Astro 404
Lecture 8
Sept. 13, 2019

Announcements:

- **Problem Set2 due online today at 5:00pm**
- Erratum: in Table, wrong absolute mag for Aldebaran
  should read $M_V = -0.64$ mag
  either value will be accepted by graders
- **Problem Set 3 out today, due next Friday**

Last time:
masses of main sequence stars: *Q: trend? implications?*
lifespans of main sequence stars: *Q: what determines? trends?*
if star cluster born with range of masses *Q: MS over time?*
Q: how to test?

for **main sequence stars**

mass–luminosity relation is roughly a power law

$$L \approx \left(\frac{M}{M_\odot}\right)^{\nu} L_\odot \qquad (1)$$

where $\nu \approx 3.5$ for masses near $1 M_\odot$

energy conservation: MS lifespan $\tau \approx E_{\mathrm{fuel}}/L$

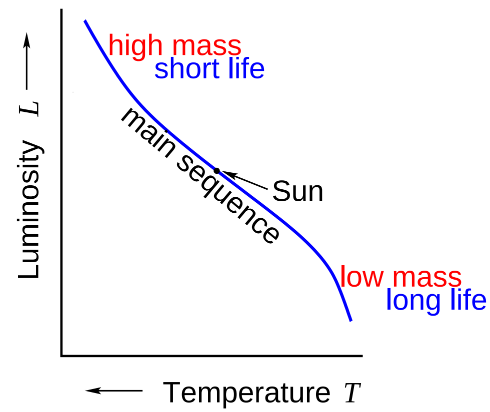
for $E_{\mathrm{fuel}} \propto M$, we infer

$$\tau(M) \approx \frac{E_{\mathrm{fuel}}}{L} \sim \frac{1}{M^{\nu-1}} \longrightarrow \frac{1}{M^{2.5}} \qquad (2)$$

star cluster evolution:

- highest mass stars $\rightarrow$ huge L $\rightarrow$ short lifespan $\rightarrow$ die first
- then next most massive stars
- process continues for all stars gone with $\tau(M) \leq$ cluster age

Star Cluster H-R Diagrams: Evolution Revealed!



our predictions confirmed!

- *young open clusters* have *full MS*
- *old globular clusters* have lower MS but *no stars with above $\sim 1 M_{\odot}$*

more clues:

- some open clusters have no giant branch!
- other open clusters lack highest masses, and have some (super)giants
- open clusters have very few white dwarfs
- globular clusters have no supergiants but full giant branch

Q: what does this suggest?

(Super)Giant Phase Follows Main Sequence

giant branch and white dwarf sequence emerge as clusters age at expense of ever more of upper main sequence

implications: *this is due to stellar evolution!*

- *highest mass stars quickly evolve from MS to supergiants*
- lower masses progressively evolve from MS to giants
- globular cluster MS "turnoff" near $1M_{\odot}$ suggests stars with $M \lesssim 1M_{\odot}$ live a very long time!

- white dwarfs also arise after MS

Decoding the H-R Diagram: Evolution Revealed!

main sequence is *ordered by mass*
and star properties do not change much during MS phase
in particular: a star does not move up or down the MS
Q: why not? what would that mean for clusters?

giants and supergiants are phase after MS
most massive MS stars $\rightarrow$ supergiants
then, ever more slowly, MS $\rightarrow$ giants
until very long MS lifespans at $M \lesssim 1 M_\odot$

and thus **stellar evolution is stellar transformation**

- *stars change color, temperature, luminosity, and size over time*
- these changes are modest and slow during main sequence
  but dramatic and increasingly rapid after MS ends

iClicker Poll: Stars and Galaxies

galaxies are made of billions of stars
at optical wavelengths, the light from a galaxy
is dominated by the combined light of its stars

elliptical galaxies are red in color

What does this suggest about elliptical galaxies?

