Astro 404

Lecture 26

Oct. 28, 2019

Announcements:

- **Problem Set 8 due Fri Nov 1**

Last time: degenerate stars – white dwarfs

- hydrostatic support from degeneracy pressure
  example of a polytrope *Q: what's that?*
- thermal structure decoupled from mechanical structure
  Q: what does this mean?
- non-relativistic degenerate stars *Q:* $P(\rho)$*?*
  Q: size R *vs mass* M*? implications?*

Polytropes, Degenerate Matter, and White Dwarfs

polytrope: matter where pressure depends only on density
that is, equation of state has form $P(\rho)$ and not $P(\rho, T)$
degenerate matter is example of this
usually polytropes are approximately power laws: $P = K\rho^{\gamma}$

in this case, hydrostatic equilibrium alone determines $\rho(r)$
that is: hydrostatic force balance alone sets mass distribution
independent of interior temperature, indeed for $T = 0$, so:
mechanical structure $\rho(r)$ independent of thermal structure $T(r)$

non-relativistic degenerate electron matter:

$$P_{e,\mathrm{nr}} = K_{e,\mathrm{nr}}\ n_{\mathrm{e}}^{5/3} = K_{\mathrm{nr,degen}}\ \rho^{5/3}$$

gives $R \propto M^{-1/3}$: higher mass $\leftrightarrow$ smaller white dwarf!

Relativistic Degenerate Stars

for (ultra)relativistic degenerate stars
electrons have Fermi momentum $p_0 \gg m_e c$, and

$$P_{e,\mathrm{rel}} = K_{e,\mathrm{rel}}\ n_{\mathrm{e}}^{4/3} = K_{\mathrm{nr,degen}}\ \rho^{4/3}$$

only one mass gives hydrostatic equilibrium
Chandrasekhar mass
$M = M_{\mathrm{Chandra}} = 1.4 M_{\odot}$ for helium-like matter
`www: Chandrasekhar (1931) paper` – you can understand this!

highest mass for which degeneracy pressure can overcome gravity
at larger masses, stars are unstable and collapse!

Q: how to test this prediction with white dwarf data?

iClicker Poll: White Dwarf Masses

Vote your conscience! *All answers get credit*

white dwarf masses are observed in hundreds of binary systems

What do we find?

A *most* masses $M_{\rm wd} < 1 M_\odot$, none $> M_{\rm Chandra}$

B *most* masses $1 M_\odot < M_{\rm wd} < M_{\rm Chandra}$, none $> M_{\rm Chandra}$

C roughly equal masses up to $M_{\rm Chandra}$

D about 10% of white dwarfs have $M_{\rm w} > M_{\rm Chandra}$

White Dwarf Masses Observed

observed white dwarf masses
- most found at $M_{\rm wd} \lesssim 1 M_\odot$

but there is an *observational bias*
- white dwarf cooling sequence: $L_{\rm wd} = 4\pi R^2 \sigma T_{\rm eff}^4 \propto M^{-2/3} T_{\rm eff}^4$
  less massive are more luminous – easier to find

but even after accounting for this bias
- true distribution dominated by $< 1 M_\odot$ white dwarfs
- *no white dwarfs found with* $M > 1.4 M_\odot$

implications:
- Chandrasekhar mass truly limits white dwarf masses
- lack of high mass WDs suggests *either*
  these are rarely made, or these don't survive when made

Dynamical Stability of Stars

Why do relativistic degenerate stars become *unstable* while non-relativistic degenerate stars do not?

