Astro 404

Lecture 16

Oct. 2, 2019

Announcements:

- **Problem Set 5 due Friday at 5:00 pm**
  see Homework Discussion on Compass for FAQs
- Instructor office hours this week:
  tomorrow (Thurs) 10–11 am, 2–3pm

Last time: solar neutrinos

Q: how do we detect neutrinos from the Sun?

Q: what is the main result of solar neutrino experiments?

Q: what does this each us about the Sun?

Solar Neutrino Results

Solar Neutrino experiments show

- the Sun shines in neutrinos
- the neutrino flux agrees with solar models!

I. proof that Sun powered by nuclear fusion

II. νs give direct view into solar core

III. these underground vats are ν telescopes!

A new window on the Universe:

Nobel Prize 2002!

Raymond Davis Jr. and Masatoshi Koshiba

Time Reversal and Particle Interactions

last time I claimed: if ν were truly *non-interacting*
and can't collide and react with ordinary particles
then they can't be made in the first place – but why?

answer: *time reversal invariance*
almost without exception: if a microscopic process can occur
then the "time reversed" process is also physically possible
$\longrightarrow$ run the movie backwards, and this must be allowed

so consider the observed reaction: $\nu_e p \rightarrow n e^+$

- *neutrino absorbed* by proton, creates neutron and positron

this requires time-reversed $n e^+ \rightarrow \nu_e p$ is possible

- *neutrino emitted*

Lesson: time reversal invariance implies that
absorbers most also be emitters
both must occur if an interaction exists

Solar Neutrino Experiments: A Deeper View

1960s: original chlorine radiochemical experiment (Ray Davis):
- sensitive only to a small component of very high-energy νs
- signal detected, but flux $\Phi_\nu^{\rm obs} \approx \Phi_\nu^{\rm predicted}/3$

  birth of **"solar neutrino problem"** – where did they go?

1990's: solar neutrino deficit confirmed

possible explanations:
- theory of solar nuclear reactions is wrong/incomplete
- neutrino theory incomplete

it was already known that: *neutrinos have 3 varieties ("flavors")*
ν_e, ν_μ, ν_τ: named for partner they appear with
solar neutrinos produced as ν_e: should remain so
$\rightarrow$ unless neutrinos can transform into different flavors!

Q: how to test for the latter possibility?

The Sun Reveals New Neutrino Physics

if neutrino flavor transformations exist

- some particles born in Sun as ν_e
- can arrive at Earth as ν_μ or ν_τ
- but radiochemical experiments only "see" ν_e

To test:
build detectors sensitive to *all flavors*
this was done: Sudbury Neutrino Observatory (SNO)

early 2000s: SNO results weigh in

- ν_μ and ν_τ *detected* from Sun!
- *total flux* for *all* ν *agrees* with Solar model!
- **confirms new neutrino physics**
- also *transformations require neutrinos have mass!*
  non-obvious property of the quantum flavor transformations

Neutrinos and Mass

neutrino flavor transformations confirmed in lab experiments:
use nuclear reactors as ν_e sources
detect neutrino disappearance with distance
characteristic of quantum "oscillation" into other flavors
`www: oscillation data`

confirms *neutrinos have mass*,
but only measures mass differences!

Using the Sun to probe neutrino transformation and mass:
Nobel Prize 2015!
Arthur MacDonald and Taakaki Kajita

iClicker Poll: How do Stars Shine?

We have proven the Sun is nuclear powered
in core: energy generated by $4p \longrightarrow {}^4\mathrm{He}$

Vote your conscience!

What can we infer about other stars?

A *all other stars* burn hydrogen $\longrightarrow$ helium

B *only* $1M_\odot$ *stars* burn hydrogen $\longrightarrow$ helium

C *all main sequence* stars burn hydrogen $\longrightarrow$ helium

D none of the above

Main Sequence: Hydrogen Burning Phase

HR diagram teaches:

- the Sun is a typical main sequence star
- main sequence is the longest phase in a star's life

energy conservation teaches:
main sequence luminosity and lifetime demand large energy source
only nuclear energy can sustain

so we infer:
all main sequence stars are nuclear reactors
converting hydrogen to helium
nuclear power makes stars shine!