A they are young and mostly made of giant stars

B they are old and mostly made of giant stars

C they are young and mostly made of white dwarfs

D they are old and mostly made of white dwarfs

The Mighty H-R Diagram

note the power of the HR diagram
and the progress we have made
still with very basic physics only

and the HR diagram has more information
which we will uncover in weeks to come

many question remain: how do stars arrive on the MS?
how and why do stars linger on the MS?
why do stars become giants? how do they leave this phase?
what is role of white dwarfs?

to make further progress – *need to develop theory of stars*

A Theory of Stars

Towards a Theory of Stars

our goal: understand the nature and evolution of stars
by building a model of stars including relevant physics

what should a model for a star describe...
Q: at a given moment of a star's life?
Q: over a star's life?

Q: what physics needed to do this?

at any moment on the star's life, account for
⋆ interior structure and composition
⋆ interior and surface motion – dynamics
⋆ light emitted from surface (flux, luminosity, spectrum)
⋆ effects of binary partner if there is one
over time account for how these change

required physics: a *full model* should include

- gravitation
- gas dynamics: compressible fluid
- thermodynamics and statistical mechanics
- electromagnetic radiation and interaction with atoms
- spoiler alert: nuclear and neutrino physics too
- rotation `www: Sun movie`
- magnetic fields `www: solar magnetogram`
- mass loss `www: solar wind`

Yikes! This is a lot! Q: simplifying assumptions?

Basic Stellar Models

begin stellar model building by making
simplest realistic model possible

simplifying assumptions:

- *a single star in isolation*: no binary partner
- *non-rotating*: angular frequency $\Omega_{\rm rot} = 0$
- *non-magnetic*: $\vec{B} = 0$ throughout

so in equilibrium, star has **spherical symmetry**

after building these basic star models
we will see effect of relaxing these assumptions

note: star not a point mass, but extended object
Q: how to physically describe the star's interior?

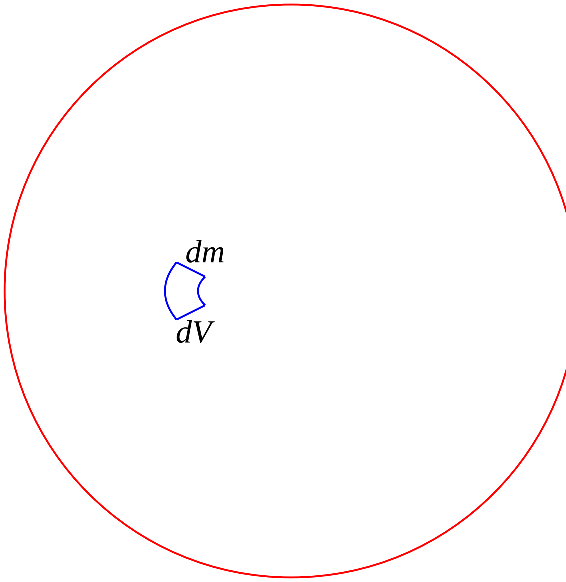
Building Models of Stars

a star is an extended object, so must physically characterize its properties at every radius r

- **mass density** ρ
- **composition** (which elements)
- **temperature** T
- **pressure** P

dependence of each on r: *radial profile*

star composed of gas: a compressible fluid



often useful to consider small parcel of gas *"fluid element"*

with mass dm and volume dV

Q: how are these related?

Mass and Density

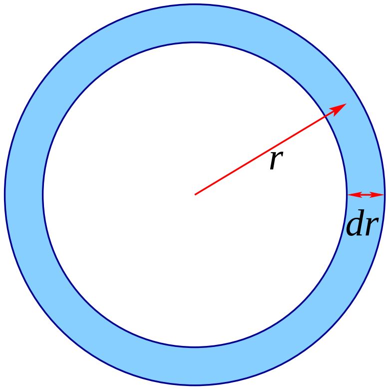
a fluid element with
- *mass* dm and
- *volume* dV

has **mass density**

$$\rho = \frac{dm}{dV}$$

which need not be uniform throughout an object!

