Astro 404
Lecture 28
Nov. 1, 2019

Announcements:

- **Problem Set 8 due today 5:00 pm**

  please proofread your upload–this is your responsibility

  Last-minute office hours: after class today
- **Problem Set 9 due Fri Nov 8**

Next week:

Astronomy Colloquium Tue Nov 5, 3:45pm
Nan Liu, Washington U. St. Louis
"Laboratory Astronomy Using Microscopes"
stardust in the lab!

Astro Courses for Spring 2019

your ASTR404 superpowers definitely qualify you for

- ASTR 405: Planetary Systems
  here especially, 404 knowhow will pay off!
- ASTR 414: Astronomical Techniques

and you might consider (talk to instructor)

- ASTR 596 AST: Fundamentals of Data Science
  Astrostatistics from Prof. Narayan
- ASTR 507: Physical Cosmology
  Prof. BDF

Last time: convection in stars

Q: what's convection? examples?

Q: when does convection occur?

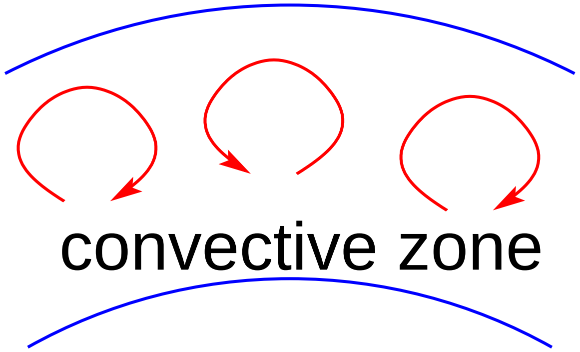
Q: why is it important for stars?

Convection in Stars: Recap

convection: instability that drives fluid circulation
due to temperature gradient
hot buoyant gas rises while cooler gas sinks

convective motions:

- mix material
- transport heat
- reduce temperature gradient



convection in stars: `www: simulation`

- can act as a fuel supply – circulating new material to burning regions
- can take the place of radiative diffusion as means of moving energy out from core
- smooths temperature structure

Adiabatic Gas

consider a *blob of gas* that *expands or contracts*
without exchanging energy with its environment
for example, rapid change, not time to radiate energy

no energy exchange: total energy (heat) constant
internal energy changes due to pdV work

$$dU = -P\,dV$$

non-relativistic, nondegenerate ideal gas: $U = 3/2\;PV$
relativistic, nondegenerate gas: $U = 3\;PV$

for $U = w\;PV$:

$$\begin{aligned} w\,d(PV) = wP\,dV + V\,dP &= -P\,dV && (1)\\ w\,V\,dP = -(w+1)\;P\,dV & && (2)\\ \frac{dP}{P} &= -\frac{w+1}{w}\frac{dV}{V} && (3) \end{aligned}$$

so for an adiabatic change (no heat exchange)

$$\frac{dP}{P} = -\frac{w+1}{w}\frac{dV}{V} \tag{4}$$

$$\log P = -\frac{w+1}{w}\log V + C \tag{5}$$

$$P \propto V^{-(w+1)/w} \tag{6}$$

$$P_{\mathrm{adiabatic}} = K\ \rho^{(w+1)/w} = \rho^{\gamma} \tag{7}$$

for adiabatic changes: pressure set by density alone!

- proportionality K depends on gas heat content
- index depends on γ on energy/pressure ratio w

Note also: *an adiabatic gas is a polytrope!*

- but doesn't have to be degenerate
- applies to ordinary, non-degenerate, ideal gasses
  as long as heat content fixed–restricts P, V, T possibilities
- so two ideal gasses with same ρ but different T
  have different heat contents ("occupy different adiabats")
- for experts: higher heat ⇔ higher entropy

Adiabatic Ideal Gas

adiabatic gas with $U/PV = w$:

$$P_{\text{adiabatic}} = K\ \rho^{(w+1)/w} = \rho^{\gamma} \tag{8}$$

non-relativistic, nondegenerate ideal gas: $w = U/PV = 3/2$

$$P_{\text{adiabatic,nr}}\ \propto\ \rho^{5/3}$$

relativistic, nondegenerate gas: $w = 3$

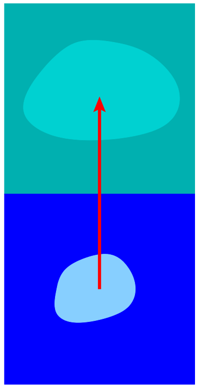
$$P_{\text{adiabatic,rel}}\ \propto\ \rho^{4/3}$$

same scalings as for degenerate cases!

