Astro 404

Lecture 18

Oct. 6, 2019

Announcements:

- **Problem Set 6 due next Friday**
- Office Hours: after class Wed, or by appointment

Last time

hydrogen fusion alternative: CNO cycle

Q: net effect of one cycle? role of C, N, O nuclei?

reaction rates

Q: what is reaction cross section?

Q: mean free path? mean free time?

CNO cycle: with any $X = \mathrm{C, N, O}$

one cycle has $4p + X \rightarrow X + {}^4\mathrm{He} + 2e^+ + 2\nu_e$

net effect: $4p \rightarrow {}^4\mathrm{He} + 2e^+ + 2\nu_e$

⋆ *same net effect as pp chain: another way to burn hydrogen!*

⋆ *total CNO unchanged:* acts as a *catalyst!*

for main sequence stars:

- pp dominates of mass $M \lesssim 1.3 M_\odot$
- CNO dominates for higher masses

interacting "see" each other

as spheres of projected area $\sigma(v)$: the

cross section

⋆ fundamental measure interaction strength/probability

Reactions: Characteristic Length and Time Scales

for general reaction $a + b \to c + d$
with projectile speed v

estimate *average time between collisions on target* b:
mean free time τ
collision rate per target b: $\Gamma = d\mathcal{N}_{\mathrm{coll}}/dt$
so wait time until next collision set by $\delta N_{\mathrm{coll}} = \Gamma_{\mathrm{per}\, b}\tau = 1$:

$$\tau = \frac{1}{\Gamma_{\mathrm{per}\, b}} = \frac{1}{n_a \sigma v} \tag{1}$$

in this time, projectile a moves distance: **mean free path**

$$\ell_{\mathrm{mpf}} = v\tau = \frac{1}{n_a \sigma} \tag{2}$$

Q: what is collision or reaction rate per volume?

Reaction Rate Per Volume

recall: collision rate *per target* b is $\Gamma_{\text{per}\,b} = n_a \sigma_{ab} v$

total collision rate *per unit volume* is

rate per volume = rate per target × targets per volume

$$r_{ab} = \frac{dn_{\text{coll}}}{dt} = \Gamma_{\text{per}\,b} n_b = \frac{1}{1+\delta_{ab}} n_a n_b \sigma v \qquad (3)$$

Kronecker δ_{ab}: 0 unless particles a & b identical

Note: *symmetric* w.r.t. the two particles

also note: $n_a n_b \propto$ *number of* ab *pairs*

reflects the fact that $ab \to cd$ reactions

are initiated by ab pairs!

Q: What if particles have more than one relative velocity?

Reaction and Energy Generation Rates

If $v \in$ distribution, rates is average over velocities:

$$\langle r_{ab} \rangle = n_a n_b \langle \sigma v \rangle \tag{4}$$

energy generation rate per volume $\mathcal{L}_{ab}$
depends on reaction rate r_{ab}
and energy release per reaction Q_{ab} :

$$\mathcal{L}_{ab} = \frac{dE_{ab}}{dV\ dt} = Q_{ab} \frac{dN}{dV\ dt} = Q_{ab}\ r_{ab} = Q_{ab}\ n_a n_b \langle \sigma v \rangle \tag{5}$$

energy generation per unit mass:

$$q_{ab} = \frac{\mathcal{L}_{ab}}{\rho} = \frac{\rho_a \rho_b}{\rho} \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{6}$$

where m_a is mass of particle a
and $X_a = \rho_a / \rho$ is fraction of mass density in a

Hydrogen Burning Rates

nuclear energy generation rate per volume:

$$q_{ab} = \frac{\dot{\epsilon}_{ab}}{\rho} = X_a X_b \frac{Q_{ab}}{m_a m_b} \rho \langle \sigma v \rangle \tag{7}$$

- proportional to *density*: $q \propto \rho$
- depends on *temperature* via particle speeds: $\langle \sigma(v)\ v \rangle$

for hydrogen burning, roughly have:

$$q_{pp} \propto X_p^2 \rho\ T^4 \tag{8}$$

$$q_{\rm CNO} \propto X_p X_{\rm CNO} \rho\ T^{16} \tag{9}$$

note strong CNO temperature dependence:

important for stars with high $T_{\rm c}$

$\Rightarrow$ huge luminosity for massive main sequence stars

Main Sequence: Core Composition Over Time

in the core of a main sequence star
hydrogen fusion ("nuclear burning") reactions: $4p \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e$

- positrons annihilate $e^+ + e^- \rightarrow \gamma + \gamma$
- neutrinos ν_e escape

so in core: net change in matter is $4p + 2e \rightarrow {}^4\text{He}$

so hydrogen burning in core:

- *reduces* the number of gas particles (electrons and nuclei)
- *increases* average mass m_g of a gas particle

iClicker Poll: Core Pressure

compare the Sun's core at start of H burning vs present day and *imagine core temperature and volume held fixed*

What effect does H burning have on core pressure?

