Announcements:
• PS2 due
• PF3 out, due next Friday noon
  short, sweet, and Nobel-packed!

Last time: 21st Century Cosmology begins

Last time: measuring the cosmic expansion history
• identify standard candle: SN Ia
• luminosity distance probes $H(z)$
• results rather...unexpected!

Q: namely? what’s directly observed? inferred?
Q: possible explanations?
Q: preliminary vote?
Faint SN Ia: Whodunit?

★ **Blame the Observations**

Maybe: SN Ia are *not* reliable standard(izable) candles

i.e., $m(\text{obs}) \neq m(\text{std candle})$

such that $L_{\text{SN}}(\text{high}z) < L_{\text{SN}}(\text{low}z)$ *systematically*

★ **Blame Einstein**

Observations correct, but

Expectations based on gravity theory $= \text{GR}$

Maybe: GR incorrect/incomplete

★ **Blame the Universe**

Observations correct, and GR correct as well, so

Infer existence of new cosmic contents which create acceleration

E.g., acceleration points to an accelerant!

Maybe: Friedmann OK, but missing terms

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I.e., beyond matter (including DM!) and radiation

New source(s) of $\rho, P$
What is to be done?

At face value
- SN Ia $\Rightarrow$ U. is accelerating
- RW$\pm$Einstein $\Rightarrow$ need new cosmic components
For now: assume these are true; then...

**Our Mission**
quantify–and ultimately identify–the new stuff
see if we can live with the consequences

But don’t forget:
- keep checking SN Ia systematics
- don’t dismiss gravity beyond Einstein:
  - GR may itself be a limiting case of larger theory
  - just as Newtonian gravity is limit of GR

First step:
*Q: Friedmann–what are conditions for acceleration?*
Acceleration in a FLRW Universe

Recall:
Cosmo principle (RW metric) + GR
= Friedmann

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right) \] (1)

But SNIa → \( \ddot{a} > 0 \):

\[ P < -\frac{1}{3} \rho c^2 \]

Q: implications? interpretation?
cosmic acceleration demands $P < -\rho c^2/3$

Cosmic pressure is
★ non-negligible
★ negative! Q: meaning?
★ (for GR experts) violation of strong energy condition $\rho + 3P \geq 0$ fails!

Exotic substance mandatory!
- NR matter and/or radiation in any form
  even weirdo particle dark matter (WIMPs, axions, ...) have $P \geq 0$: inadequate!
- new accelerant must be dark
  i.e., has not been undetected in EM radiation
- simplest solution is oldest...
Acceleration and the Cosmological Constant

Originally: Einstein modification of GR to allow for static universe: \( \ddot{a} = \dot{a} = 0 \)

- forced to introduce new constant of nature \textbf{cosmological constant} \( \Lambda \)
- \( [\Lambda] = [\text{length}^{-2}] \); alters cosmic geometry
- spoils GR \( \rightarrow \) Newtonian limit: instead,

\[
\nabla^2 \phi = 4\pi G \rho - \frac{c^2}{3} \Lambda
\]

\( Q: \) why isn’t this immediately fatal?
Cosmo-Sociology: The Checkered History of $\Lambda$

$\Lambda$ often invoked to solve cosmo problems, then abandoned when observations improved

“My greatest blunder.”
— A. Einstein, allegedly, on inventing $\Lambda$

“The cosmological constant is the last refuge of scoundrels.”
— famous Chicago cosmologist and current $\Lambda$ enthusiast, circa 1990
Living with Λ

With Λ ≠ 0, new term in both Friedmann eqs

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R^2 a^2} + \frac{c^2}{3} \Lambda \\
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + \frac{3P}{c^2} \right) + \frac{c^2}{3} \Lambda
\]  

(2)  

(3)

Note appearance & sign in acceleration  
⇒ Λ an “accelerant” → “antigravity”  

Q: intuitive reason? Hint: original purpose?

convenient to introduce \( \Omega_\Lambda = \Lambda c^2 / 3H^2 \)  
allows easy comparison of Λ term with others  

Q: but you can guess which larger, based on observed accel?
The Data: $\Lambda$ Looms Large

SN Ia data in $\Lambda$ cosmology:

- allow for $\Omega_\Lambda = \Lambda c^2 / 3 H^2 \neq 0$
- find best fit to $d_L$ data:
  
  “concordance universe”

\[
\Omega_\Lambda \approx 0.7 \quad \Omega_m \approx 0.3
\]  (4)

- not only is $\Omega_\Lambda \neq 0$, but
- $\Omega_\Lambda \gtrsim 2\Omega_m$: $\Lambda$ dominated by $\Lambda$ now!

Q: if this is all true, cosmic fate?
\( \Lambda \) and Cosmic Fate: Big Chill and Dark Sky

*If* acceleration is truly due to \( \Lambda \) then:
- already dominates Friedmann
- as \( a \) increases, matter & curvature terms drop
  \( \rightarrow \Lambda \) dominates even more!

