Astro 404

Lecture 13

Sept. 25, 2019

Announcements:

- **Problem Set 4 due Friday**
  instructor office hours: Today 11am-noon or by appt
- PS 4 typos:
  Q2f should read: $^4\text{He} = \boxed{2p, 2n}$
  Q3c should read: $s = -2$, and eq. (6) is correct
  also, FAQ responses posted on Compass HW Discussion

Last time: Virial theorem

Q: what's that? when does it apply? Lessons?

Virial Theorem: Lessons

equilibrium links thermal and gravitational energy
more compact ↔ hotter

- stellar interiors much hotter than T_{eff}

- expect stellar evolution to include
  phases of “burning” non-gravitational “fuel”
  between phases of gravitational contraction

- (non-relativistic) ideal gas stars are self-regulating: stable

- (non-relativistic) ideal gas stars require time to evolve

- relativistic stars are barely bound, can evolve rapidly
  these stars are unstable!

Stars: Energy Generation

How Does the Sun Shine?

The Sun radiates: shines from thermal radiation

- recall: surface flux $F_{\text{surf},\odot} = \sigma T^4_{\text{surf},\odot} = 60 \text{ MWatt/m}^2$
- total power output = rate of energy emission = luminosity

$$L_\odot = 4\pi R^2_{1 \text{ AU}} F_\odot(1 \text{ AU}) = 3.85 \times 10^{26} \text{ Watts} \tag{1}$$

$\rightarrow$ the Sun is a 4×10^{26}-Watt lightbulb

- But also: the Sun has *constant* temperature, luminosity (over human timescales $\gtrsim$ centuries)

Q: how is the Sun unlike a cup of coffee?

The Sun is Not a Cup of Coffee

Coffee Thermodynamics

Demo: cup of coffee: cools

thermodynamic lesson:

- left alone, hot coffee cools (surprise!)
  $\longrightarrow$ energy radiated, not replaced
- to keep your double-shot soy latte from cooling
  need Mr. Coffee$^{\rm TM}$ machine–energy (heat) source

Contrast with the Sun

- surface $T_{\odot}$ constant over human lifetimes
  but energy *is* radiated, at enormous rate
- ergo: something must replace the lost energy

▷ What is solar heat source (fuel supply)?
  $\longrightarrow$ a mystery in Astronomy until the 20th century

Q: all possible energy/heat sources which Sun taps?

Q: how to test/compare which are important?

Energy Conservation and the Sun

recall: power is energy flow rate $L = dE/dt$

assume:

- Sun always emits energy at today's rate (L constant)
- radiation lasts for time $\tau_\odot =$ "lifetime" of Sun

  Q: what is a minimum value for $\tau_\odot$?

energy output over Sun's lifetime:

$$E_{\text{out}} = L\tau$$

Energy conservation:

solar energy supply = lifelong energy output

Solar Batteries: Required Lifetime

from radioactive dating of meteorites:
the solar system is very old: age $t_{\rm ss} = 4.55 \times 10^9$ yr
Sun's present age essentially the same:
$t_{\odot,\rm now} = t_{\rm ss} = 4.55$ billion years

total energy output over this time is huuuge!
$\longrightarrow$ required huge energy reservoir

Q: possible sources–not just right answer, but any energy reservoirs?

iClicker Poll: Rank the Energy Sources

Vote your conscience!

Of the proposed solar energy reservoirs

Which one is the largest, i.e., can power the Sun longest?

Which one is the smallest?

Q: how to sort the candidates? how to tell which is right?

Energy Sources in the Sun

to evaluate energy sources, need to study energy "budget"

- output: energy supply required to power Solar luminosity
- input: available energy sources that might act as fuel

PS4: sort sources by *time* they can power the Sun

Here: look at energy budget directly

Solar Energy: Required Supply

Sun must shine for at least the age of Solar System, emitting

$$E_{\rm emit} = L_\odot \; t_{\rm ss} \approx 6 \times 10^{50} \text{ erg} = 6 \times 10^{43} \text{ Joule} \tag{2}$$

this a lot! but also huge mass → huge fuel supply solar emitted energy to date

$$E_{\text{emit}} = L_\odot \; t_{\text{ss}} \approx 6 \times 10^{50} \text{ erg} = 6 \times 10^{43} \text{ Joule}$$

per unit mass

$$u_{\text{emit}} = \frac{E_{\text{emit}}}{M_\odot} = 3 \times 10^{17} \text{ erg/g} = 3 \times 10^{14} \text{ J/kg}$$

and *Sun is mostly hydrogen*, with $N_{\text{p}} \approx M_\odot / m_p \approx 10^{57}$ protons
so emitted energy per proton, is

$$\epsilon_{\text{emit}} = \frac{E_{\text{emit}}}{N_{\text{p}}} = 5 \times 10^{-7} \text{ erg/proton} = 3 \times 10^{5} \text{ eV/proton}s$$

Q: typical chemical energy per proton? Hint: bonds?

Q: average Sun's thermal energy per proton?

Q: and so?

solar energy emitted to date, per proton:

$$\epsilon_{\text{emit}} = \frac{E_{\text{emit}}}{N_{\text{p}}} = 5 \times 10^{-7} \text{ erg/proton} = 3 \times 10^{5} \text{ eV/proton}$$

if fuel is chemical: power from exothermic chemical reactions
energy comes from atomic bonds
typical scale: binding energy of hydrogen, $\epsilon_{\text{H}} = B_{\text{H}} = 13.6 \text{ eV/proton}$
to compare, TNT has $\epsilon_{\text{TNT}} = u_{\text{TNT}} m_p = 0.05 \text{ eV/proton}$

if fuel is thermal: power from cooling
ideal gas: per proton, energy is $\epsilon_{\text{therm}} \sim \langle kT \rangle$
and from Virial theorem we found: $\langle kT_{\odot} \rangle \approx 10^3 \text{ eV}$
Q: what does this say about gravitational potential energy supply?

these are woefully inadequate! PS4: so is gravitation, rotation
Lesson: **a huge non-gravitational energy source needed**

historically, a mystery! no feasible energy source known

so 19th century astronomers argued (incorrectly) that age of Earth must be much shorter than geologists and biologists thought. D'oh!

