Astro 404

Lecture 27

Oct. 30, 2019

Announcements:

- **Problem Set 8 due Fri Nov 1**
- Typo Fixed Tue Morning Q1(b) should read "find $d/dr(r^2\, d\rho/dr)$" – correct version posted
- Office Hours: Instructor – after class or by appointment
- TA: Thursday noon-1pm or by appointment

Last time: star formation

Q: what's the fuel for making stars? why?

Molecular Gas is Star Formation Fuel

lessons:

- molecular hydrogen has smallest binding energy
  requires coldest temperatures to survive collisions
- as T rises, molecules $\rightarrow$ torn to atoms $\rightarrow$ torn to ions
- *collapse and star formation most likely in* molecular gas

our Galaxy and other galaxies contain **giant molecular clouds**

- made mostly of molecular hydrogen H_2
- but most easily seen via CO carbon monoxide molecules
- typical giant molecular cloud conditions
- mass $M \sim 10^5 M_\odot$, size $R \sim 10$ pc, temperature $T \sim 20$ K
  can be opaque to optical light, visible in IR and radio

`www: Orion giant molecular cloud`

Conditions for Collapse

consider a cloud of mass M, radius R, temperature T
with average particle mass $m_{\rm g}$

Sir James Jeans (1902): when does collapse occur?

if hydrostatic equilibrium $\longrightarrow$ Virial theorem

$$\frac{GM^2}{R} \sim NkT = \frac{M}{m_{\rm g}} kT$$

Q: condition for gravitational collapse?
Q: critical radius? critical density?
Q: which is easier to collapse–large cloud or small?

Gravitational Instability

condition for equilibrium: Virial theorem

$$\frac{GM^2}{R} \sim NkT = \frac{M}{m_{\mathrm{g}}}kT$$

gravitational collapse requires *dis*equilibrium: Jeans instability

$$\begin{aligned} \frac{GM^2}{R} &\gg NkT = \frac{M}{m_{\mathrm{g}}}kT \\ R &\ll R_{\mathrm{J}} = \frac{Gm_{\mathrm{g}}M}{kT} & (1) \\ \rho &\gg \rho_{\mathrm{J}} \sim \frac{M}{R_{\mathrm{J}}^3} \sim \left(\frac{kT}{Gm_p}\right)^3 \frac{1}{M^2} & (2) \end{aligned}$$

Jean mass, radius, and density

$\rho_{\mathrm{J}} \propto 1/M^2$: highest mass has lowest critical density

Q: timescale for collapse?

Initial Collapse: Freefall

Initially, Jeans unstable cloud:

- has large gravitational potential energy
- by definition, has negligible thermal pressure
- has low density: long mean free path $\ell_{\rm mfp} = 1/n\sigma$ for photons inside cloud

so collapse begins in free fall – gravity unopposed with gravitational (dynamic) timescale (PS2)

$$\tau_{\rm ff} \sim \frac{1}{\sqrt{G\rho}}$$

real interstellar clouds have *nonuniform density*

Q: if there are density fluctuations, how does collapse proceed?

Q: what does this mean for the collapsing cloud?

Fragmentation: Birth of Protostars

freefall time: $\tau_{\rm ff} \sim 1/\sqrt{G\rho}$
for non-uniform density cloud:

- high-ρ regions have shortest $\tau_{\rm ff}$: collapse fastest
- in these high-ρ regions, collapse makes density even higher even faster collapse
- and high-ρ substructures collapse faster still

overall picture: cloud **fragmentation**
into many smaller collapsing objects
and highest density knots collapse fastest $\rightarrow$ **protostars**
`www: protostars in Eagle Nebula`

frefall continues until gravitational energy trapped
and turned into random motions $\rightarrow$ thermalized
Q: condition for trapping energy/heat?
Q: other nonthermal work the released energy can do?

From Freefall to Thermalization

collapse $\rightarrow$ heating: higher T $\rightarrow$ blackbody flux $F \propto T^4$
but at first, photon mean free path $\ell = 1/n\sigma \gtrsim R$
"optically thin" $\rightarrow$ radiation escapes: cloud cools

when density increases, $\ell \lesssim R$ and energy trapped
but can be used to break bounds
unbind H_s and ionized H

if a fraction $X \approx 0.75$ of gas mass is hydrogen

- energy to dissociate H_2 molecules: $E(\mathrm{H_2}) = XM/2m_p \; B(\mathrm{H_2})$
- energy to ionize H atoms: $E(\mathrm{H}) = XM/m_p \; B(\mathrm{H})$
- total energy to reach full ionization $E_{\rm ion} = E(\mathrm{H_2}) + E(\mathrm{H})$
- leaves gas at temperature set by $E_{\rm ion} N \; kT = M \, kT/m_{\rm g}$

$$kT_{\rm ionized} \sim X \left(\frac{1}{2} B(\mathrm{H_2}) + B(\mathrm{H}) \right) \sim k \times 30{,}000 \ \mathrm{K} \qquad (3)$$

The Hayashi Limit

Chushiro Hayashi (1960's):
as protostars collapse in near-freefall
interior well-mixed $\longrightarrow$ uniform temperature
[star is fully convective – see below]

while gravitational energy release used to ionize star
temperature remains nearly constant until T_{ionized}

iClicker Poll: Protostars and the H-R Diagram

while protostars in freefall
temperature nearly uniform out to photosphere
and nearly constant despite contraction

How will protostar luminosity change?