The bleak \( \Lambda \)-dominated future:
- future \( a(t) \approx e^{\sqrt{\Omega_\Lambda}H_0(t-t_0)} \rightarrow \) exponential expansion *forever!*
  fate is not only *big chill* but *supercooling*
- *event horizon* exists: \( d_{\text{event, comov}}(t_0) \approx \Omega_\Lambda^{-1/2}d_H \approx 6400 \text{ Mpc} \)
  we will *never* see beyond this!
- worse still: later on,
  \[
d_{\text{event, comov}}(t_0 + \Delta t) = e^{-\sqrt{\Omega_\Lambda}H_0\Delta t}d_{\text{event, comov}}(t_0)
\]
  event horizon shrinks exponentially with time!
- observational astronomy from data mining only!
\( \Lambda \) as Vacuum Energy

Can rewrite \( \Lambda \) as energy density: \( \rho_\Lambda \):

in Friedmann, put

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R^2 a^2} \left[ 3 \rho - \frac{\kappa c^2}{R^2 a^2} + \frac{\Lambda c^2}{3} \right] \equiv \frac{8\pi G}{3} (\rho + \rho_\Lambda) - \frac{\kappa c^2}{R^2 a^2}
\]

so that

\[
\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad \text{and} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\text{crit}}}
\]

Then introduce pressure \( P_\Lambda \) in Fried accel:

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3P) + \frac{\Lambda c^2}{3} \equiv -\frac{4\pi G}{3} (\rho + \rho_\Lambda + 3P + 3P_\Lambda)
\]

can show:

\[
P_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\rho_\Lambda
\]

i.e., \( P_\Lambda = w \rho_\Lambda \), with \( w = -1 \)
Note:

- $\Lambda$ is strict constant $\rightarrow \rho_\Lambda$ constant in space and time
  “energy density of the vacuum” $\rightarrow$ dark energy
- $P_\Lambda < 0$: as needed for acceleration
- equation of state parameter $w = -1$ preserves $\Lambda$ constancy

So: $\Lambda$ is equivalently a length scale
or an energy density

Q: what sets its value?
\[ \Lambda \text{ and its Discontents} \]

In Classical GR:

- \( \Lambda \) is a (optional) parameter to be measured
- no \textit{a priori} insight as to its value
  (beyond escaping solar system limits)

But quantum mechanics & particle physics offer a new perspective on vacuum energy

Recall: blackbody radiation
usually write total energy density:

\[
\varepsilon_{\text{blackbody}}(T) = \frac{1}{2\pi^2 c^2} \int_{\omega=0}^{\infty} \frac{\hbar \omega}{e^{\hbar \omega/kT} - 1} \omega^2 \ d\omega = a_{\text{Boltz}} T^4
\]

note that \( \varepsilon \rightarrow 0 \) as \( T \rightarrow 0 \): vacuum has no energy
...but (\( \Lambda \) aside) this was always a cheat!

\textit{Q: why? what omitted?}
Uncertainty principle → nothing “at rest”
→ ground state “zero point motion”
→ zero point modes have energy $E_0 \neq 0$

Blackbody result: treats photon modes as harmonic oscillators
but threw away zero point energy $E_0 = \hbar \omega/2$!
Cheated!
● handwaving excuse:
  $E_0$ cost of “assembling” oscillators/quanta
  ...and then only energy differences count
● in practice, usual Planck result is really
  $\epsilon_{\text{usual}} = \epsilon_{\text{tot}}(T) - \epsilon_{T=0} = \epsilon_{\text{tot}}(T) - \epsilon_{\text{zeropoint}}$
● but in GR: curvature $\leftrightarrow$ mass-energy density
  absolute energy scales matter!
e.g., $(\dot{a}/a)^2 \sim 8\pi G/3 \epsilon/c^2$

Q: what if we keep the zero-point energy?
Try keeping zero point energy:

\[ \varepsilon \sim \int_0^\infty \langle E(\omega) \rangle \omega^2 \, d\omega \]  

\[ = \int_0^\infty \left( \frac{\hbar \omega}{e^{\frac{\hbar \omega}{kT}} - 1} + E_0 \right) \omega^2 \, d\omega \]  

\[ = \varepsilon_{\text{usual}} + \varepsilon_{\text{zeropoint}} \]  

where the zero point contribution is

\[ \varepsilon_{\text{zeropoint}} \sim \int_0^\infty \omega^3 \, d\omega = \infty^4 \]

“ultraviolet catastrophe”!

Q: possible cures?
Vacuum Energy in Particle Physics

what is cause of catastrophe?

\[ \varepsilon_{\text{zeropoint}} \sim \int_0^{\omega_{\max}} \omega^3 \, d\omega \sim \omega_{\max}^4 \]

allowed \( \omega_{\max} \to \infty \)
\( \to \) included modes of arbitrarily high energy
\( \quad \) arbitrarily small wavelength

If quanta energy has upper limit \( E_{\max} \)
i.e., a minimum wavelength \( \lambda_{\min} = \frac{\hbar c}{E_{\max}} \)
then \( \varepsilon_{\text{zeropoint}} \neq \infty \)

Q: what might such a limit be?
Q: i.e., at what scale might energies “max out”?
The Planck Scale and $\Lambda$

Highest known energy scale in physics: Planck Scale
when quantum effects become important for gravity

a particle of mass $m$, energy $mc^2$
has quantum scale $\lambda_{\text{quantum}} = \frac{\hbar}{mc}$ (Compton wavelength)
equal to GR scale $\lambda_{\text{GR}} = \frac{2Gm}{c^2}$ (Schwarzschild radius)
if $m = M_{\text{Pl}}$: the Planck mass

$$M_{\text{Pl}}c^2 = \sqrt{\frac{c}{G\hbar}}c^2 \sim 10^{19} \text{ GeV}$$  \hspace{1cm} (8)
$$\ell_{\text{Pl}} = \frac{\hbar}{M_{\text{Pl}}c} \sim 10^{-33} \text{ cm}$$  \hspace{1cm} (9)

if quanta have $E_{\text{max}} = M_{\text{Pl}}$ and $\lambda_{\text{min}} = \ell_{\text{Pl}}$
then estimate vacuum energy density

$$\rho_{\text{vac,Pl}} \sim M_{\text{Pl}}^4 \sim 10^{110} \text{ erg/cm}^3 \sim 10^{89} \text{ g/cm}^3$$

Q: implications?