Astro 404
Lecture 19
Oct. 9, 2019

Announcements:

- **Problem Set 6 due next Friday**
- Office Hours:
  instructor: Wed 11am, Friday after class, or by appointment
  TA: Thursday noon-1pm, or by appointment
- Hour Exam Friday Oct 18. Information on Compass

2019 Nobel Prize in Physics announced yesterday!

- Extrasolar Planets: Michel Mayor, Didier Queloz
  discovers of "hot Jupiters" that launched exoplanet revolution
- Physical Cosmology: James Peebles
  pioneer of modern theoretical cosmology
- ...but not Vera Rubin, discoverer of dark matter in galaxies
  who died in 2016. Nobel Prize has issues.

Last Time

nuclear reaction rates: there was a lot of formalism!
easy to get lost! but basic ideas are simple

key point:
almost all important nuclear reactions in stars are the result of *two particles colliding: 2-body reactions*
that is: *reactions occur between pairs of particles*

a general example is $a + b \to c + d + \cdots$, with a

$$\text{reaction rate} \propto \text{number of pairs} \propto n_a n_b$$

and the number densities and mass densities are related

$$\text{number density } n_a \propto (\text{abundance of } a) \times \text{mass density } \rho \quad (1)$$

Q: so how does the rate of reactions depend on density?

rate of reactions: focusing on density, we have

$$\text{reaction rate} \propto \text{number of pairs} \propto \rho^2 \tag{2}$$

energy generation: each $ab \to cd \cdots$ reaction releases a fixed amount of nuclear energy, so

$$\mathcal{L} = \frac{dE}{dt\ dV} = \text{energy generation rate per volume} \propto \rho^2 \tag{3}$$

lesson: *2-body reaction* gives $\mathcal{L} \propto \rho^2$ density dependence

Q: what would we get for a 3-body reaction?
Q: what about a 1-body reaction, e.g., a decay $a \to cd \cdots$?

so far: only addressed density dependence
reaction rates also depend on *cross section and velocity*
through the average value of $\langle \sigma\, v \rangle$
Q: what conditions in a star determine this?

Energy Generation Rates

reaction cross sections and speeds depend on *temperature* T

so **nuclear energy generation rate per volume** has

$$\mathcal{L} \propto \rho^2 \langle \sigma v \rangle \tag{4}$$

- in nuclear reactions $\langle \sigma v \rangle$ *grows strongly with* T
- can approximate as $\mathcal{L} \propto \rho^2 T^s$
  with value of s specific to each nuclear reaction
  but generally $s \geq 4$, that is, $\mathcal{L} \propto \rho^2 T^4$ at least

energy generation rate is not uniform inside star
Q: where the highest? lowest?

we have seen: nuclear reactions determine
energy production rate per volume $\mathcal{L} = dE/dt\, dV$

Q: so what is physical significance of this?

$$l(r) = \int_0^r \mathcal{L}\ dV = 4\pi \int_0^r r^2\ \mathcal{L}(r)\ dr \quad (5)$$

Q: and of this?

$$l(R) = 4\pi \int_0^R r^2\ \mathcal{L}(r)\ dr \quad (6)$$

Summing Up: Power Generation

energy production rate per volume $\mathcal{L} = dE/dt\,dV$
is *power per volume* produced by nuclear reactions

so a volume integral out to radius r

$$l(r) = \int_0^r \mathcal{L}\ dV = 4\pi \int_0^r r^2\ \mathcal{L}(r)\ dr \tag{7}$$

is the total power generated within radius r
i.e., the *enclosed power* generated!

and $l(R)$ is *total nuclear power made in the star!*

but if the star is in hydrostatic equilibrium
it has fixed potential and internal energy *Q: and so?*

Nuclear Power and Stellar Luminosity

in *hydrostatic equilibrium*, potential and internal energy constant and so *total energy in star constant*

but then energy conservation demands:
all nuclear energy created inside the star
must be lost from its surface at exactly the same rate!

so *the total nuclear power generated* must be precisely
the total power radiated away
which is exactly the star's *luminosity*: $l(R) = L$

this also means that $l(r)$ is the **enclosed luminosity**

note also that $\mathcal{L}$ is luminosity per unit volume!
also known as luminosity density

note the close similarity with *mass density and mass*

$$\rho(r) = \frac{dM}{dV} = \text{mass per volume} \tag{8}$$

and so the *enclosed mass* within radius r is

$$m(r) = \int_0^r \rho \; dV = 4\pi \int_0^r r^2 \rho(r) \; dr \tag{9}$$

in fact we can write *enclosed luminosity* as

$$l(r) = \int \mathcal{L} \; dV = \int \frac{\mathcal{L}}{\rho} \; \rho \; dV = \int_0^{m(r)} q \; dm \tag{10}$$

where $q = \mathcal{L}/\rho \propto \rho T^s$ is the *power per unit mass*
and where total luminosity $L = \int_0^M q \; dm$

