Astro 596/496 PC
Lecture 16
Feb. 24, 2010

Announcements:

- PF3 due Friday noon

Last time: ignorance parameterized–*dark energy*
Q: why dark energy?
Q: connection between Λ *and dark energy?*
Q: definition, units, significance of w*?*
Q: current limits on w, Ω_w*?*
Q: why would it be a Big Deal if we prove, e.g., $w = -0.9$*?*
or $w_{z=1} - w_{z=0} = 0.1$*?*
Q: why are scalar fields appealing dark energy candidates?

The Physics of Scalar Fields

scalar field: $\phi(\vec{x}, t)$

scalar $\rightarrow$ single-valued object = *function*

no directionality $\rightarrow$ kosher with cosmo principle

field $\rightarrow$ function takes values at all points in space(time)

Scalar fields abound in all areas of physics

Q: examples of known, physical scalar fields?

in particle physics, scalar fields arise in

force unification, origin of mass

in cosmology: DE, inflation $\rightarrow$ can't avoid!

"Scalar fields are the cosmologist's blunt instrument."

– J. Frieman

Scalar Fields: Motion and Energy

need equations to govern scalar field $\phi(\vec{x}, t)$

- field "motion" = variation in space & time
  will seek 2nd-order eq of motion
  $\rightarrow$ schematically, " $\partial\partial\phi =$ *stuff* "

- energy: "kinetic" and "potential"

Q: Relativistic considerations? Impact on eq of motion?

Scalar Field Dynamics

Special Relativity demands:

- ϕ disturbances ("signals") propagate at speeds $\leq c$
- space & time on equal footing

$\Rightarrow$ if second-order equation of motion, must have

$$\partial^2\phi = \partial_t^2\phi - \nabla^2\phi = \text{sources/interactions of } \phi \tag{1}$$

lead to ϕ **energy density**

$$\varepsilon_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \varepsilon_{\text{int}} = \frac{1}{2}\dot{\phi}^2 + \nabla\phi^2 + \varepsilon_{\text{int}} \tag{2}$$

Simplest **interactions**: only with self $\rightarrow$ $\varepsilon_{\text{int}} = V(\phi)$
gives equation of motion with "force" $-\partial_\phi V \equiv -V'(\phi)$

$$\partial^2\phi = \partial_t^2\phi - \nabla^2\phi = -\partial_\phi V(\phi) \tag{3}$$

Q: cosmological considerations?

Cosmological Scalar Fields

Equation of Motion

in Minkowski space: relativistic scalar field

$$\partial_t^2 \phi - \nabla^2 \phi = \text{sources/interactions}$$

in FRW, include coupling to gravity $\rightarrow$ redshifting effects

$$\partial_t^2 \phi - \nabla^2 \phi = -3\frac{\dot{a}}{a}\dot{\phi} + \text{sources/interactions}$$

but cosmo principle $\rightarrow$ at any time ϕ homogeneous:
same at all x, so no space derivatives

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V' = 0$$

where $V'(\phi) \equiv \partial_\phi V$

Q: similarities with Newtonian particle equation of motion?

Field energy density:

$$\varepsilon = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

note kinetic, potential terms

Field momentum density $\rightarrow$ pressure

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Q: crucial, important difference between these expressions?
Q: how could we exploit this?

Scalar Field "Equation of State"

For cosmic ϕ field,
can write expression for $w = P/\rho$ parameter:

$$w_\phi \equiv \frac{P_\phi}{\varepsilon_\phi} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \tag{4}$$

Limiting cases:

- if kinetic term dominates: $\frac{1}{2}\dot{\phi}^2 \gg V(\phi)$
  Q: then $w_\phi \rightarrow$ *?*
- if the potential term dominates:
  Q: then $w_\phi \rightarrow$ *?*

Q: implications?

Scalar Potentials as Accelerants

Relativistic scalar field potential contributes negative pressure!

$$w_\phi = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \xrightarrow{V \gg \dot{\phi}^2} -1 \qquad (5)$$

candidate for dark energy

Good news:

$w_\phi \rightarrow -1$ *independent of details of V!*

as long as $V \gg \dot{\phi}^2$: potential dominates

cosmic acceleration "natural" if

▷ scalars present and

▷ ε_ϕ large

Q: bad news?

