Astro 596/496 PC
Lecture 14
Feb. 19, 2010

Announcements:
- PS2 due
- PF3 out, due next Friday noon
  short, sweet, and Nobel-packed!

Last time: 21st Century Cosmology begins

Last time: measuring the cosmic expansion history
- identify standard candle: SN Ia
- luminosity distance probes $H(z)$
- results rather...unexpected!

`www: unexpected results`

Q: namely? what's directly observed? inferred?
Q: possible explanations?
Q: preliminary vote?

Faint SN Ia: Whodunit?

⋆ **Blame the Observations**
mabye: SN Ia are *not* reliable standard(izable) candles
i.e., $m(\text{obs}) \neq m(\text{std candle})$
such that $L_{\rm SN}(\text{high}z) < L_{\rm SN}(\text{low}z)$ *systematically*

⋆ **Blame Einstein**
observations correct, but
expectations based on gravity theory = GR
maybe: GR incorrect/incomplete

⋆ **Blame the Universe**
observations correct, and GR correct as well, so
infer existence of new cosmic contents which create acceleration
e.g., acceleration points to an accelerant!
maybe: Friedmann OK, but missing terms
i.e., beyond matter (including DM!) and radiation
new source(s) of ρ, P

What is to be done?

At face value
- SN Ia $\Rightarrow$ U. is accelerating
- RW+Einstein $\Rightarrow$ need new cosmic components

For now: assume these are true; then...

Our Mission

quantify–and ultimately identify–the new stuff
see if we can live with the consequences

But don't forget:

▷ keep checking SN Ia systematics

▷ don't dismiss gravity beyond Einstein:
GR may itself be a limiting case of larger theory
just as Newtonian gravity is limit of GR

First step:
Q: Friedmann–what are conditions for acceleration?

Acceleration in a FLRW Universe

Recall:
Cosmo principle (RW metric) + GR
= Friedmann

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{1}$$

But SNIa $\rightarrow$ $\ddot{a} > 0$:

$$P < -\frac{1}{3}\rho c^2$$

Q: implications? interpretation?

cosmic acceleration demands $\boxed{P < -\rho c^2/3}$

Cosmic pressure is

★ non-negligible

★ negative! *Q: meaning?*

★ (for GR experts) violation of strong energy condition $\rho + 3P \geq 0$ fails!

Exotic substance mandatory!

- NR matter and/or radiation in *any* form even wierdo particle dark matter (WIMPs, axions, ...) have $P \geq 0$: inadequate!
- new accelerant must be *dark* i.e., has not been undetected in EM radiation
- simplest solution is oldest...

Acceleration and the Cosmological Constant

Originally: Einstein modification of GR
to allow for *static* universe: $\ddot{a} = \dot{a} = 0$

- forced to introduce new constant of nature
  cosmological constant $\boxed{\Lambda}$

- $[\Lambda] = [\text{length}^{-2}]$; alters cosmic geometry

- spoils GR $\rightarrow$ Newtonian limit: instead,

$$\nabla^2 \phi = 4\pi G \rho - \frac{c^2}{3}\Lambda$$

Q: why isn't this immediately fatal?

Cosmo-Sociology: The Checkered History of Λ

Λ often invoked to solve cosmo problems, then abandoned when observations improved

"My greatest blunder."
– A. Einstein, allegedly, on inventing Λ

"The cosmological constant is the last refuge of scoundrels."
– famous Chicago cosmologist and current Λ enthusiast, circa 1990

Living with Λ

With $\Lambda \neq 0$, new term in both Friedmann eqs

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2} + \frac{c^2}{3}\Lambda \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda \tag{3}$$

Note appearance & sign in acceleration

$\Rightarrow$ Λ an "accelerant" $\rightarrow$ "antigravity"

Q: intuitive reason? Hint: original purpose?

convenient to introduce $\Omega_\Lambda = \Lambda c^2/3H^2$

allows easy comparison of Λ term with others

∞ *Q: but you can guess which larger, based on observed accel?*

The Data: Λ Looms Large

SN Ia data in Λ cosmology:

- allow for $\Omega_\Lambda = \Lambda c^2/3H^2 \neq 0$
- find best fit to d_L data:

"concordance universe"

`www:` $\Omega_\Lambda - \Omega_{\rm m}$ `plane`

$$\Omega_\Lambda \simeq 0.7 \qquad \Omega_{\rm m} \simeq 0.3 \tag{4}$$

- not only is $\Omega_\Lambda \neq 0$, but
- $\Omega_\Lambda \gtrsim 2\Omega_{\rm m}$: U *dominated* by Λ *now*!

Q: if this is all true, cosmic fate?

Λ and Cosmic Fate: Big Chill and Dark Sky

if acceleration is truly due to Λ then:

- already dominates Friedmann
- as a increases, matter & curvature terms drop
  $\rightarrow \Lambda$ dominates even more!

The bleak Λ-dominated future:

⋆ future $a(t) \simeq e^{\sqrt{\Omega_\Lambda} H_0 (t-t_0)}$ $\rightarrow$ exponential expansion *forever*!
fate is not only *big chill* but *supercooling*

⋆ *event horizon* exists: $d_{\text{event,comov}}(t_0) \simeq \Omega_\Lambda^{-1/2} d_H \sim 6400$ Mpc
we will *never* see beyond this!
worse still: later on,

$$d_{\text{event,comov}}(t_0 + \Delta t) = e^{-\sqrt{\Omega_\Lambda} H_0 \Delta t} d_{\text{event,comov}}(t_0)$$

event horizon shrinks exponentially with time!
observational astronomy from data mining only!