look at *response to perturbations*

consider a star, mass M and radius R

in *hydrostatic equilibrium*

mass shell m at radius $r(m)$ feels weight per area

$$P = P_{\mathrm{grav}} = \int_m^M \frac{m\ dm}{4\pi r^4}$$

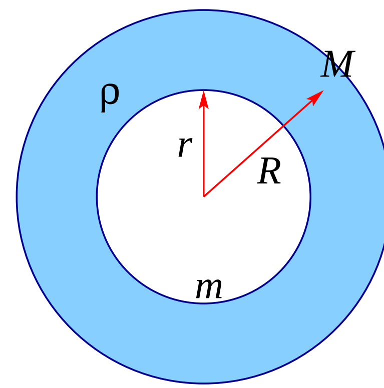
where $dm = 4\pi r^2\ \rho\ dr$ or

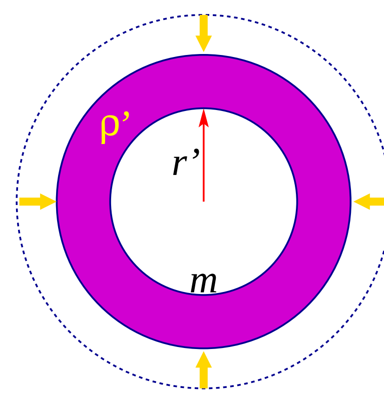
$$\rho = \frac{1}{4\pi r^2}\frac{dm}{dr}$$

compress shell a small amount:

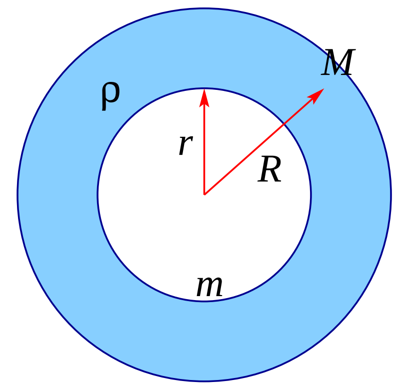
$r' = r - \epsilon r = (1-\epsilon) r$

Q: response of gas P_{gas}? P_{grav}





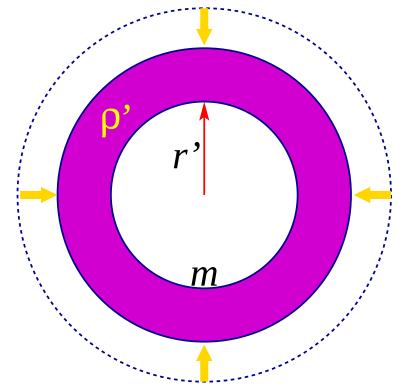
compress shell a small amount $r' = r - \epsilon r = (1-\epsilon)$
while hold shell enclosed mass m fixed



gravitational response: weight per area

$$P'_{\text{grav}} = \int_m^M \frac{m\ dm}{4\pi(1-\epsilon)^4 r^4} = (1-\epsilon)^{-4} P \approx (1+4\epsilon)P$$

where we used $(1-\epsilon)^s \approx 1 - s\epsilon + \cdots$



gas pressure response:

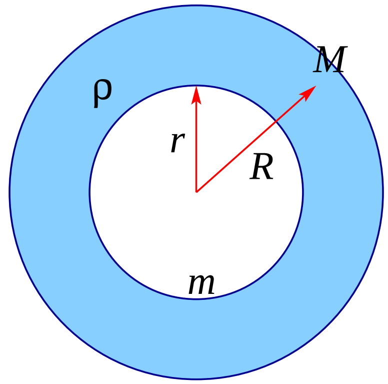
$$\rho' = \frac{1}{4\pi(1-\epsilon)^2 r^2}\frac{dm}{dr'} = (1-\epsilon)^{-2}\frac{1}{4\pi r^2}\frac{dm}{dr}\frac{dr}{dr'} = \frac{\rho}{(1-\epsilon)^3} \approx (1+3\epsilon)\rho$$

for a polytrope: $P = K\rho^\gamma$, so

$$P'_{\text{gas}} = K(\rho')^\gamma \approx (1-\epsilon)^{3\gamma} P \approx (1+3\gamma\epsilon)P \qquad (1)$$