in spherical symmetry: volume element in thin shell of radius r and thickness dr is



$$dV \stackrel{\text{sph}}{=} 4\pi r^2 dr = A_{\text{shell}}\ dr$$

and thus

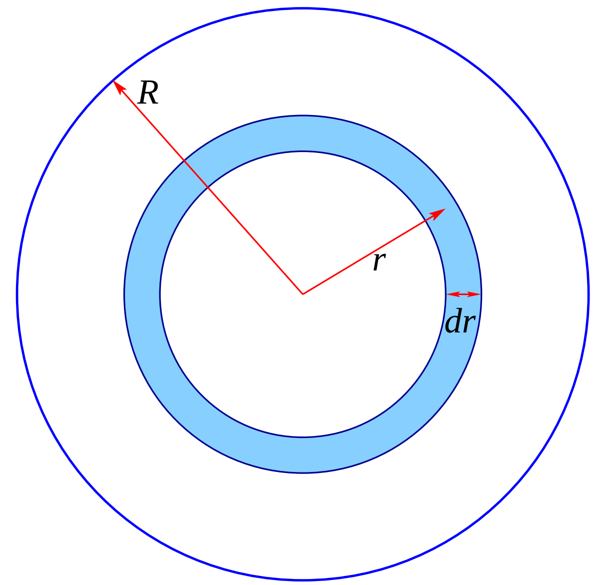
$$\rho \stackrel{\text{sph}}{=} \frac{1}{4\pi r^2}\frac{dm}{dr}$$

Enclosed Mass

for spherical mass distribution:

enclosed mass defined as

mass inside radius r



$$m(r) = \int_0^r \rho \; dV = 4\pi \int_0^r \rho(r) \; r^2 \; dr$$

Q: what is $m(0)$?

Q: as r increases, $m(r)$ behavior?

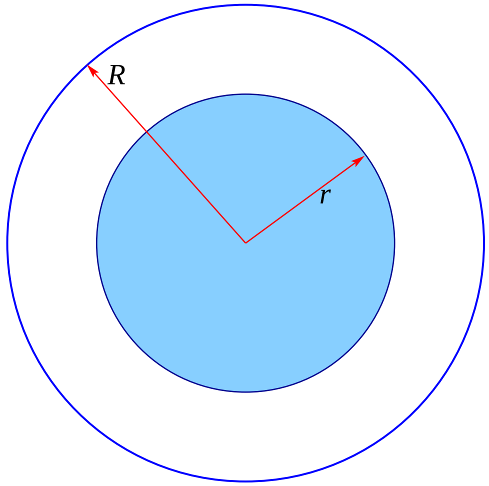
for star of radius R:

Q: what is $m(R)$? what is $m(r)$ for $r > R$?

Q: what changes if star expands or contracts?

Q: what doesn't change?

Mass Coordinate



enclosed mass for *star of radius* R:

$m(r) = 4\pi \int_0^r \rho(r)\ r^2\ dr$

$\rho > 0$, so $m(r)$ grows monotonically with r

- $m(0) = 0$: nothing to enclose at center
- $m(R) = M$: *total mass of star*
- for $r > R$, still $m(r) = M$

for stars: **total mass** M fixed (when mass loss negligible)
but expansion/contraction changes density profile $\rho(r)$

lesson: can label star interior regions with r
but also useful to label star interior via $m(r)$

- $m(r)$ sometimes called **"mass coordinate"**
- tracks stellar matter if expansion or contraction
- sometimes called a *Lagrangian* coordinate ("follows" fluid)

Newtonian Gravitational Field

for point mass M:

- acceleration independent of test mass
- thus only depends on "source" M

formally: can write test mass force $\vec{F}_m = m\vec{g}$
and thus in the presence of a gravity source M
i.e., given the existence and amount *mass*
any and all test particles at point $\vec{r}$ feel acceleration

$$\vec{a} = \vec{g}(\vec{r}) \tag{3}$$

$\Rightarrow$ physical interpretation: each mass M sets up
its own **gravitational field $\vec{g}$** throughout space

Gravity from many sources: Superposition

Thus far: only considered single point masses
what if we add more gravity sources–i.e., more masses?

