Astronomy 596 PC Spring 2010
Problem Set #7: Erratum

Thanks to an alert student, I became aware of a mistake in problem 2(a). I will not grade this problem, so if you already turned it in, no need to revise your response. Furthermore, if you haven't yet turned it in, you may simply skip 2(a). For completeness included a updated version of 2(a) in this erratum.

2(a) [**1 point**] Consider a quasar at redshift z_q, with rest-frame spectrum which includes photons at the Lyα rest wavelength λ_α and at shorter wavelengths. Hydrogen along the line of sight to the quasar will suffer Lyα absorption for quasar radiation which is at λ_α *in the absorber rest frame,*

The cross section for Lyα absorption, as a function of absorber frequency $\nu = c/\lambda$, is very narrowly peaked around $\nu_\alpha = c/\lambda_\alpha$, so we take:

$$\sigma(\nu) = \sigma_0 \; \nu_\alpha \; \delta(\nu - \nu_\alpha) \tag{1}$$

where $\sigma_0 \sim 8\times10^{-18}$ cm^2, and where δ is a Dirac delta function, such that $\int \delta(\nu')\, d\nu' = 1$. Consequently, if we at $z = 0$ observe at some frequency $\nu > (1+z_q)\nu_\alpha$ (i.e., blueward of the quasar Lyα) then some of the radiation will be absorbed. For an observer frequency ν the absorption will be due to a shell of material at redshift $z = \nu/\nu_\alpha - 1$.

Show that, in a matter-dominated universe, the optical depth for Lyα absorption out to some redshift z is

$$\tau_{\rm HI}(z) = \int n_{\rm HI,phys} \; \sigma\left[(1+z)\nu - \nu_\alpha\right] \; d\ell \tag{2}$$

$$= \frac{1}{\Omega_{\rm m}^{1/2}} \; \sigma_0 \; n_{\rm HI}(z) \; \frac{c}{H_0}(1+z)^{-3/2} \tag{3}$$

where $n_{\rm HI,phys}(z)$ is the (assumed homogeneous) physical density of neutral hydrogen at redshift z, and the photon pathlength per unit redshift is $dl = cdt$.