∞

our enclosed luminosity satisfies

$$\frac{dl}{dr} = 4\pi \ r^2 \ \mathcal{L}(r) = 4\pi \ r^2 \ q(r) \ \rho(r) \tag{11}$$

this is the *second equation of stellar structure!*

- *relates nuclear energy generation to luminosity*
- expresses energy conservation

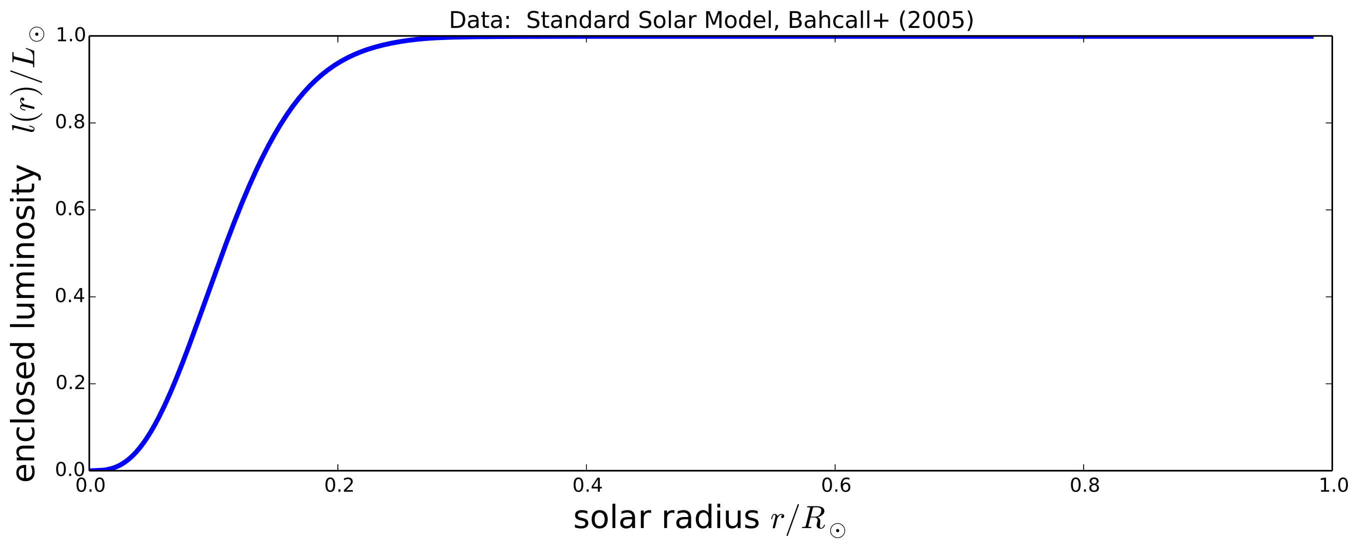
note that if we know the "local luminosity" $l(r)$
we can also find the "local energy flux"

$$F(r) = \frac{l(r)}{4\pi r^2} \tag{12}$$

which correctly gives $F(R) = L/4\pi R^2$ as the surface flux

Energy Generation: The Sun

models of the Sun show that
enclosed luminosity $l(r)$ *grows rapidly with radius*
and with enclosed mass



in the *innermost 20% of the Sun's radius*, $r = 0.2\, R_\odot$:

- enclosed mass $m(r) = 0.34\, M_\odot$
- enclosed luminosity $l(r) = 0.94\, L_\odot$

Q: lessons?

Solar Energy Comes from the Inner Core

in the *innermost 20% of the Sun's radius*, $r = 0.2\,R_\odot$:

- enclosed mass $m(r) = 0.34\,M_\odot$
- enclosed luminosity $l(r) = 0.94\,L_\odot$

mass is concentrated at center (density highest)

energy generation is entirely from innermost region
this define the **inner core**

in this core region temperature is close to our Virial estimate:
$T(0.2 R_\odot) = 9 \times 10^6$ K

the outer bulk of the Sun is cooler and generates little power
but acts to compress the core enough to sustain nuclear reactions

Stellar Structure: The Story Thus Far

thus far, great progress!

