Astro 596/496 PC
Lecture 12
Feb. 15, 2010

Announcements:

- PS2 due Friday in class
  first Problems 1 & 2 wordy but fun and not difficult
  today's Director's Cut relevant to Problem 1

Last time: FLRW lifestyles

$\triangleright$ cosmic time dilation *Q: what's that?*
how big for a SN at $z = 1$?

$\triangleright$ cosmic causality

$\triangleright$ particle horizon *Q: what's that? why important?*

Today: last day of boot camp!

- cosmic distance measures
  Q: disance $d(z)$ in Newtonian cosmology? validity range?

“Newtonian Distance”

Newtonian cosmology:

- small speeds, weak gravity
  ignore curvature

Hubble’s Law: $H_0 d_{\mathrm{Newt}} \equiv v \simeq cz$

applicability: $z \ll 1$

solve:

$$d_{\mathrm{Newt}} = z \frac{c}{H_0} = z\, d_H$$

Distances and Relativity

Basic but crucial distinction, important to remember:

In *Newtonian/pre-Relativity* physics: space is *absolute*
- "distance" has unique, well-defined meaning:
  $\Rightarrow$ Euclidean separation between points
- can think of as "intrinsic" to objects and points

In *Special and General Relativity*: space *not* absolute
- distance observer-dependent, not intrinsic to objects, events
- different well-defined measurements can lead to different results for distance

In FLRW universe, "distance" not unique: answer depends on
- *what you measure*
- *how you measure it*

Proper Distance

So far: have constructed *comoving* coordinates
which expand with Universe ("home" of FOs)

RW metric: encodes **proper distance**
i.e., *physical* separations as measured by metersticks/tapemeasures:
▷ in RW frame i.e., by comoving observers=FOs
▷ *at one* fixed cosmic instant t

$$d\ell_{\rm prop}^2 = a(t)^2 d\ell_{\rm com}^2 = a(t)^2 \left(\frac{dr^2}{1-\kappa r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right)$$

Can read off proper distances for small displacements
as measured by FOs at time t:

- $d\ell_r^{\rm prop} = a(t)\, d\ell_r^{\rm com} = a(t)\, dr/\sqrt{1-\kappa r^2/R^2}$
- $d\ell_\theta^{\rm prop} = a(t)\, d\ell_\theta^{\rm com} = a(t)\, r d\theta$
- $d\ell_\phi^{\rm prop} = a(t)\, d\ell_\phi^{\rm com} = a(t)\, r \sin\theta d\phi$

Q: how to find distance for finite displacements?

for finite displacements: integrate small ones

e.g., radial distance (at t) from $r = 0$ to r is

$$\ell_r^{\text{prop}} = a(t)\ell_r^{\text{com}} = a(t)\int_0^r d\zeta/\sqrt{1 - \kappa\zeta^2/R^2} \qquad (1)$$

Note: $d\ell_r^{\text{prop}}/dt = \dot{a}\,\ell_r^{\text{com}} = H\,\ell_r^{\text{prop}}$ exactly!

$\rightarrow$ i.e., at a *fixed cosmic time* t

proper distances increase exactly according to Hubble!

Q: what does this mean for points with $\ell_{\text{r}}^{\text{prop}} > d_H$*?*

Q: is this a problem?

Q: how would you in practice measure ℓ_r^{prop} *for large* r*?*

Luminosity Distance

for a point source (unresolved), observables:
1. redshift z
2. flux (apparent brightness) F
summed over all wavelengths: "bolometric"

input/assumption: "standard candle"
known (the tricky part!) rest-frame luminosity
$L_{\rm em} = dE_{\rm em}/dt_{\rm em}$

Goal: for std candles, want to relate
observed z and F

Q: physical effects: "normal" environment?
Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

My preferred strategy: start with observation, work back
$\Rightarrow$ measurement result is invariant
i.e., all agree on what detector registers
even if some observers think it's crazy

Observation: FO with telescope, area $A_{\rm det}$
in time interval $\delta t_{\rm obs}$
measures energy $\delta\epsilon_{\rm obs}$

observed flux (bolometric, λ-integrated) given by
$\delta\epsilon_{\rm obs} = F_{\rm obs} A_{\rm det} \delta t_{\rm obs}$
$\rightarrow$ $F_{\rm obs}$ is rate of energy flow per unit area

Connect to std candle emitter: $L_{\rm em}$ at $(t_{\rm em}, r_{\rm em} = r)$
- choose $r_{\rm em} = 0$ as center
- light "cone" (sphere) today reaches us, has present area $A_{\rm sph} = 4\pi r^2$

Q: energy conservation?