Another Way to Burn Hydrogen

the Sun and other stars are mostly made of hydrogen
with about 28% helium by mass (less by number–*Q: why?*)
and about 2% by mass of heavier elements

some of most abundant heavy elements ("metals")
are carbon, oxygen, nitrogen (CNO)
these allow for another set of reactions

$$
\begin{array}{rcll}
{}^{12}\mathrm{C} + p & \rightarrow & {}^{13}\mathrm{N} + \gamma & \\
{}^{13}\mathrm{N} & \rightarrow & {}^{13}\mathrm{C} + e^{+} + \nu_e & \text{radioactive decay} \\
{}^{13}\mathrm{C} + p & \rightarrow & {}^{14}\mathrm{N} + \gamma & \\
{}^{14}\mathrm{N} + p & \rightarrow & {}^{15}\mathrm{O} + \gamma & \\
{}^{15}\mathrm{O} & \rightarrow & {}^{15}\mathrm{N} + e^{+} + \nu_e & \text{radioactive decay} \\
{}^{15}\mathrm{N} + p & \rightarrow & {}^{12}\mathrm{C} + {}^{4}\mathrm{He} &
\end{array}
$$

Q: what is total net input? total net output?
Q: what is the role of CNO?

The CNO Cycle

$$
\begin{aligned}
{}^{12}\mathrm{C} + p &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{13}\mathrm{C} + p &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + p &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + e^{+} + \nu_e \quad \text{radioactive decay} \\
{}^{15}\mathrm{N} + p &\rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}
\end{aligned}
$$

then repeat–recycle the ^{12}C!

⋆ *same net effect as pp chain: another way to burn hydrogen!*

⋆ *total CNO unchanged:* acts as a *catalyst!*

⋆ CNO morphs to different forms but comes back: cyclic!
can start anywhere in the cycle!

this chain: the **CNO cycle**

iClicker Poll

Vote your conscience!

Q: Which chain dominates hydrogen production in stars?

A pp dominates for all stars

B CNO cycle dominates for all stars

C pp dominates for stars with cooler cores, CNO for hotter

D pp dominates for stars with hotter cores, CNO for cooler

Hydrogen Burning: pp versus CNO

reaction chain speed/importance set by slowest link
the most difficult and thus "*rate limiting step*"

pp *chain:* rate limited by $pp \to de^+\nu_e$
- weak reaction required
- three body final state disfavored

CNO cycle: rate limited by $p + {}^{14}\mathrm{N} \to {}^{15}\mathrm{O} + \gamma$
- large Coulomb repulsion due to ^{14}N charge $Z = 7$
- but CNO has no weak reactions, only weak decays

which is dominant depends on star core temperature!
cooler stars can't overcome large CNO Coulomb barrier
but hot stars can, then can burn fast

for main sequence stars:
- pp dominates of mass $M \lesssim 1.3 M_\odot$
- CNO dominates for higher masses

Reaction Rates and Cross Sections

Imagine some general reaction: $a + b \rightarrow c + d$

Consider particle beam:
"projectiles," number density n_a
incident w/ velocity v
on targets of number density n_b

Due to interactions, targets and projectiles "see" each other as spheres of projected area $\sigma(v)$: the

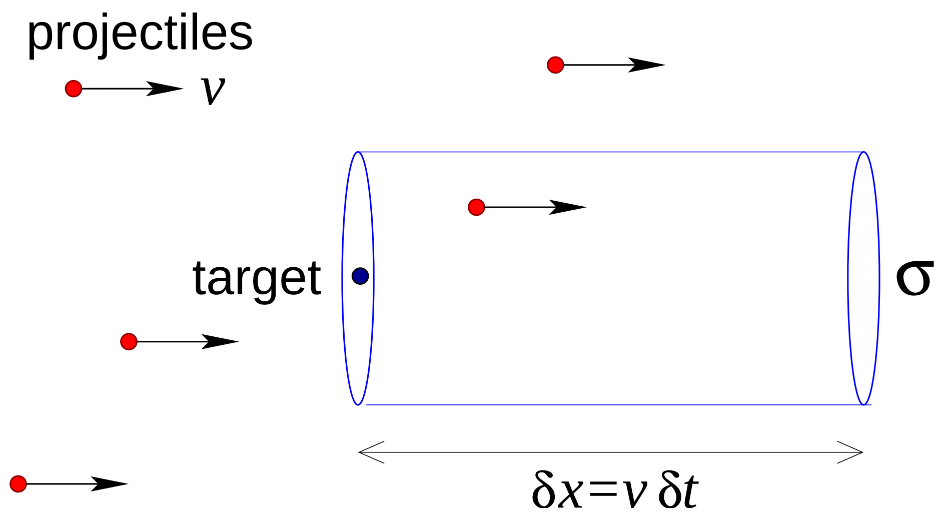
cross section

⋆ fundamental measure interaction strength/probability
⋆ *microphysics meets astrophysics via* σ

in time δt, what is avg # collisions on one target?
Q: what defines "interaction zone" around target?

