Astronomy 596/496 PC Spring 2010
Problem Set #1

Due in class: Friday, Feb. 5

Total points: 10+2

Note: Your homework solutions should be legible and include all calculations, diagrams, and explanations. The TA is not responsible for deciphering unreadable or illegible problem sets! Also, homework is graded on the method of solution, not just the final answer; you may not get any credit if you just state the final answer!

Note to ASTR496PC Students: You may drop choose to drop all of Problem 2.

1. *Sloan Digital Sky Survey (SDSS): Galaxy Redshifts, Distances, and Densities.* The Sloan survey contains a tremendous amount of cosmological data, most of it publicly available online. From the course `links` page, follow the *navigate* link to go to the SDSS `Navigate Tool`, which will start you on a nearby (and thus large and bright) spiral galaxy; this will be the starting point of your journey. Gathering a bit of data, you can quickly arrive at some interesting cosmological conclusions.

   In the navigate tool, wander around the digital sky. Note that you can zoom in and out, and that you can (and should) add a grid and labels to the field of view.

   You will gather and manipulate data for several galaxies; I recommend using a spreadsheet or a simple computer program to keep track of the bookkeeping.

   (a) **[1 point]** Your first task as a cosmologist is to compute the redshifts of a handful of real galaxies. To do this, you will need galaxy spectra. Not all SDSS galaxies have spectra, but using the `SpecObjs` option you can identify those that do. Find at least 5 such galaxies randomly (be sure they are galaxies and not stars!). For each, click on the galaxy in the image, then click the `Explore` button to find data and a spectrum for the galaxy. Using rest wavelengths from the SDSS line list (also linked from the course page), compute redshifts for two lines for each galaxy, and show that these agree with each other, and with the value computed by the automated software. Also for each galaxy, make note of its r-band magnitude, the middle value in the $ugriz$ entries.

   For each galaxy, use Hubble's law to find its recession speed, and distance in Mpc. Compute the average distance to your set of galaxies. About what fraction of the Hubble length is probed by SDSS?

   (b) **[1 point]** The flux from each galaxy (in different wavelength bands–i.e., colors) are given as the values of $ugriz$, which are expressed as apparent magnitudes.[1] For each galaxy in (a), use the observed r-band magnitude, which is centered at 625 nm and thus "red." Together with the absolute r-band magnitude of

[1] If you have not yet encountered the charms of the astronomical magnitude system, you are in for a treat. Peacock has a discussion of magnitudes, but this skips the basics. To summarize, the magnitude system is just a logarithmic scale for flux, but with a crazy base (2.5) and the "wrong" sign. Two objects with observed fluxes F_1 and F_2 have ("apparent") magnitudes which differ by $m_2 - m_1 = -2.5 \log_{10}(F_2/F_1)$. In addition, an object's luminosity is effectively defined via its *absolute* magnitude which is the apparent magnitude one *would* observe if the object were placed at distance $r = 10$ pc, that is, $M = m(10\,\mathrm{pc}) = m - 5\log_{10}(r/10\,\mathrm{pc})$. Any introductory astronomy text (.e.,g Carroll & Ostlie, on reserve) discusses these issues in detail.

the Sun $M_{\odot,r} = 4.4$ mag, to compute the galaxy's r-band luminosity in units of $L_{\odot,r}$. Then compute the average galaxy luminosity $\langle L \rangle$, in units of $L_{\odot,r}$.
Compare your results with the Milky Way luminosity, $L_{\mathrm{MW},r} \sim 2 \times 10^{10} L_{\odot}$, and comment.

(c) **[1 point]** On the SDSS site, use the `Navigate` tool to find a random spot in the sky. Then use the `Chart` tool to identify all galaxies *brighter* (in the r-band) than your measured average–i.e., show all galaxies with magnitude $m_r < \langle m_r \rangle$ (watch out for the fiendish magnitude sign convention!). Use the zoom feature to adjust the field until there are a manageable (say, 10–20) number $N(< \langle m_r \rangle)$ of such galaxies. Finally, use the label feature to show the angular size per pixel along with the width and height parameters to determine the angular area $\Delta\Omega$ in the field of view. Use this to compute a sky surface density $dN_{\mathrm{gal,SDSS}}/d\Omega$ of galaxies per square arcmin.

(d) **[1 point]** Use your average distance to SDSS galaxies, your value for $dN_{\mathrm{gal,SDSS}}/d\Omega$, and homogeneity to estimate the number density $n_{\mathrm{gal}}(< \langle m_r \rangle)$ of galaxies brighter than $\langle m_r \rangle$ in the universe today. Express your answer in objects Mpc^{-3}. Then go on to estimate the total number of such galaxies in the entire observable universe. This is a handy number to keep in mind. How does your estimate compare to the total number of human beings alive on Planet Earth today?