A core pressure *reduced*

B core pressure *increased*

C core pressure *constant*

Main Sequence Evolution

hydrogen burning $4p + 2e \to {}^4\text{He}$
reduces number of gas particles in Sun's core
and increases average gas particle mass m_{g}

core pressure: ideal gas law

$$P = \frac{N\ kT}{V} = n\ kT \tag{10}$$

if core T and volume V fixed:
core pressure P decreases!

but pressure supports the core against gravity
reduced pressure $\to$ can't maintain hydrostatic equilibrium!

Q: how would the star respond?

Main Sequence Evolution

Virial theorem, ideal gas

$$U = \frac{3}{2}\frac{M}{m_{\rm g}}\langle kT\rangle \;=\; -\frac{1}{2}\Omega \sim \frac{GM^2}{R} \qquad (11)$$

$$\langle kT\rangle \;\sim\; \frac{GMm_{\rm g}}{R} \qquad (12)$$

main sequence: H$\rightarrow$^{4}He burning gives $m_{\rm g}$ increase
$\rightarrow$ *contraction*: core density increase
$\rightarrow$ $\langle kT\rangle$ increase

recall pp chain energy release per mass: $q_{pp} \propto \rho T^4$
core increase in ρ, T $\rightarrow$ *higher energy production!*

star luminosity increases – "main sequence brightening"

Main Sequence Evolution: H–R Diagram

main sequence core H burning: luminosity increase with time

in detailed models of the Sun

- initial *zero age main sequence* luminosity

$$L_{\odot,\mathrm{init}} = 0.7 L_{\odot,\mathrm{today}} \tag{13}$$

- turns out: star radius R increases too
  small change in T_{eff} – in Sun, slight increase

Q: consequences for HR diagram? how to test?

Main Sequence Evolution on the H–R Diagram

Sun in H–R diagram over time:
Sun point moves upward on main sequence
other stars evolve similarly but sometimes change in T_{eff}
for a group of stars with mixed ages
"smears out" the main sequence width

to test: find $1 M_{\odot}$ *"solar twins" in young star clusters*
these indeed show lower L!

Q: implications of 30% less luminous young Sun for Earthlings?

The Faint Young Sun

consequences of Sun's main sequence brightening

in the past the Sun was less luminous $L_{\odot,\mathrm{init}} = 0.7 L_{\odot,\mathrm{today}}$
so lower flux $F_{\odot}(1\ \mathrm{au})$: "faint young Sun"

but this sets Earth's temperature, so: cooler early Earth!
if Earth absorbs same sunlight as now (same albedo)

$$T_{\mathrm{Earth,init}} = \left(\frac{L_{\odot,\mathrm{init}}}{L_{\odot,\mathrm{today}}}\right)^{1/4} T_{\mathrm{Earth,today}} \approx 263\ \mathrm{K} = -12^{\circ}\ \mathrm{C} \quad (14)$$

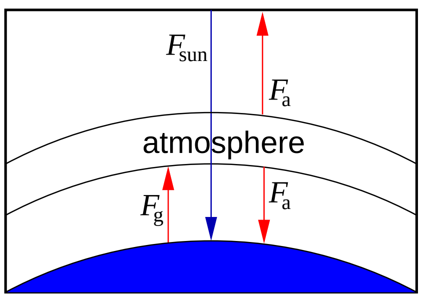
Cold enough to freeze seawater!
Earth frozen for first 2 Gyr (2.5 Gyr ago)! Yikes.

but: evidence for liquid water, and even life, up to 3.8 Gyr ago
Q: possible explanations?

Fain Young Sun Problem: Possible Solutions

liquid water on Earth back to Archean era 2.5 – 3.8 Gyr ago
how to reconcile with Solar evolution?

Greenhouse Effect – add blanket to Earth
atmospheric greenhouse gases trap Earth's thermal IR radiation
warm Earth's surface above airless temperature



today: greenhouse effect warms Earth by $\sim 30^\circ$
in past: *if thicker greenhouse gases, Earth warmer*
e.g., Sagan & Mullen (1972) proposed ammonia in early Earth
later shown unlikely, but basic idea remains

Solar Mass Loss – move the Earth
if early Sun had stronger mass loss than today
then *initial solar mass larger*

but Earth's angular momentum $J_{\rm Earth} = M_{\rm Earth} v a$ conserved
PS6: this and Kepler's laws say
higher $M_\odot \to$ smaller semimajor axis a: Earth closer!
and sees *higher solar flux* $F = L_\odot / 4\pi a^2$: hotter!

PS6: find needed mass loss today
would also affect orbits of other planets
good: lots of evidence early Mars had liquid water

Q: what about the future Sun? mitigation?

The Future Sun

main sequence brightening will continue in the future
unmeasurably small changes on human timescales
but eventually will profoundly affect the Earth

1 Gyr from now: Sun 10% more luminous
heating $\rightarrow$ evaporation of water vapor $\rightarrow$ adds to greenhouse
in upper atmosphere, UV from Sun breaks up H_2O molecules
and H lost to space:

- Earth hot and dry
- and losing water

3.5 Gyr from now: Sun 40% more luminous
oceans evaporated, hydrogen lost to space
runaway greenhouse effect
Uh oh. probably no life unless mitigation.

Mitigation?

- move Earth's orbit outward
  perhaps by exchanging energy with Jupiter
- a huge task, but we have lots of time