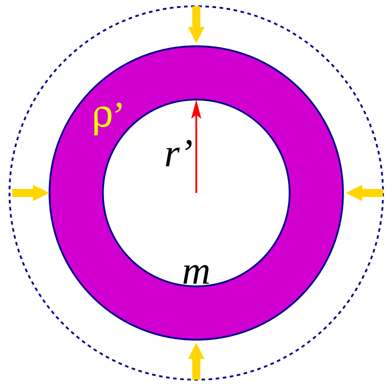
so after perturbation $r' = (1-\epsilon)r$

new pressures are in general no longer the same



so look at pressure difference

$$P'_{\text{gas}} - P'_{\text{grav}} \approx (1+3\gamma\epsilon)P - (1+4\epsilon)P$$
$$= \left(\gamma - \frac{4}{3}\right) 3\epsilon P$$



Q: response if $\epsilon = 0$*?*

Q: response if $\gamma > 4/3$*?*

∞ *Q: response if* $\gamma < 4/3$*?*

Q: implications?

Polytropic Index and Dynamical Stability

after *radial compression* $r'(m) = (1 - \epsilon) r(m)$:

$$P'_{\mathrm{gas}} - P'_{\mathrm{grav}} \approx (1 + 3\gamma\epsilon)P - (1 + 4\epsilon)P = \left(\gamma - \frac{4}{3}\right) 3\epsilon P \qquad (2)$$

if $\gamma > 4/3$

outward gas pressure grows faster

than inward weight per area due to gravity of outer layers

net outward pressure: gas expands back to original size

restoring force opposes perturbation → *stable equilibrium*

if $\gamma < 4/3$

net inward pressure: restoring force enhances perturbation

runaway → equilibrium not restored → *dynamically unstable!*

if $\gamma = 4/3$
no restoring force at all!
but this means *perturbation not undone*

lesson: instability if $\gamma \leq 4/3$!

so for white dwarfs: degenerate stars

- low mass $\leftrightarrow$ Fermi momentum $p_0 < m_e c$
  degenerate electrons are non-relativistic
  $P \propto \rho^{5/3}$: $\gamma = 5/3$ means stability!
- but as mass increases, Fermi momentum p_0 increases
  to $p_0 \geq m_e c$: electrons become relativistic
- as $M \to M_{\text{Chandra}}$, then $\gamma \to 4/3$
  fully relativistic $\to$ unstable!

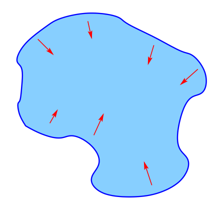
Star Formation: Birth to Main Sequence

Star Formation

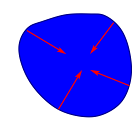
the formation of stars (and planets) remains mysterious
deserves its own course: **Astronomy 405**
offered this coming semester!

here: we sketch some highlights
so you see how (proto)stars approach the main sequence

basic idea:



- huge, *low-density interstellar gas clouds*
- collapse under their own gravity



- and likely fragment into "protostellar cores"
- which contract until nuclear reactions ignite to signal marking the *zero age main sequence*

iClicker Poll: Fuel for Star Formation

interstellar gas clouds exist in several forms

Which of these is most favorable to gravitational collapse?

Hint: we now want to be *out* of hydrostatic equilibrium!

A ionized gas: mostly free p and e

B atomic gas: mostly H = pe atoms

C molecular gas: mostly H_2 = HH molecules

Conditions for Cloud Collapse

to collapse, clouds must *not* be in hydrostatic equilibrium!
gravity must overwhelm pressure gradients

at low interstellar densities: classical ideal gas (non-degenerate)
$P = n\ kT$: low T means low P

but kT also sets particle kinetic energy scales
compare to **binding energy**
B = energy to tear gas particle apart

- molecular hydrogen: $B(\mathrm{H_2}) = 4.5$ eV
- atomic hydrogen: $B(\mathrm{H}) = 13.6$ eV
- ionized hydrogen: already torn apart—unbound!