If one point particle of mass m at $\vec{r}$
gravity is

$$\vec{g} = -\frac{Gm}{r^2}\hat{r} \tag{4}$$

for many particles: use principle of superposition
$\Rightarrow$ take vector sum of gravitational acceleration

bad news: this can be complicated!
good news: spherical symmetry drastically simplifies
best news: you already have the technology in hand
Q: what's that? hint–it was in PHYS 212

Gravitation and Electrostatics: Family Resemblance

how sum up? how do the integral?

You already have the technology! Notice similarity:

	Electrostatics	*Gravity*
"charge"	q	m
force	$qQ/4\pi\epsilon_0 r^2\ \hat{r}$	$-GmM/r^2\ \hat{r}$
field	$\vec{F}_q = q\vec{E}$	$\vec{F}_m = m\vec{g}$

formally identical inverse square law forces!
(except sign, and $\pm q$ allowed, $m \geq 0$)

So: can import electrostatics technology
Memory lane: Gauss' Law from EM
`www: PHYS 212`

Gauss' Law in E&M

consider a point charge Q

enclose in sphere: $\vec{E}$ normal to surface $\vec{S}$

$$\int_S \vec{E} \cdot d\vec{S} = E \int_S dS = \frac{Q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0} \tag{5}$$

miracle: holds for all $\vec{E}$ and surfaces $\vec{S}$

$$\text{electric flux} = \int_S \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\epsilon_0} \tag{6}$$

where q_{enc} is total charge enclosed in surface S

Gauss' Law for gravity: for point mass M

$$\int_S \vec{g} \cdot d\vec{S} = -\frac{GM}{r^2} 4\pi r^2 = -4\pi GM \tag{7}$$

and in general:

$\int_S \vec{g} \cdot d\vec{S} = -4\pi G M_{\text{enc}}$

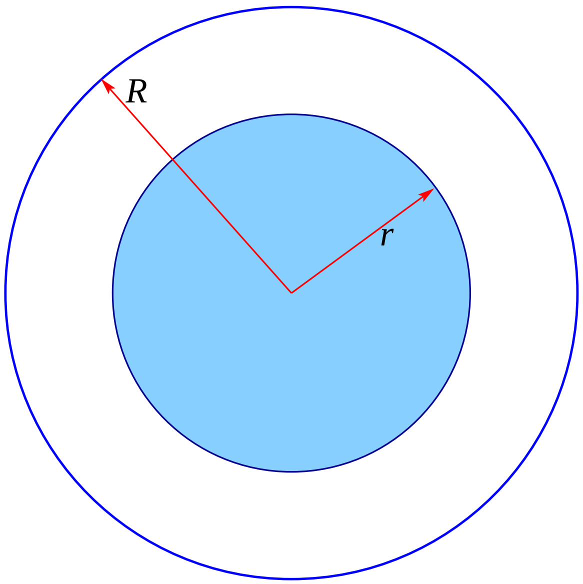
A Gravitating Sphere

spherical mass distribution $\rho(r)$
and $\vec{g}(r, \theta, \phi) = \vec{g}(r)$

Gauss' Law: choose spherical surface

$$\int_S \vec{g} \cdot d\vec{S} = 4\pi r^2 g(r) = -4\pi G m(r) \quad (8)$$

where $m(r) = 4\pi \int dr r^2 \rho(r)$
is the *enclosed mass!*



solve:

$$\vec{g}(r) = -\frac{Gm(r)}{r^2}\hat{r} \tag{9}$$

note similarity to point-source formula

but this works for *any* spherical mass distribution

and works inside, outside mass distribution!

Q: field at center?

Q: field if hollow out inside and you're there?

$\Rightarrow$ field is same as if interior mass concentrated at center!

iClicker Poll: Maximal Gravity

imagine the Earth's density were uniform (constant)

Where would the gravitational acceleration be the strongest?

A at the center

B at the surface

C at the Moon's distance

D none of the above