Bad news:

$w_\phi \rightarrow -1$ *independent of details of* V

▷ not guided as to details, physics of V

▷ need to know/guess/measure interactions with itself, other matter/energy

Quintessence

Simplest assumption: ϕ feels only itself
i.e., interacts only with itself
$\rightarrow V(\phi)$ independent of matter, radiation
but: still has gravitational interactions
$\rightarrow$ indirectly communicates with matter, rad
via $-3H\dot{\phi}$ term

Also: will see later–we *need* past epochs of
matter, radiation dom: else no light elements,
no grow of cosmic structures
$\rightarrow$ in past, $\rho_\phi \ll \rho_{\rm tot}$

Quintessence: Ingredients

- cosmic scale field ϕ (or Q)
- gravity + self-interactions $V = V(\phi)$ only

Outcome (for some choices of V):

- ϕ evolves slowly, small at early times: $\rho_\phi/\rho_{\text{tot}} \ll 1$
- ϕ evolution coupled to matter, rad
  $\rightarrow$ "attractor" solutions for ρ_ϕ/ρ
  ⋆ ρ_ϕ slowly becomes dominant
  ⋆ acceleration sets in
  `www: tracker model`
- addresses the coincidence problem
  or at least trades it for ϕ and V

Phantom Energy

If allow $w < -1$, i.e., $\|w\| > 1$
- consistent with SN+other dat
- in some analyses, even gives best fit!

But this violates "dominant energy condition"
i.e., $\rho + P > 0$ fails
acts to, e.g., prevent energy flows moving locally $> c$(!)
"phantom energy"
allowed in some quantum gravity models

what's life like if $w < -1$?

energy density $\rho \sim a^{-3(1+w)}$ *increases* w/ expansion!
Q: implications?

when phantom energy dominates

$$
\begin{aligned}
(\dot{a}/a)^2 &\approx \Omega_w H_0^2 a^{3\|w+1\|} && (6)\\
a^{-3\|w+1\|/2}\, da/a &= \sqrt{\Omega_w} H_0 dt && (7)
\end{aligned}
$$

integrate to get future evolution:

$$a(t) = \left(\frac{t_{\rm r}}{t_{\rm r} - \Delta t}\right)^{2/3\|w+1\|} \tag{8}$$

where $\Delta t = t - t_0$ is *time from now*; i.e., $\Delta t = 0$ today and

$$t_{\rm r} = \frac{2H_0^{-1}}{3\|w+1\|\sqrt{\Omega_w}} \tag{9}$$

is a timescale

Q: implications?

Q: how differs from, say, Λ case?

The Big Rip

Phantom energy domination

$$a(t) = \left(\frac{t_{\mathrm{r}}}{t_{\mathrm{r}} - \Delta t}\right)^{2/3\|w+1\|} \quad (10)$$

has $a \to \infty$ when $\Delta t = t_{\mathrm{r}} \sim 11\,\|w+1\|^{-1}$ Gyr
i.e., infinite expansion occurs a finite time from now!

it gets worse...
as t_{r} approaches, $\rho_w \to \infty$ everywhere
overwhelms binding energies $\rightarrow$ bound structures torn apart:
first clusters, then galaxies, planets, people, atoms, nuclei...
$\rightarrow$ all particles separated from all others
"cosmic doomsday" $\rightarrow$ **Big Rip**

the big rip foretold:
cosmologist W. Allen, *Annie Hall* (1977)

Director's Cut Extras: Scalar Field Dynamics

Scalar Field Dynamics: Plausibility Argument

from A. Zee, *Quantum Field Theory in a Nutshell*

Picture an array of masses connected by springs
"bedspring" model of a field

N-oscillator model: discrete version
masses m, positions q_n,
equilibrium when $q_n = na$: lattice spacing a
acceleration due to offset from equilibrium:

$$m\ddot{q}_n = k(q_{n+1} - q_n) - k(q_n - q_{n-1}) \tag{11}$$

continuum limit: $N \to \infty$, $a \to 0$, and then

$$\rho \frac{\partial^2 \phi}{\partial_t^2} = Y \frac{\partial^2 \phi}{\partial_x^2} \tag{12}$$

put $c^2 = Y/\rho$: "sound speed"

$$\partial_t^2 \phi - 1/c^2 \partial_x^2 \phi = 0 \tag{13}$$

smoothed version of oscillator network:

$$\partial_t^2 \phi - \frac{1}{c^2} \partial_x^2 \phi = 0 \tag{14}$$

"equation of motion" for ϕ

- note equal footing of x, t
- supports wave solutions: $\phi = \phi(x \pm c_s t)$

if additional forces between masses

$$\partial_t^2 \phi - 1/c^2 \partial_x^2 \phi = F = -\partial V / \partial_\phi \tag{15}$$

where **potential** V can depend on ϕ,
other matter fields

harmonic oscillator model obviously not literal
cosmos not made of bedsprings!
Q: why is it still useful?