Q: lessons?

Molecular Gas is Star Formation Fuel

lessons:

- molecular hydrogen has smallest binding energy
  requires coldest temperatures to survive collisions
- as T rises, molecules $\rightarrow$ torn to atoms $\rightarrow$ torn to ions
- *collapse and star formation most likely in* molecular gas

our Galaxy and other galaxies contain **giant molecular clouds**

- made mostly of molecular hydrogen H_2
- but most easily seen via CO carbon monoxide molecules
- typical giant molecular cloud conditions
- mass $M \sim 10^5 M_\odot$, size $R \sim 10$ pc, temperature $T \sim 20$ K
  can be opaque to optical light, visible in IR and radio

`www: molecular clouds`

Conditions for Collapse

consider a cloud of mass M, radius R, temperature T
with average particle mass $m_{\rm g}$

Sir James Jeans (1902): when does collapse occur?

if hydrostatic equilibrium $\longrightarrow$ Virial theorem

$$\frac{GM^2}{R} \sim NkT = \frac{M}{m_{\rm g}} kT$$

Q: condition for gravitational collapse?
Q: critical radius? critical density?
Q: which is easier to collapse–large cloud or small?

Gravitational Instability

condition for equilibrium: Virial theorem

$$\frac{GM^2}{R} \sim NkT = \frac{M}{m_\mathrm{g}} kT$$

gravitational collapse requires *dis*equilibrium: Jeans instability

$$\begin{aligned} \frac{GM^2}{R} &\gg NkT = \frac{M}{m_\mathrm{g}} kT \\ R &\ll R_\mathrm{J} = \frac{G m_\mathrm{g} M}{kT} \qquad (3) \\ \rho &\gg \rho_\mathrm{J} \sim \frac{M}{R_\mathrm{J}^3} \sim \left(\frac{kT}{G m_p}\right)^3 \frac{1}{M^2} \qquad (4) \end{aligned}$$

Jean mass, radius, and density

$\rho_\mathrm{J} \propto 1/M^2$: highest mass has lowest critical density

Q: timescale for collapse?

Initial Collapse: Freefall

Initially, Jeans unstable cloud:

- has large gravitational potential energy
- by definition, has negligible thermal pressure
- has low density: long mean free path $\ell_{\rm mfp} = 1/n\sigma$ for photons inside cloud

so collapse begins in free fall – gravity unopposed with gravitational (dynamic) timescale (PS2)

$$\tau_{\rm ff} \sim \frac{1}{\sqrt{G\rho}}$$

frefall continues until gravitational energy trapped and turned into random motions → thermalized

Q: condition for trapping energy/heat?

Q: other nonthermal work the released energy can do?

From Freefall to Thermalization

collapse $\rightarrow$ heating: higher T $\rightarrow$ blackbody flux $F \propto T^4$
but at first, photon mean free path $\ell = 1/n\sigma \gtrsim R$
"optically thin" $\rightarrow$ radiation escapes: cloud cools

when density increases, $\ell \lesssim R$ and energy trapped
but can be used to break bounds
unbind $\mathrm{H_s}$ and ionized H

if a fraction $X \approx 0.75$ of gas mass is hydrogen
- energy to dissociate H_2 molecules: $E(\mathrm{H_2}) = XM/2m_p \; B(\mathrm{H_2})$
- energy to ionize H atoms: $E(\mathrm{H}) = XM/m_p \; B(\mathrm{H})$
- total energy to reach full ionization $E_{\mathrm{ion}} = E(\mathrm{H_2}) + E(\mathrm{H})$
- leaves gas at temperature set by $E_{\mathrm{ion}}N \; kT = M \, kT/m_{\mathrm{g}}$

$$kT \sim X \left(\frac{1}{2}B(\mathrm{H_2}) + B(\mathrm{H})\right) \sim k \times 30{,}000 \text{ K} \tag{5}$$