Energy conservation:

summed over sphere, $\delta E_{\text{obs}} = F_{\text{obs}} A_{\text{sph}} \delta t_{\text{obs}}$

and so

$$F_{\text{obs}} = \frac{\delta E_{\text{obs}}/\delta t_{\text{obs}}}{A_{\text{sph}}} = \frac{\delta E_{\text{obs}}/\delta t_{\text{obs}}}{4\pi r^2}$$

but expansion $\rightarrow$ $\delta E_{\text{obs}}/\delta t_{\text{obs}} \neq L_{\text{em}}$!

- energy redshifting $\delta E_{\text{obs}} = a_{\text{em}} \delta E_{\text{em}}$
- time dilation $\delta t_{\text{obs}} = \delta t_{\text{em}}/a_{\text{em}}$

$$\Rightarrow \quad \frac{\delta E_{\text{obs}}}{\delta t_{\text{obs}}} = a_{\text{em}}^2 \frac{\delta E_{\text{em}}}{\delta t_{\text{em}}} = a_{\text{em}}^2 L_{\text{em}} = \frac{L_{\text{em}}}{(1+z)^2}$$

So we have

$$F_{\text{obs}} = a_{\text{em}}^2 \frac{L_{\text{em}}}{4\pi r^2} = \frac{L_{\text{em}}}{4\pi (1+z)^2 r^2} \qquad (2)$$

Observed flux is

$$F_{\text{obs}} = a_{\text{em}}^2 \frac{L_{\text{em}}}{4\pi r^2} = \frac{L_{\text{em}}}{4\pi(1+z)^2 r^2} \tag{3}$$

identify **luminosity distance** via Newtonian/Euclidean result:

$$d_L \equiv \sqrt{\frac{L_{\text{em}}}{4\pi F_{\text{obs}}}} \tag{4}$$

and so

$$d_L = \frac{r}{a_{\text{em}}} = (1+z)r$$

Q: why interesting?

Q: r unmeasured–how relate to observables?

Q: sanity checks? non-expanding? small z?

Q: why is $d_L \neq \ell_{\text{com}}$?

Q: why is $d_L > r$?

Q: what if measure spectrum $F_\nu = dF/d\nu$?

luminosity distance: $d_L = (1+z)r$

Note: relate r to emission redshift z via trusty photon propagation eq:

$$\int_0^{r_{\rm em}} \frac{dr}{\sqrt{1-\kappa r^2/R^2}} = \int_{t_{\rm em}}^{t_{\rm obs}} \frac{dt}{a(t)}$$

$$= \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a\dot{a}} = \int_{a_{\rm em}}^{a_{\rm obs}} \frac{da}{a^2 H(a)}$$

$$= \int_0^{z_{\rm em}} \frac{dz}{H(z)}$$

where Friedmann gives $H(z)$

$\rightarrow$ r and thus d_L manifestly depends on cosmology (i.e., cosmic geometry, parameters)

★ d_L for SN Ia $\rightarrow$ cosmic acceleration!

Note: for alt radial variable χ

$d_L = (1+z) R S_\kappa(\chi)$

Extended Objects:
Angular Diameter Distance

if object resolved as extended source on sky, new observable

⋆ *angular size* $\delta\theta$

- and as usually, redshift z
  and flux (apparent bolometric brightness) F

input/assumption: "standard ruler"
known (the tricky part!) rest-frame size: diameter $D_{\rm em}$

Goal: for std rulers, want to relate
observed z and $\delta\theta$

Q: physical effects: "normal" environment?
Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

To visualize, consider closed universe

- observer at $r = 0$
- a pair of radial photon on opposite edges of source trace longitudes

diagram: sphere sketch

Invariant:
angular (longitude) separation $\delta\theta$ remains same
...while physical separation evolves, due to propagation and cosmic expansion

At *emission* epoch, physical separation of photons is standard ruler size $D_{\rm em}$
but also related to $\delta\theta$ and $r = r_{\rm em}$ via RW metric
Q: how?