density determines enclosed mass (*mass conservation*)

$$\frac{dm}{dr} = 4\pi r^2 \rho(r) \tag{13}$$

$$m(r) = 4\pi \int_0^r r^2 \rho(r)\, dr \tag{14}$$

density and and enclosed mass determine pressure
due to hydrostatic equilibrium (force balance)

$$\frac{dP}{dr} = -\frac{Gm(r)\, \rho(r)}{r^2} \tag{15}$$

$$P(r) = P_c - \int_0^r \frac{Gm\, dm}{dr} \tag{16}$$

density and temperature determine nuclear reaction rates
and thus determine *luminosity* (energy conservation)

$$\frac{dl}{dr} = 4\pi r^2\,\mathcal{L} = 4\pi r^2\; q\;\rho \tag{17}$$

$$l(r) = 4\pi \int_0^r r^2\,\mathcal{L} = \int_0^{m(r)} q\; dm \tag{18}$$

one thing remains: temperature
comes from considering how *energy is transported*
from the core to the surface of the star
that is, how heat flows outward

Radiative Energy Transport

temperature set by heat flow, i.e., energy transport

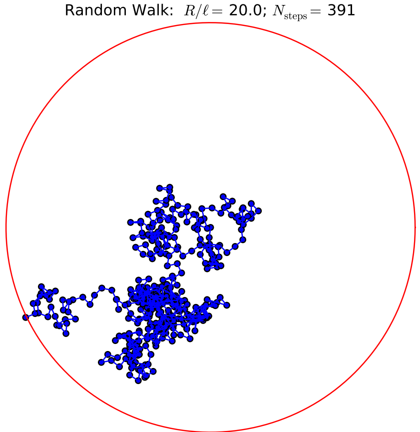
for now: assume all energy flow due to photons
"**radiative energy transport**"

star interior is *opaque:* photons scatter frequently
with **mean free path** $\ell_{\rm mfp}$ very short

key idea: each scattering event randomizes photon direction
photon "forgets" previous history: "**random walk**"

Q: how can photons ever escape if each step randomized?

PS6: shows how random walking photons escape



PS6 warmup: imagine *photon born at center* $\vec{r}_0 = 0$
first step has displacement $\vec{r}_1$
where $r_1^2 = |\vec{r}_1|^2 = \vec{r}_1 \cdot \vec{r}_1 = \ell_{\rm mfp}^2$
but if we *average over very many newborn photons*
going randomly with all directions chosen equally
then: $\langle \vec{r}_1 \rangle = 0$ *Q: why?*

simplified random walk: 1-dimensional case
photon only moves on x-axis

then random walk is like *flipping coins:*
on average, each step has equal chance
of "heads" $+\ell_{\mathrm{mfp}}$ and "tails" $-\ell_{\mathrm{mfp}}$
so if *flip many coins for one step, averages to zero*

but if *flip one coin many times*, usually develop
random excess of heads over tails, or vice versa
which means *net progress away from origin!*

when net displacement gets to edge of star, escape!

Photon Mean Free Paths

photon mean free path $\ell_{\mathrm{mfp}} = 1/n_{\mathrm{sc}}\sigma_{\mathrm{sc}}$
where n_{sc} is the number density of scatters
and σ_{sc} is photon scattering cross section

recall that number and mass densities related by
$\rho_{\mathrm{sc}} = m_{\mathrm{sc}} n_{\mathrm{sc}}$, with scatterer mass m_{sc}
so useful to define **opacity**

$$\kappa = \frac{\sigma_{\mathrm{sc}}}{m_{\mathrm{sc}}} \tag{19}$$

measures cross section per unit scatter mass, and

$$\ell_{\mathrm{mfp}} = \frac{1}{n_{\mathrm{sc}}\sigma_{\mathrm{sc}}} = \frac{1}{\kappa \rho_{\mathrm{sc}}} \tag{20}$$

iClicker Poll: Mean Free Paths in Stars

consider photons in the real Sun

How does photon mean free path change in Sun?

A $\ell_{\rm mfp}$ longest in Sun's center, shortest at surface

B $\ell_{\rm mfp}$ shortest in Sun's center, longest at surface

C $\ell_{\rm mfp}$ is uniform in the Sun photon mean free path is

$$\ell_{\rm mfp} = \frac{1}{n_{\rm sc}\sigma_{\rm sc}} = \frac{1}{\kappa\rho_{\rm sc}} \tag{21}$$

in Sun:

- $\rho(r)$ decreases from center to surface
- and in addition sometimes κ also deceases towards surface

so: mean free path goes from short to long
solar "fog" thins as we go out

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our *Sun is a gas*, density smoothly drops with radius
it does not really have a surface!
yet it does show a *sharp edge* in images
Q: how's that? what is the surface really?

The Solar Photosphere

the surface of the Sun appears sharp
despite random scattering of photons and smooth density profile

photons we see are **not scattered** between Sun and us
and so originate from *final scattering* events in Sun

apparent edge of Sun is **surface of last scattering**
also know as the solar **photosphere**

sharpness of photosphere must mean:

- density drops very rapidly near apparent surface
- mean free path changes rapidly from short to long
  until $\ell_{\rm mfp} > R_\odot$: *escape!*