lambda_2$

→ radial velocity $v_r/c = \Delta\lambda/\lambda_0$

then $v_1 = r_1\omega = 2\pi r_1/P$

can solve for r!

iClicker Poll: Star Masses

Vote your conscience!

Measure *mass* M and and *luminosity* L for main sequence stars plot L vs M; each star is one (M, L) point
What trend(s) will we find?

A M and L tightly related: L increases with M
(more massive = more luminous)

B M and L tightly related: L decreases with M
(more massive = less luminous)

C M and L unrelated: large spread in L for each M

D none of the above

Q: what does this mean for H-R diagram?

Star Masses

For main sequence stars:

data show very tight trend: $\log L = \alpha \log M + const$, $\alpha \approx 3.5$

$\rightarrow$ solve to find $L \propto M^{\alpha}$, and use $L(M_{\odot}) = L_{\odot}$:

$$L = \left(\frac{M}{M_{\odot}}\right)^{\alpha} L_{\odot} \approx \left(\frac{M}{M_{\odot}}\right)^{3.5} L_{\odot} \qquad (12)$$

very strong increase of L with M!

example: $L(2M_{\odot}) = 2^{3.5} L_{\odot} = 11 L_{\odot}$

Q: which has more total lum?

2 stars at $1M_{\odot}$, or 1 at $2M_{\odot}$?

Lesson for H-R diagram: main sequence is really
a sequence in mass