⇝ expect non-relativistic ideal gasses to be dynamically stable and relativistic gasses to be dynamically unstable

Convection in Stars

When does convection set in?
Depends on pressure gradient



consider blob a radius r with $\rho(r)$ and $P(r)$
displaced upward: $r \to r' = r + \delta r$

- rapid motion → *adiabatic change*
- expands to pressure equilibrium at new location
  new pressure $P_{\text{blob}} = P(r')$

Q: if star region has $P = K\rho^{\gamma}$, what does blob do?
Q: what if region has $P < K\rho^{\gamma}$?
Q: what if region has $P > K\rho^{\gamma}$?

blob initially has $\rho(r)$ and $P(r)$
displaced, new regions has $P(r') = P(r + \delta r)$
adiabatic expansion: $P(r') = P_{\text{blob}} = K\rho_{\text{blob}}^{\gamma}$

if star region has $P = K\rho^{\gamma}$, then:

- $P(r') = K\rho(r')^{\gamma}$
- so surrounding medium has $\rho(r') = \rho_{\text{blob}}$
- *neutrally buoyant* – *no further motion*

if $P(r') > K\rho(r')^{\gamma}$ then
$\rho_{\text{blob}}^{\gamma} > \rho^{\gamma}(r')$ so $\rho_{\text{blob}} > \rho(r')$
negatively buoyant → convectively stable

if $P(r') < K\rho(r')^{\gamma}$ then $\rho_{\text{blob}} < \rho(r')$
positively buoyant → convectively unstable!
Q: conclusion–when does convection occur?

Convection and Adiabatic Gradients

lesson:

- convection occurs when $P(r') < K\rho(r')^{\gamma}$
- when P decreases with r more steeply than adiabatic

ideal gas: $P = \rho\, kT/m_{\mathrm{g}}$

so adibatic gas with $P_{\mathrm{ad}} \propto \rho^{\gamma} \propto (P_{\mathrm{ad}}/T)^{\gamma}$ has $P_{\mathrm{ad}} \propto T^{\gamma/(\gamma-1)}$

convection condition:

$dP/P < dP_{\mathrm{ad}}/P_{\mathrm{ad}} = \gamma/(\gamma-1)\ dT/T$, so temperature gradient

$$\frac{dT}{dr} > \frac{\gamma-1}{\gamma}\,\frac{T}{P}\,\frac{dP}{dr} \tag{9}$$

so *steep temperature gradient leads to convection*

and then flows mix material, smooth the temperature gradient

⇒ alters the structure of stars

⇒ also introduces uncertainty – hard to model accurately!

iClicker Poll: Convection and Main Sequence Stars

turns out: some main sequence stars have convective cores and some do not

For a given star, what difference does convective core make?

A extends the main sequence lifetime of the star

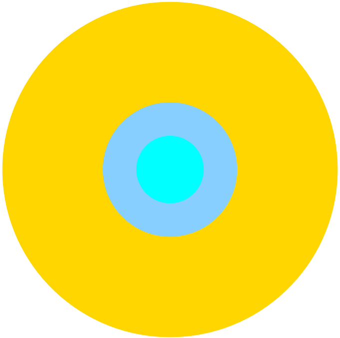
B increases mass of helium made during main sequence

C makes core temperature more uniform

D more than one of the above

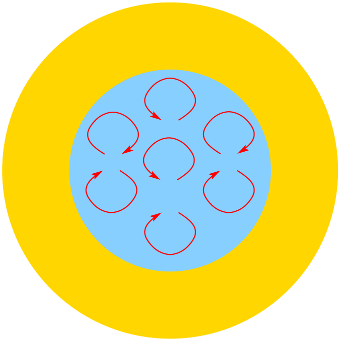
E none of the above

Convective Cores of Stars



if stellar core is *not convective*

- core gas is not stirred
- helium ash remains where formed
- no new fuel available when H depleted



if core *is convective:*

- gas circulates through entire convective zone
- hydrogen fuel and helium ash mixed
- new fuel brought downward
- so all hydrogen in convective zone available to burn

result: a star with convective core
burns more hydrogen, makes more helium, and lives longer than a star without convection

iClicker Poll: Guess the Convection

Which main sequence stars have convective cores?

Hint: what drives convection, and how is H burned?

A high-mass main sequence stars

B low-mass main sequence stars

Convection on the Main Sequence

convection driven by strong temperature gradients
think soup on a stove!

high-mass stars:
- have high $\langle kT \rangle \sim GMm_{\mathrm{g}}/R$
- burn H in CNO cycle: energy generation $q_{\mathrm{CNO}} \propto T^{16}$!

severe T dependence drives T gradient and thus convection

so high-mass main sequence stars have convective cores
- burn fuel more efficiently, extends still-short lifetime
- at end of main sequence, convection sets mass and size of helium core

but *core of low-mass stars (including Sun) not convective*
- energy transport is radiative diffusion
- helium core size set by region of helium production