(e) **[1 point]** Use the average luminosity $\langle L \rangle$ of your galaxies, and your estimate of their number density, to estimate the galaxy luminosity density $\mathcal{L}$ in units of $L_{\odot}\ \mathrm{Mpc}^{-3}$. (This is not a hard problem!)
Then use $\mathcal{L}$ to find the mass density ρ of the universe, in $M_{\odot}\ \mathrm{Mpc}^{-3}$. Do this assuming that the average mass-to-light ratio Υ is that of

i. local stars
ii. galaxy halos
iii. galaxy clusters

Comment on the impact of the difference in these mass-to-light ratios. Which one do you think is the most appropriate for determining the mass density of the universe?
The mass density derived from the mass-to-light ratio for local stars is sometimes called the density of luminous matter ρ_{lum}. Use your value of ρ_{lum} to compute Ω_{lum}. Compare your answer with Ω_{matter} and Ω_{baryon}, and comment.

(f) **[1 bonus point]** For a bonus point, comment on complications that would be involved in firming up some of the estimates we have made in this problem.[2]

2. *SDSS Part Deux: "Size" Distribution*

(a) **[1 point]** Consider the number $N_{\mathrm{gal}}(< d)$ of galaxies found by a survey which reaches out to some distance d, the survey "depth." This is sometimes known as the cumulative "size" distribution since you should find that the result increases with the survey depth and hence the size of the telescope used for the survey.

[2] Note for example that SDSS does not randomly choose galaxies for spectroscopic redshifts, but among other things demands that a galaxy is bright enough so that it will yield sufficient photons to make a meaningful spectrum.

Assuming homogeneity, how should $N_{\rm gal}(< d)$ scale with d?

Also assume for the moment that all galaxies have the same luminosity L. In this case, the survey depth is given by the minimum flux $F_{\rm min}$ the survey instrument can detect; show how d scales with the flux limit $F_{\rm min}$. Then show how the number $N_{\rm gal}(> F)$ of observed galaxies brighter than F scales with $F = F_{\rm min}$. Finally, if $m_{\rm max}$ is the magnitude limit that corresponds to flux limit $F_{\rm min}$, show how $N_{\rm gal}(< m)$ scales with $m = m_{\rm max}$.

(b) **[1 point]** Now apply your results: still using the `Finding Chart` tool, find a random field and plot it with an image scale of about 1.58 arcsec/pix. Then instead of plotting all galaxies, plot only the ones with r magnitude between, say, 0 and 15 mag. Then increase the maximum magnitude, and watch the change in the number of objects. Comment on the behavior you see. How well does your prediction work? When does it break down, and why?

3. *The Friedmann Equation: Limiting Cases.* In class, we studied a *matter-dominated* universe, and explicitly solved for some properties of this universe. Here we will generalize this treatment.

(a) **[1 point]** Using the Friedmann equation in a "generic" universe with matter, radiation, Λ, and curvature, show that time and the scale factor are related by

$$t/t_H = \int_0^a \frac{u\,du}{\sqrt{\Omega_{\rm m}u + \Omega_{\rm r} + \Omega_\Lambda u^4 + (1-\Omega_0)u^2}} \tag{1}$$

i.e., the age t is given by the present Hubble time $t_H = H_0^{-1}$ times a integral which depends upon the cosmological parameters

(b) **[1 point]** Find expressions for $a(t)$ for epochs in which the universe is dominated by either matter, or radiation, or curvature, or Λ. Note that these components do not necessarily dominate today, so that you should not assume any $\Omega_i = 1$. That is, each of you answers should show a dependence on the present value of the relevant Ω_i.

(c) **[1 point]** Using the $a(t)$ solutions above, find $\rho(t)$ the case of a radiation-dominated and then in the case of a matter-dominated universe. In both cases, express your answer in terms of the dimensionless parameters $\Omega_{\rm m}$ or $\Omega_{\rm r}$, and only the cosmic time t and physical constants like G (but not including H_0 and ρ_0). Also show the relationship between ρ_0 and t_0 for each case. *Hint:* consider the Friedmann equation as an expression for ρ.

(d) **[1 bonus point]** To a very good approximation, we can regard the present-day universe as having (for dynamical purposes) only matter and Λ. For this case, show that we can exactly solve for the age of the universe as

$$H_0 t_0 = \frac{2}{3\sqrt{\Omega_\Lambda}} \sinh^{-1}\sqrt{\frac{\Omega_\Lambda}{\Omega_{\rm m}}} \tag{2}$$

For the present "concordance" values $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$, evaluate[3] t_0 in Gyr, compare to the Hubble time H_0^{-1}, and comment.

[3] If you calculator doesn't have a $\sinh^{-1}$ key, one can show that $\sinh^{-1}(x) = \ln(x + \sqrt{1+x^2})$.