A L *increases* with collapse

B L *decreases* with collapse

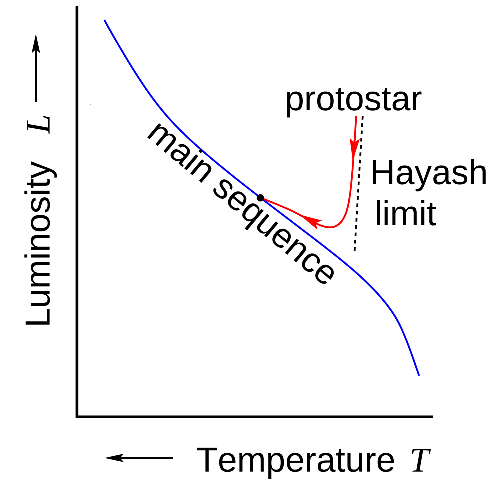
C L *nearly constant* with collapse

Q: how will this appear on H-R diagram?

Protostars on the H-R Diagram

if T uniform and nearly constant during collapse

- the $L = 4\pi R^2 \sigma T^4 \;\propto\; R^2$:
  contraction decreases L
- on H-R diagram (T_{eff}, L):
  nearly vertical drop on "Hayashi track"
- until minimum L when fully ionized



then: further collapse raises temperature (and density)
until nuclear reactions begin

- temperature becomes non-uniform (hotter in core)
- protostar luminosity gradually increases
- until collapse halted entirely: hydrostatic equilibrium at last!
- star joins main sequence! "**zero age main sequence**"

the Main Sequence Across Stellar Masses

main sequence recap:

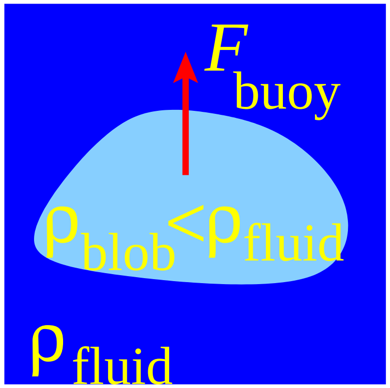
- longest-lived stellar phase
- in hydrostatic equilibrium
- pressure support – non-degenerate
- low mass stars: gas pressure dominates
  high mass stars: radiation pressure dominates
- luminosity powered by core hydrogen fusion

evolution on main sequence:

- as core hydrogen depleted
- $(\rho_{\mathrm{C}}, T_{\mathrm{C}})$ increase $\rightarrow L$ increase
- main sequence brightening

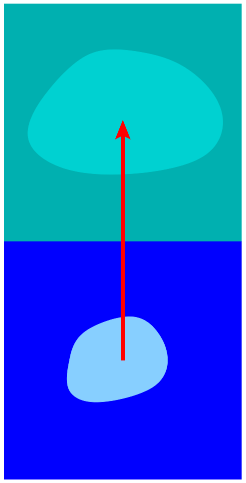
future star evolution depends crucially on size and mass
of **helium core** ash of hydrogen burning
which is depends on mixing during main sequence phase

Buoyancy and Gas Transport



Archimedes principle: buoyancy of object in fluid
equal to *weight of displaced fluid*
so a low-density blob
has less mass and weight for its volume
$\rightarrow$ positively buoyant $\rightarrow$ rises

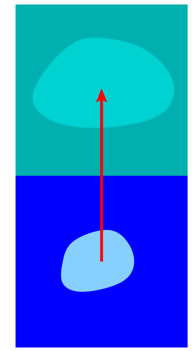
we have seen: stars stratified
inward towards core: increased ρ, T, P



consider blob of gas in star, displaced upward
expands to match lower surrounding pressure
Q: what if new blob density higher than surroudings?
Q: and if it is lower?
Q: condition for stability?
Q: effect of instability?