Spoiler Alert!

there is **only one** viable candidate:

- Nuclear Energy

The Sun is a vast nuclear reactor
in hot core, hydrogen converted to helium
by nuclear reactions

Note: needed *quantitative* estimates of burn times
to answer *qualitative* question "What powers the Sun?"
$\longrightarrow$ the power of (and necessity of) number crunching!

Overview: the Structure of Matter

Zooming into microscopic structure of matter:

matter
↓
molecules
↓
atoms
↙ ↘
nucleus   electrons
↙ ↘
protons   neutrons

atoms come in **elements** `www: periodic table`

92 natural, 23+ artificial

element determined by *nuclear charge* $Z = \#$ protons

e.g., hydrogen H: $Z = 1$, uranium U: $Z = 92$

same element (same # p) can have different # neutrons
$\rightarrow$ "isotopes"
examples: most hydrogen is ^{1}H $= \boxed{1p,\ 0n}$
but $\sim 10^{-4}$ of hydrogen is deuterium ^{2}H $= \boxed{1p,\ 1n}$
most U is ^{238}U $= \boxed{92p,\ 146n}$; about $\sim$ 1% is ^{235}U $= \boxed{92p,\ 143n}$

atom net charge fixed by # electrons
e = # $p \rightarrow$ neutral
e = # $p - 1 \rightarrow$ singly ionized

Note: all p, n, e are absolutely *identical* and *indistinguishable*
this turns out to be crucial for the understanding of matter
in a quantum mechanical way

iClicker Poll: Forces in Nuclei

Consider a nucleus, say $^{4}\mathrm{He} = \boxed{2p+2n}$
maintains same size: not imploding, exploding

How many forces act on each proton?

A exactly 1

B more than 1

C fewer than 1

Forces in Nuclei

nuclei made of *protons and neutrons:* **"nucleons"**
sizes similar:
$r \approx 1.4 \times 10^{-15}$ m $= 1.4$ femtometer $= 1.4$ fermi

in nucleus: nucleons are touching!
- nuclear size $\ll$ atom size
- protons very close $\rightarrow$ *huge electrostatic repulsion!*

electrostatic (Coulomb) energy between two protons in nucleus

$$E_{\mathrm{C}} = \frac{e^2}{[4\pi\epsilon_0]r} \approx 1 \text{ MeV} \tag{3}$$

if this is unopposed, nuclei would fly apart!
nuclear stability requires attractive force between nucleons

Nuclear Forces

thus: existence of nuclei demands a stabilizing force

the **nuclear interaction / nuclear force**

- attractive at short distances
- stronger than Coulomb force at short distances
- with $\sim$ MeV scale strength
- weakens at long distances or all nuclei would merge to one!

Nuclear Binding

stable atomic nuclei are bound states of nucleons

- that is: they can't "fall apart" on their own
- the same way bound atoms, planetary systems, binary stars don't fall apart

so nucleus A, with Z protons and N neutrons
has **binding energy** B_A = energy required to rip apart
this means that

$$E_A + B_A = ZE_{\rm p} + NE_{\rm n} \tag{4}$$

that is

$$\begin{aligned} \text{binding} &= \text{parts} - \text{whole} & (5) \\ B_A &= ZE_{\rm p} + NE_{\rm n} - E_A & (6) \\ &= > 0 & (7) \end{aligned}$$

so energy of parts is more than whole!

but Einstein says $E = mc^2$
Q: what does this mean generally? implications for nuclei?

Nuclear Binding

Einstein $E = mc^2$ says: an object at rest, with mass m contains energy $E = mc^2$ simply by having mass

- mass is a form of energy!
- not due to motion: "rest mass energy"

for nuclei (and similar any other bound system), binding energy

$$B_A = ZE_{\mathrm{p}} + NE_{\mathrm{n}} - E_A \; > \; 0 \tag{8}$$

implies a *mass difference*

$$B_A = Zm_{\mathrm{p}}c^2 + Nm_{\mathrm{n}}c^2 - m_A c^2 = (Zm_p + Nm_n - m_A)c^2 \; > \; 0 \tag{9}$$

- mass of parts > mass of whole
- mass difference measures binding energy

bound system	binding energy B	binding energy per nucleon $B/(Z+N)$
hydrogen atom $\boxed{pe}$	13.6 eV	13.6 eV/nucleon
^{4}He nucleus $\boxed{2p, 2n}$	28.3 MeV	7.07 MeV/nucleon
^{56}Fe nucleus $\boxed{26p, 30n}$	492 MeV	8.79 MeV/nucleon
^{238}U nucleus $\boxed{92p, 146n}$	1801 MeV	7.57 MeV/nucleon

Q: atoms vs nuclei comparison?

Q: comparison among nuclei?

Q: lessons for Sun?

Lessons: Nuclear Energy as Stellar Fuel

nuclei vastly more tightly bound than atoms

- much harder to rip apart
- but much more energy at play when this happens

nuclear reactions: transformation of one set of nuclei to another

- leads to changes in sum of binding energies
- can require net energy input (endothermic)
- or can release net energy (exothermic)

typical reaction energy per nuclear particle (nucleon $= n, p$):
$B/(Z+N) \sim few$ Mev/nucleon
this is more than enough to power the Sun!

raises the question: how does the Sun–and all stars–do this?