Astro 596/496 PC
Lecture 11
Feb. 12, 2010

Announcements:
- PF2 was due at noon
- PS2 out, due next Friday in class
  first Problems 1 & 2 wordy but fun and not difficult

Last time:
Friedmann-Lemaître-Robertson-Walker (FLRW) metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - \kappa r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

Q: parameters? variables?
Q: physical significance of ds?

Today:
- lifestyles in a relativistic FLRW universe

Worked Example: Photon Propagation

photon path: radial null trajectory $ds = 0$ (Fermat)
⋆ emitted at $r_{\rm em}$, $t_{\rm em}$
⋆ observed at $r_{\rm obs} = 0$, $t_{\rm obs}$

for FOs at $r_{\rm em}$ and $r_{\rm obs} = 0$,
any $t_{\rm em}$ and $t_{\rm obs}$ pairs have

$$\int_{t_{\rm em}}^{t_{\rm obs}} \frac{dt}{a(t)} = \int_0^{r_{\rm em}} \frac{dr}{\sqrt{1 - \kappa r^2/R^2}}$$

time-dep time-indep

Since RHS is time-independent *Q: why?*
then *any* two pairs of emission/observation events
between comoving points $r \to 0$ must have

$$\int_{t_{\rm em,1}}^{t_{\rm obs,1}} \frac{dt}{a(t)} = \int_{t_{\rm em,2}}^{t_{\rm obs,2}} \frac{dt}{a(t)} \qquad (1)$$

consider two sequential emission events, lagged by δt_{em}
subsequently seen as sequential observation events with δt_{obs}

time-independence of propagation integral means

$$\int_{t_{\text{em}}}^{t_{\text{obs}}} \frac{dt}{a(t)} \;=\; \int_{t_{\text{em}}+\delta t_{\text{em}}}^{t_{\text{obs}}+\delta t_{\text{obs}}} \frac{dt}{a(t)}$$

rearranging...

$$\int_{t_{\text{em}}}^{t_{\text{em}}+\delta t_{\text{em}}} \frac{dt}{a(t)} \;=\; \int_{t_{\text{obs}}}^{t_{\text{obs}}+\delta t_{\text{obs}}} \frac{dt}{a(t)}$$

if δt small (*Q: compared to what?*)
then $\delta t_{\text{em}}/a(t_{\text{em}}) = \delta t_{\text{obs}}/a(t_{\text{obs}})$ and so

$$\frac{\delta t_{\text{obs}}}{\delta t_{\text{em}}} = \frac{a(t_{\text{obs}})}{a(t_{\text{em}})}$$

Observational implications:

⋆ for *any* pairs of photons

$$\frac{\delta t_{\mathrm{obs}}}{\delta t_{\mathrm{em}}} = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{em}})} = \frac{1+z_{\mathrm{em}}}{1+z_{\mathrm{obs}}}$$

and since $a(t_{\mathrm{obs}}) > a(t_{\mathrm{em}})$

$\rightarrow$ $\delta t_{\mathrm{obs}} > \delta t_{\mathrm{em}}$

$\rightarrow$ time dilation!

cosmic time dilation recently observed!

Q: how would effect show up?

Q: why non-trivial to observationally confirm?

`www: cosmic time dilation evidence`

Cosmological Redshifts Revisited

consider light with wavelength λ, frequency $f = c/\lambda$
FO emits wavecrests with period $\delta t_{\rm em} = 1/f = \lambda/c$

⋆ if photon pairs are wavecrests, then

$$\frac{\delta t_{\rm obs}}{\delta t_{\rm em}} = \frac{\lambda_{\rm obs}}{\lambda_{\rm em}}$$

and thus

$$\frac{\lambda_{\rm obs}}{\lambda_{\rm em}} = \frac{a(t_{\rm obs})}{a(t_{\rm em})} = \frac{1+z_{\rm em}}{1+z_{\rm obs}}$$

$\rightarrow$ $\lambda_{\rm obs} > \lambda_{\rm em}$
$\rightarrow$ **cosmic redshifting!**