Convection and Stability

upward displaced blob comes into pressure equilibrium:



- if new blob density $>$ surrounding fluid:
  negatively buoyant $\rightarrow$ sinks back down: stable
- if new blob density $<$ surrounding fluid:
  positively buoyant $\rightarrow$ continues to rise: unstable

convection: `www: solar granulation`

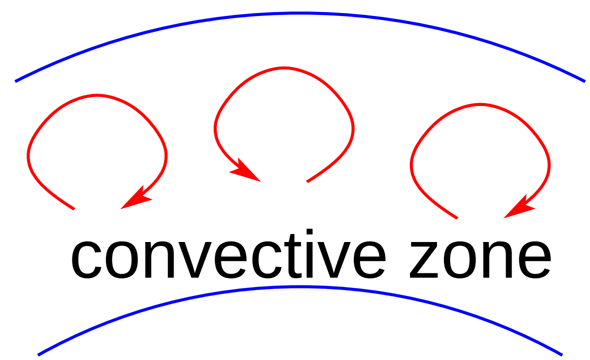
rising hot blob, sinking cooler blobs

examples: air above flame, soup on stove – T high at base

instability due to strong temperature gradient

convective motions:

- mix material
- transport heat
- reduce temperature gradient



Adiabatic Gas

consider a *blob of gas* that *expands or contracts without exchanging energy with its environment*
for example, rapid change, not time to radiate energy

no energy exchange: total energy (heat) constant
internal energy changes due to pdV work

$$dU = -P\,dV$$

non-relativistic, nondegenerate ideal gas: $U = 3/2\ PV$
relativistic, nondegenerate gas: $U = 3\ PV$

for $U = w\ PV$:

$$w\,d(PV) = wP\,dV + V\,dP \;=\; -P\,dV \tag{4}$$

$$w\,V\,dP = -(w+1)\ P\,dV \tag{5}$$

$$\frac{dP}{P} \;=\; -\frac{w+1}{w}\frac{dV}{V} \tag{6}$$

so for an adibatic change (no heat exchange)

$$\frac{dP}{P} = -\frac{w+1}{w}\frac{dV}{V} \tag{7}$$

$$\log P = -\frac{w+1}{w}\log V + C \tag{8}$$

$$P \propto V^{-(w+1)/w} \tag{9}$$

$$P_{\rm adiabatic} = K\ \rho^{(w+1)/w} \tag{10}$$

for adiabatic changes: pressure set by density alone!

proportionality K depends on gas heat content

non-relativistic, nondegenerate ideal gas: $w = U/PV = 3/2$

$$P_{\rm adiabatic,nr} \propto \rho^{5/3}$$

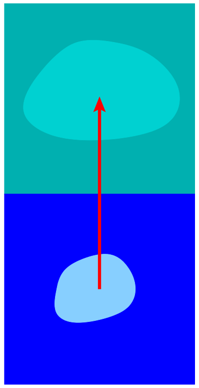
relativistic, nondegenerate gas: $w = 3$

$$P_{\rm adiabatic,rel} \propto \rho^{4/3}$$

same scalings as for degenerate cases!

Convection in Stars

When does convection set in?
Depends on pressure gradient



consider blob a radius r with $\rho(r)$ and $P(r)$
displaced upward: $r \to r' = r + \delta r$

- rapid motion $\to$ *adiabatic change*
- expands to pressure equilibrium at new location
  new pressure $P_{\text{blob}} = P(r')$

Q: if star region has $P = K\rho^\gamma$, what does blob do?
Q: what if region has $P < K\rho^\gamma$?
Q: what if region has $P > K\rho^\gamma$?

blob initially has $\rho(r)$ and $P(r)$
displaced, new regions has $P(r') = P(r + \delta r)$
adiabatic expansion: $P(r') = P_{\text{blob}} = K\rho_{\text{blob}}^{\gamma}$

if star region has $P = K\rho^{\gamma}$, then:

- $P(r') = K\rho(r')^{\gamma}$
- so surrounding medium has $\rho(r') = \rho_{\text{blob}}$
- *neutrally buoyant* – *no further motion*

if $P(r') > K\rho(r')^{\gamma}$ then
$\rho_{\text{blob}}^{\gamma} > \rho^{\gamma}(r')$ so $\rho_{\text{blob}} > \rho(r')$
negatively buoyant $\rightarrow$ convectively stable

if $P(r') < K\rho(r')^{\gamma}$ then $\rho_{\text{blob}} < \rho(r')$
positively buoyant $\rightarrow$ convectively unstable!
Q: conclusion–when does convection occur?

Convection and Adiabatic Gradients

lesson:

- convection occurs when $P(r') < K\rho(r')^{\gamma}$
- when P decreases with r more steeply than adiabatic

ideal gas: $P = \rho\, kT/m_{\mathrm{g}}$

so adibatic gas with $P_{\mathrm{ad}} \propto \rho^{\gamma} \propto (P_{\mathrm{ad}}/T)^{\gamma}$ has $P_{\mathrm{ad}} \propto T^{\gamma/(\gamma-1)}$

convection condition:

$dP/P < dP_{\mathrm{ad}}/P_{\mathrm{ad}} = \gamma/(\gamma-1)\ dT/T$, so temperature gradient

$$\frac{dT}{dr} > \frac{\gamma-1}{\gamma}\ \frac{T}{P}\ \frac{dP}{dr} \tag{11}$$

so *steep temperature gradient leads to convection*

and then flows mix material, smooth the temperature gradient