Beyond the Main Sequence

Post-Main Sequence Stellar Evolution

recall: energy conservation teaches: *all stars must die!*

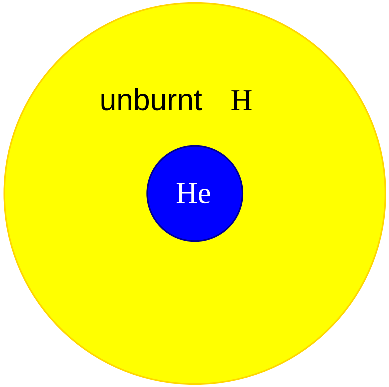
how a star dies is controlled primarily by its **mass**
though binarity, rotation, composition also important

at end of main sequence: *helium core is about 10% of star mass*

we first consider effects for low-mass (solar-type) stars

iClicker Poll: A Helium-Core Sun

What happens when *all* core H converted to He?



A the Sun's core expands

B the Sun's core contracts

C the Sun begins to burn helium

D the Sun ignites unburnt hydrogen outside core

Low Mass Stars: Core Hydrogen Exhaustion

as core hydrogen exhausted, core fusion ends
heat loss → lower temperature → lower pressure

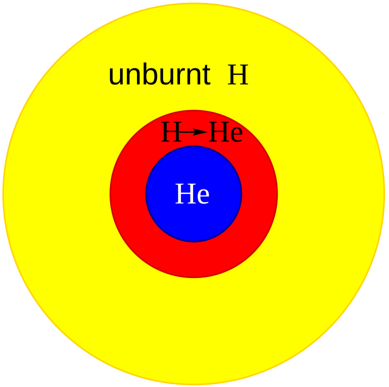
hydrostatic equilibrium lost: *core contracts*

- core density rises until degeneracy sets in

- contraction halts when degeneracy pressure supports core

Q: effect on region surrounding helium core?

Hydrogen Shell Burning



as helium core contracts

- H material overlying core also contracts, heats new fuel, can begin to fuse!
  → **H burning in shell around core**

H shell burning occurs above degenerate core

- high density and temperature: high L
- increases mass of He core, shell thins and propagates out

Q: response of outer layers–envelope?

Red Giant Phase

injection of energy in shell throws envelope out of equilibrium

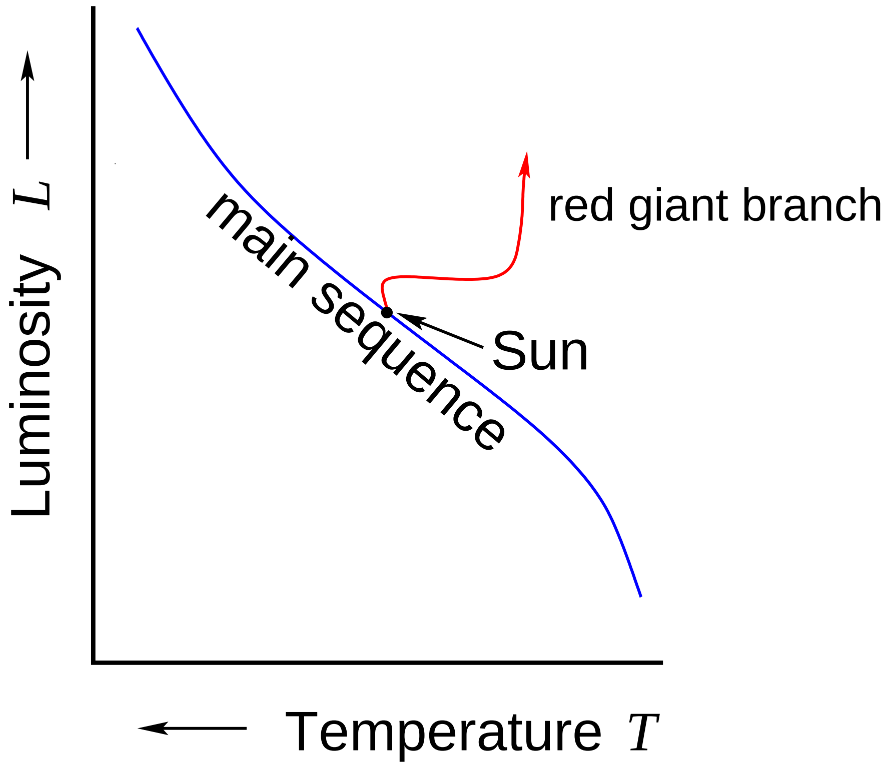
- outer layers of star expand by factor ~ 100
- so surface $T_{\text{eff}} \propto L^{1/4}/R^{1/2}$ drops
- star becomes **red giant**

note "mirror" effect of shell burning

core contraction, envelope expansion

Q: movement of star on HR diagram?

HR Diagram: Red Giant Phase



`www: Gaia observed HR diagram for field stars`

Q: how to test?