Astro 404
Lecture 22
Oct. 16, 2019

Announcements:

- Good news: *no homework due next Friday!*
- *Bad news:* **Hour Exam Friday Oct 18.** Info on Compass
  all homework solutions posted
- Review: class Wednesday and today will have some review
  but new material today and this week is not only exam
  and BDF will answer Compass Exam Discussion Forum

- More good news: instructor upgrade today
  Prof. Paul Ricker
  theoretical and computational astrophysicist extraordinarie
  collider of stars, planets, and moons
  blaster of black hole blowtorches into galaxy clusters

Q: recall what happens to star core during main sequence?

Main Sequence Evolution: Recap

Main sequence ultrafast review:

- main sequence powered by *nuclear fusion in core*
- nuclear reaction net effect $4p + 2e^- \rightarrow {}^4\text{He} + 2\nu_e$ escape
- so *number* N *of gas particles in core decreases*

  while average gas particle mass m_{g} increases

But for ideal gas, pressure in core

$$P_{\text{ideal}} = \frac{N\,kT}{V} = n\;kT = \frac{\rho\,kT}{m_{\text{g}}} \tag{1}$$

so if core T and V fixed, then P would decrease!

Q: but what really happens?

nuclear reactions reduce particle number in star core
this reduces pressure if core T and V fixed

but to keep hydrostatic equilibrium: core gets hotter!
Virial theorem say mean star temperature

$$\langle kT \rangle \sim \frac{GMm_{\mathrm{p}}}{R} \tag{2}$$

core density and temperature increases on main sequence
this leads faster nuclear reactions since $\mathcal{L}_{pp} \propto \rho^2 T^4$
which generates more energy
and leads to *main sequence brightening*

but this process of burning hotter and brighter is not forever!
Q: when does main sequence end?

Main Sequence Evolution: The Last Gasps

Main sequence hydrogen burning $4p + 2e^- \rightarrow {}^4\text{He} + 2\nu_e$
required hydrogen exists in star core

but main sequence burning constantly removes hydrogen!

Sun age t	core H fraction $X_{\text{H,core}}$	core He fraction $Y_{\text{He,core}}$
initial: 0 Gyr	70%	28%
today: 4.55 Gyr	36%	62%

Lesson: the core already has less H than He!
Sun's fuel gauge is 1/2 of its initial value!
→ the Sun is middle aged!

Q: what happens when Sun's core runs out of hydrogen entirely?

Stars Under Pressure

core hydrogen burning ends when fuel supply consumed
that is, when core hydrogen exhausted: $X_{\text{core}} \rightarrow 0$
and turned into helium “ash”

with no nuclear reactions, core enclosed luminosity $l_{\text{core}} \rightarrow 0$
equilibrium lost!

- heat in core still diffuses/random walks outwards
- loses thermal pressure
- gravitation drives contraction → compression

re-pressurizes core

- ...but core heat continues to diffuse out!
  repeat cycle: continue to contract!

Q: what finally stops this process?

Halting Contraction

after core hydrogen burning stops
contract to ever higher density and temperature
until/unless some new form of pressure emerges

several factors are possible

- Coulomb repulsion between the charged particles
- if hot enough and dense enough, helium “ash” can “ignite” and begin as fuel for new phase of nuclear burning
- some new effect becomes important in the core matter

iClicker Poll: Fate of the Future Sun

Vote your conscience! *All answers get credit*

When the Sun's core hydrogen is exhausted:

What's the main thing eventually stopping core contraction?

A Coulomb repulsion between charged particles

B core helium begins nuclear fusion

C some new effect becomes important in matter of the core

Losing Idealism

in fact, all above effects are important for some stars
and all have significance for the future Sun

but the dominant thing arresting core contraction after main sequence
is that the *matter in the core (*^{4}He *nuclei and electrons* e^-*)*
stops behaving as a classical ideal gas!
and a new pressure source emerges!

the origin of this transition
is due to the quantum nature of matter
and quantum effects at high density

Matter and Light: Old School

pre-quantum (“**classical**”) picture of matter and light

- *matter* composed of *particles*
  with definite positions and momenta

- *light* composed of *waves*
  with definite wavelengths, but not localized positions
  and that can diffract and interfere

these familiar distinctions fell apart
in earth 20th century

The Quantum Duality of Matter and Light

early 20th century: quantum revolution

- *matter sometimes shows wave properties!*

for particle of mass m, momentum $p = mv$: **de Broglie wavelength** is

$$\lambda_{\text{deB}} = \frac{h}{p} = \frac{h}{mv} \tag{3}$$

with Planck's constant

$$h = 6.626 \times 10^{-34}\ \text{Joule sec} = 4.14 \times 10^{-15}\ \text{eV sec}$$

- *light sometimes acts as particle: photon* $m_\gamma = 0$

for wavelength λ, photon momentum and energy are

$$p_\gamma = \frac{h}{\lambda} \quad E_\gamma = cp_\gamma = \frac{hc}{\lambda} \tag{4}$$

Q: so what's really going on?