interaction zone: particles sweep out "*scattering tube*"

- tube area σ
- length $\delta x = v \delta t$



scattering tube volume around target:

$\delta V = \sigma \delta x = \sigma v \delta t$

collide if a projectile is in the volume

Cross Section, Flux, and Collision Rate

in scattering tube volume $\delta V = \sigma v \, \delta t$,
average number of projectiles in tube $= \mathcal{N}_{\text{proj,tube}} = n_a \delta V$
so: *average number of collisions in δt:*

$$\delta \mathcal{N}_{\text{coll}} = \mathcal{N}_{\text{proj,tube}} = n_{\text{a}} \sigma v \delta t \tag{1}$$

so $\delta \mathcal{N}_{\text{coll}} / \delta t$ gives

$$\text{avg collision rate } \textit{per target } b \quad \boxed{\Gamma_{\text{per}\, b} = n_a \sigma_{ab} v} \tag{2}$$

Q: Γ units? sensible scalings n_a, σ, v? why no n_b?

Q: average target collision time interval?
Q: average projectile distance traveled in this time?

Reactions: Characteristic Length and Time Scales

estimate *average time between collisions on target b*:
mean free time τ
collision rate: $\Gamma = d\mathcal{N}_{\text{coll}}/dt$
so wait time until next collision set by $\delta N_{\text{coll}} = \Gamma_{\text{per}\,b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\text{per}\,b}} = \frac{1}{n_a \sigma v} \tag{3}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\text{mpf}} = v\tau = \frac{1}{n_a \sigma} \tag{4}$$

no explicit v dep, but still $\ell(E) \propto 1/\sigma(E)$
Q: physically, why the scalings with n, σ?

PS5: alternative derivation of mean free path

Q: what sets σ for billiard balls?
Q: what set σ for $e^- + e^-$ scattering?

Cross Section vs Particle "Size"

if particles interact only by "touching"
(e.g., billiard balls)
then $\sigma \leftrightarrow$ particle radii

but: if interact by force field
(e.g., gravity, EM, nuclear, weak)
cross section σ *unrelated* to physical size!

For example: e^- has $r_e = 0$ (as far as we know!)
but electrons scatter via Coulomb (and weak) interaction
"touch-free scattering"

Q: what is collision or reaction rate per volume?

Reaction Rate Per Volume

recall: collision rate *per target b* is $\Gamma_{\text{per}\, b} = n_a \sigma_{ab} v$
total collision rate *per unit volume* is

$$r_{ab} = \frac{dn_{\text{coll}}}{dt} = \Gamma_{\text{per}\, b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \sigma v \quad (5)$$

Kronecker δ_{ab}: 0 unless particles a & b identical
Note: *symmetric* w.r.t. the two particles

also note: $n_a n_b \propto$ *number of ab pairs*
reflects the fact that $ab \to cd$ reactions
are initiated by ab pairs!

Q: What if particles have more than one relative velocity?
What is energy generation rate per volume?

Reaction and Energy Generation Rates

If $v \in$ distribution, rates is average over velocities:

$$\langle r_{ab} \rangle = n_a n_b \langle \sigma v \rangle \tag{6}$$

energy generation rate per volume:
depends on reaction rate r_{ab}
and energy release per reaction Q_{ab} :

$$\dot{\epsilon}_{ab} = \frac{dE_{ab}}{dV\ dt} = Q_{ab} \frac{dN}{dV\ dt} = Q_{ab}\ r_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle \tag{7}$$

energy generation per unit mass:

$$q_{ab} = \frac{\dot{\epsilon}_{ab}}{\rho} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{8}$$

where m_a is mass of particle a
and $X_a = \rho_a / \rho$ is fraction of mass density in a

Hydrogen Burning Rates

nuclear energy generation rate per volume:

$$q_{ab} = \frac{\dot{\epsilon}_{ab}}{\rho} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{9}$$

- proportional to *density*: $q \propto \rho$
- depends on *temperature* via particle speeds: $\langle \sigma(v)\ v \rangle$

for hydrogen burning, roughly have:

$$q_{pp} \propto X_p^2 \rho\ T^4 \tag{10}$$

$$q_{\rm CNO} \propto X_p X_{\rm CNO} \rho\ T^{16} \tag{11}$$

note strong CNO temperature dependence:
important for stars with high $T_{\rm c}$
$\Rightarrow$ huge luminosity for massive main sequence stars