At *emission* epoch, standard ruler size D_{em}
at emission point r fixes angular separation $\delta\theta$:

$$D_{\text{em}} = \delta\ell_\theta^{\text{prop,em}} = a_{\text{em}}\delta\ell_\theta^{\text{com}} = a_{\text{em}} r \delta\theta \tag{5}$$

But $\delta\theta$ remains fixed over propagation
so today we observe

$$\delta\theta = \frac{D_{\text{em}}}{a_{\text{em}} r}$$

identify angular diameter distance
via Newtonian/Euclidean result:

$$d_A \equiv \frac{D_{\text{em}}}{\delta\theta} \tag{6}$$

and so

$$d_A = a_{\text{em}} r = \frac{r}{1+z} = \frac{S_\kappa(\chi)}{1+z}$$

Angular diameter distance: $d_A = r/(1+z)$

Q: sanity checks?

Q: why is $d_A < r$?

Q: what if resolve at different λ?

Note:

- $d_A = a_{\rm em}^2 d_L = d_L/(1+z)^2$ different measures!
- d_A also depends on cosmo (but d_A/d_L doesn't!)

  Q: implications for CMB fluctuations?

  `www: WMAP`

Director's Cut Extras: Surface Brightness

Extended Objects Part Deux: Surface Brightness

if object is resolved, can determine
surface brightness $I =$ flux/(angular area $\Delta\Omega$)

Q: physical effects: "normal" environment?
Q: effects in cosmological setting?
Q: relevant equations? calculation strategies?
Q: sanity check(s)?

Newtonian/Euclidean Surface Brightness

For intuition: review Newtonian/Euclidean result

- flat space
- no redshifting, time dilation

consider an **extended source**, i.e., not pointline
which is **resolved** by your telescope
i.e., apparent angular size $>$ point spread function

observables:

- flux F as before, but also
- angular dimensions $\rightarrow$ angular area $\Delta\Omega$

Wavelength-integrated (bolometric) surface brightness
is wavelength-integrated flux per unit soruce angular area:

$$I_{\text{obs}} = \frac{F_{\text{obs}}}{\Delta\Omega}$$

Dependence on source distance?

- as usual, $F = L/4\pi d^2$
- source sky area $\Delta\Omega \Rightarrow$ physical area $S = d^2\Delta\Omega$, so

$$I_{\rm obs} = \frac{F_{\rm obs}}{\Delta\Omega} = \frac{L/4\pi d^2}{S/d^2} = \frac{L}{4\pi S}$$

Newtonian/Euclidean result *independent* of source distance!

“conservation of surface brightness”

Want (bolometric) surface brightness:

$$I_{\rm obs} = \frac{F_{\rm obs}}{\Delta\Omega_{\rm obs}}$$

1. already know $F_{\rm obs} = a_{\rm em}^2 L_{\rm em}/4\pi r^2$

2. RW metric says angular area

$$\Delta\Omega_{\rm obs} \simeq \frac{\delta\ell_\theta^2}{4\pi r^2} = \frac{D_{\rm em}^2}{4\pi a_{\rm em}^2 r} = \frac{A_{\rm em}}{4\pi a_{\rm em}^2 r^2}$$

Combine:

$$I_{\rm obs} = \frac{a_{\rm em}^2 L_{\rm em}/4\pi r^2}{4\pi A_{\rm em}/a_{\rm em}^2 r^2} = a_{\rm em}^4 \frac{L_{\rm em}}{A_{\rm em}} \tag{7}$$

$$= a_{\rm em}^4 I_{\rm em} = \frac{I_{\rm em}}{(1+z)^4} \tag{8}$$

Note:

- cosmic dimming $\propto (1+z)^4$
- no explicit dependence on distance r:
  "cons of surf brightness"
  true nonrel too *Q: examples?*
- indep of cosmology!
  useful consistency check!

Q: implications for CMB brightness?

CMB implications:
for blackbody, Stefan-Boltzmann sez

$$I = \sigma T^4$$

consider CMB, emitted at z_{em}
with temperature T_{em}

today, observe surface brightness

$$I_{\mathrm{obs}} = (1+z_{\mathrm{em}})^{-4} I_{\mathrm{em}} = (1+z_{\mathrm{em}})^{-4} \sigma T_{\mathrm{em}}^4 = \sigma \left(\frac{T_{\mathrm{em}}}{1+z_{\mathrm{em}}} \right)^4$$

still follows blackbody law, but with

$$T_{\mathrm{obs}} = \frac{T_{\mathrm{em}}}{1+z_{\mathrm{em}}}$$

which we have already derived by other means!