Note: one-to-one relationships
redshift $z \leftrightarrow$ emission time $t_{\rm em} \leftrightarrow$ comov. dist. at emission $r_{\rm em}$
any/all of these denote a cosmic **epoch**

Cosmic Causality

Recall special relativity (Minkowski space)
$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$
light: $ds = 0 \to$ cone $dt^2 = dx^2 + dy^2 + dz^2$
diagram: spacetime sketch

Now RW metric: $ds^2 = dt^2 - a^2 d\ell^2_{\rm com}$
introduce new time variable η: **conformal time**
defined by $d\eta = dt/a(t)$ (see PS2)

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\ell^2_{\rm com}\right) = a(\eta)^2 \ \times \ \text{(Minkowski structure)}$$

diagram: spacetime sketch–η vs $\ell_{\rm com}$

For a flat universe ($\kappa = 0$), it's even better:

$$ds^2 = a(\eta)^2 \left(d\eta^2 - dr_{\text{com}}^2\right) = a(\eta)^2 \ \times \ \text{(exact Minkowski form)}$$

In either case $\rightarrow$ spacelike, timelike, lightlike divisions same and in $(\eta, \ell_{\text{com}})$ space:
light cone structure the same $\Rightarrow$ *causal structure the same*!

Namely:

- a spacetime point can only be influenced by events in past light cone
- a spactime point can only influence events in future light cone

So far: like Minkowski
New cosmic twist: finite cosmic age
Q: implications for causality?

Causality: Particle Horizon

past light cone at t defined by
photon propagation over cosmic history:

$$\int_{t_{\rm em}=0}^{t_{\rm obs}=t_0} \frac{d\tau}{a(\tau)} = \int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} \equiv d_{\rm hor,com}(t_0)$$

where $d_{\rm hor,com}$ is comoving distance
photon has traveled since big bang

if $d_{\rm hor,com} = \int_0^t d\tau/a(\tau)$ converges
then only a finite part of U has affected us
$\rightarrow$ $d_{\rm hor}$ defines *causal boundary*
$\rightarrow$ "**particle horizon**"

Q: physical implications of a particle horizon?
∞
Q: role of finite age?
Q: sanity check–simple limiting case with obvious result?

Particle Horizons: Implications

our view of the Universe:

diagram: our spacetime, our particle horizon, our worldline

⋆ astronomical info comes from events along past light cone

⋆ geological info comes from past world line

if particle horizon finite (i.e., $\neq \infty$), then $d_{\mathrm{horiz,com}}$:

- gives comoving size of **observable universe**
- encloses region which can communicate over cosmic time
  $\rightarrow$ causally connected region
- sets "zone of influence" over which particles can
  "notice" and/or affect each each other
  and local physical processes can "organize" themselves
  e.g., shouldn't see bound structures large than particle horizon!

So *is* $d_{\rm hor}$ finite?
depends on details of $a(t)$ evolution as $t \to 0$:
behavior near singularity crucial

will see in PS2:

- ▷ for matter, radiation domination: $d_{\rm hor}$ finite
  and $d_{\rm hor} \to 0$ for $t \to 0$
  Q: implications for CMB?
  Hint: observed $T_{\rm CMB}(\theta, \phi)$ isotropic to 5th decimal place...

will see in coming weeks

- ▷ inflation (if real!) adds twist!

7th Inning Stretch

...a good time for questions...

Cosmic Distance Measures

More examples of how spacetime properties
impose relationships among observables

Warmup: Newtonian cosmology
another sanity check, limiting case
Q: validity range?

Consider Newtonian cosmo:

- given observed z, what is distance $d_{\rm Newt}$?
- *Q: good for which z?*
- *Q: complications in full FLRW universe?*

"Newtonian Distance"

Newtonian cosmology:

- small speeds, weak gravity
  ignore curvature

Hubble's Law: $H_0 d_{\mathrm{Newt}} \equiv v \simeq cz$
applicability: $z \ll 1$
solve:

$$d_{\mathrm{Newt}} = z\frac{c}{H_0} = z\, d_H$$

In full FLRW, "distance" not unique
answer depends on

- what you measure
- how you measure it