Is it really a particle or a wave?

So both light and matter can sometimes show wave properties
and other times show particle properties

what you see depends on the experiment
a full discussion of which is the fun of taking Quantum Mechanics

for our purposes, roughly speaking:
when matter in a star is not too dense
that is, when particle spacing is more than de Broglie $\lambda_{\rm deB}$
then quantum effects not important
matter acts like ordinary ("classical") gas of particles
→ *ordinary classical ideal gas approximation is good!*

but what happens in relativistic case (photons)?
and what happens when matter becomes very dense?

Thermal Gas of Photons

important special case: **thermodynamic equilibrium**
where massive particles and photons interact
until all have thermal distributions with same temperature T

consider a thermal bath of *photons*

- temperature T
- particle mass $m_\gamma = 0$
  which means particle energy $E = \sqrt{(mc^2)^2 + (cp)^2} = cp$
  and also that $v = p/E = c$ ("relativistic particles")

want to know: what is distribution of photon wavelengths/energies
and overall average energy density and pressure

full derivation: Director's Cut Extras (next time)
quick and dirty: *dimensional analysis*

Thermal Photons: Dimensional Analysis

for a bath of photon T, we want:

- number density n_γ
- energy density ε_γ
- pressure P_γ

scales in the problem:

- thermal energy kT
- $m = 0$ so no mc^2 rest energy
  but special relativity relevant: c
- quantum effects relevant: h

Q: how to construct number density n_γ, ε_γ, P_γ*?*

Hint: $hc \approx 1000$ MeV fm has dimensions [energy × length]

Hint: we already have discussed this situation weeks ago!

Q: what is the name for a thermal distribution of photons?

Thermal Photons: Blackbody Radiation

dimensional analysis: kT, h, c form one *length*

$$\ell = \frac{hc}{kT} = \frac{h}{p_T} \tag{5}$$

the *thermal de Broglie length*

from this we estimate **number density**

$$n_\gamma \sim \ell^{-3} \sim \left(\frac{kT}{hc}\right)^3 \tag{6}$$

energy density

$$\varepsilon_\gamma \sim kT\ell^{-3} \sim \frac{(kT)^4}{(h^3c^3)} \tag{7}$$

pressure has dimensions of energy density, so

$$P_\gamma \sim \varepsilon \tag{8}$$

of course we know thermal photons result: blackbody radiation!

Blackbody Radiation: Exact Results

for blackbody photons at T:

$$
\begin{aligned}
n_\gamma &= g\frac{\zeta(3)}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 \propto T^3 \\
\varepsilon_\gamma &= g\frac{\pi^2}{30}\frac{(kT)^4}{(\hbar c)^3} \propto T^4 \\
P_\gamma &= \frac{1}{3}\varepsilon_\gamma \propto T^4
\end{aligned}
$$

where $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots = 1.20206\ldots$

and where $\hbar = h/2\pi$ is the chic "reduced Planck's constant"

Note: already saw that relativistic gas has $P = \varepsilon/3$

also note: energy flux is roughly $F \sim c\varepsilon \sim T^4$

which is Stefan-Boltzmann result!

next time: move to non-relativistic case

Hour Exam Review

Exam Overview

Exam Information Posted on Compass

highlights:

- Hour Exam is in two days – **Friday Oct 18**
- Time: 50 minutes, usual class time 10:00 - 10:50 am
- exam is **closed book, closed notes, closed internet**
  but you may bring *a calculator*
  and one ordinary 8.5 in × 11 in sheet of paper
  with your name and anything you can write by hand
- covers everything through: Fri Oct 11 class and HW6

Free Advice

be prepared to draw an HR diagram from memory
- without the aid of any notes
- labeling axes
- drawing and labeling major regions
- showing the location the Sun

you could imagine being asked something like
indicate main sequence stars with the shortest lifespans
indicate main sequence stars with the longest lifespans

Practice now!

Exam Topics: Some Main Themes

star masses, distances, luminosity: what are they? how to measure?

HR diagram: main regions? how does it encode stellar evolution? results for star clusters vs all stars?

hydrostatic equilibrium: what is it? why important? equation?

Virial theorem: what is it? when/where does it apply? main lessons?

main sequence: what controls the sequence? which stars live there? lifespans?

nuclear reactions and hydrogen burning: main effect? how do we know?

radiative heat transfer: how does nuclear energy get out of the Sun? why doesn't it come out immediately?

Questions about any of these?