Λ as Vacuum Energy

Can rewrite Λ as energy density: ρ_Λ:
in Friedmann, put

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2 a^2} + \frac{\Lambda c^2}{3} \equiv \frac{8\pi G}{3}(\rho + \rho_\Lambda) - \frac{\kappa c^2}{R^2 a^2}$$

so that

$$\rho_\Lambda = \frac{\Lambda c^2}{8\pi G} \quad \text{and} \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\text{crit}}}$$

Then introduce pressure P_Λ in Fried accel:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda c^2}{3} \equiv -\frac{4\pi G}{3}(\rho + \rho_\Lambda + 3P + 3P_\Lambda)$$

can show:

$$P_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\rho_\Lambda$$

i.e., $P_\Lambda = w\rho_\Lambda$, with $w = -1$

Note:

- Λ is strict constant $\to$ ρ_Λ constant in space and time
  "energy density of the vacuum" $\to$ **dark energy**
- $P_\Lambda < 0$: as needed for acceleration
- equation of state parameter $w = -1$ preserves Λ constancy

So: Λ is equivalently a length scale
or an energy density
Q: what sets its value?

Λ and its Discontents

In Classical GR:

- ▷ Λ is a (optional) parameter to be measured
- ▷ no *a priori* insight as to its value
  (beyond escaping solar system limits)

But quantum mechanics & particle physics
offer a new perspective on vacuum energy

Recall: blackbody radiation
usually write total energy density:

$$\varepsilon_{\text{blackbody}}(T) = \frac{1}{2\pi^2 c^2} \int_{\omega=0}^{\infty} \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} \omega^2 \, d\omega = a_{\text{Boltz}} T^4$$

note that $\varepsilon \rightarrow 0$ as $T \rightarrow 0$: vacuum has no energy
...but (Λ aside) this was always a cheat!
Q: why? what omitted?

Uncertainty principle $\rightarrow$ nothing "at rest"
$\rightarrow$ ground state "zero point motion"
$\rightarrow$ zero point modes have energy $E_0 \neq 0$

Blackbody result: treats photon modes
as harmonic oscillators
but threw away zero point energy $E_0 = \hbar\omega/2$!
Cheated!

- handwaving excuse:
  E_0 cost of "assembling" oscillators/quanta
  ...and then only energy *differences* count
- in practice, usual Planck result is really
  $\varepsilon_{\rm usual} = \varepsilon_{\rm tot}(T) - \varepsilon_{T=0} = \varepsilon_{\rm tot}(T) - \varepsilon_{\rm zeropoint}$
- but in GR: curvature $\leftrightarrow$ mass-energy density
  absolute energy scales matter!
  e.g., $(\dot{a}/a)^2 \sim 8\pi G/3\ \varepsilon/c^2$

Q: what if we keep the zero-point energy?

Try keeping zero point energy:

$$\begin{aligned}
\varepsilon &\sim \int_0^\infty \langle E(\omega)\rangle\ \omega^2\ d\omega && (5)\\
&= \int_0^\infty \left(\frac{\hbar\omega}{e^{\hbar\omega/kT}-1}+E_0\right)\omega^2\ d\omega && (6)\\
&= \varepsilon_{\text{usual}}+\varepsilon_{\text{zeropoint}} && (7)
\end{aligned}$$

where the zero pont contribution is

$$\varepsilon_{\text{zeropoint}} \sim \int_0^\infty \omega^3\ d\omega = \infty^4$$

"ultraviolet catastrophe"!

Q: possible cures?

Vacuum Energy in Particle Physics

what is cause of catastrophe?

$$\varepsilon_{\text{zeropoint}} \sim \int_0^{\omega_{\text{max}}} \omega^3 \, d\omega \sim \omega_{\text{max}}^4$$

allowed $\omega_{\text{max}} \to \infty$

$\to$ included modes of arbitrarily high energy

arbitrarily small wavelength

If quanta energy has upper limit E_{max}

i.e., a minimum wavelength $\lambda_{\text{min}} = \hbar c / E_{\text{max}}$

then $\varepsilon_{\text{zeropoint}} \neq \infty$

Q: what might such a limit be?

Q: i.e., at what scale might energies "max out"?

The Planck Scale and Λ

Highest known energy scale in physics: Planck Scale
when quantum effects become important for gravity

a particle of mass m, energy mc^2
has quantum scale $\lambda_{\rm quantum} = \hbar/mc$ (Compton wavelength)
equal to GR scale $\lambda_{\rm GR} = 2Gm/c^2$ (Schwarzchild radius)
if $m = M_{\rm Pl}$: the Planck mass

$$M_{\rm Pl}c^2 = \sqrt{\frac{c}{G\hbar}}c^2 \sim 10^{19} \text{ GeV} \quad (8)$$

$$\ell_{\rm Pl} = \frac{\hbar}{M_{\rm Pl}c} \sim 10^{-33} \text{ cm} \quad (9)$$

if quanta have $E_{\rm max} = M_{\rm Pl}$ and $\lambda_{\rm min} = \ell_{\rm Pl}$
then estimate vacuum energy density

$$\rho_{\rm vac,Pl} \sim M_{\rm Pl}^4 \sim 10^{110} \text{ erg/cm}^3 \sim 10^{89} \text{ g/cm}^3$$

Q: implications?