Astro 596/496 PC
Lecture 21
March 8, 2010

Announcements:

- PF 4 due next Friday noon
- Cosmology bigshot in the house!
  seminar *right after class* today, Loomis 464
  Pierre Sikivie, U Florida
  "Bose-Einstein Condensation of Dark Matter Axions"

Last time:
recombination $\rightarrow$ huge drop in free e^- $\rightarrow$ CMB freeze/decouple
to calculate in detail: need cosmic statistical mechanics
key inputs: uncertainty principle, Boltzmann factor
key outputs: non-rel, non-degen $n = g(mT/2\pi\hbar^2)^{3/2} e^{-(m-\mu)/T}$
for reaction in ("chemical") equilibrium: $\sum \mu_i = \sum \mu_f$

Q: apply to recombination?

Recombination: Equilibrium Thermodynamics

dominant cosmic plasma components γ, p, e, H (ignore He, Li)
equilibrium: equal rates for

$$p + e \leftrightarrow \mathrm{H} + \gamma$$

and so chem potentials have

$$\mu_p + \mu_e = \mu_{\mathrm{H}} \tag{1}$$

recall: for non-rel species $n = g(mT/2\pi\hbar^2)^{3/2} e^{-(m-\mu)/T}$
thus we have **Saha equation**

$$\frac{n_e n_p}{n_{\mathrm{H}}} = \frac{g_e g_p}{g_{\mathrm{H}}} \left(\frac{m_e m_p}{m_{\mathrm{H}}}\right)^{3/2} \left(\frac{T}{2\pi\hbar^2}\right)^{3/2} e^{-(m_e+m_p-m_{\mathrm{H}})/T} \tag{2}$$

$$\approx \left(\frac{m_e T}{2\pi\hbar^2}\right)^{3/2} e^{-B/T} \tag{3}$$

where $B \equiv m_e + m_p - m_{\mathrm{H}} = 13.6$ eV introduce "free electron fraction" $X_e = n_e/n_B$

use $n_B = \eta n_\gamma \propto \eta T^3$

from Extras last time: $n_\gamma = 2\zeta(3)/\pi^2\, T^3$, with $\zeta(3) = \sum_1^\infty 1/n^3 = 1.20206\ldots$

and note that $n_p = n_e$ *Q: why?*, so

$$\frac{n_e^2}{n_{\rm H} n_B} = \frac{X_e^2}{1 - X_e} = \frac{\sqrt{\pi}}{4\sqrt{2}\zeta(3)} \frac{1}{\eta} \left(\frac{m_e}{T}\right)^{3/2} e^{-B/T} \tag{4}$$

Q: sanity checks? what sets characteristic T scale?

Q: when is $X_e = 0$ (exactly)?

At last–recombination!

Q: how define physically?

Q: how define operationally, in terms of X_e?

Q: given some $X_{e,\rm rec}$, how to get $z_{\rm rec}$?

The Epoch of Recombination

Saha gives

$$\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\pi^{1/2}} \; \eta \; \left(\frac{B}{m_e}\right)^{3/2} \left(\frac{T}{B}\right)^{3/2} e^{B/T} \tag{5}$$

if always equilib, then strictly $X_e = 0$ only at $T = 0$
but note $e^{B/T}$: X_e exponentially small when $T \ll B$

viewed as a function of $B/T \equiv u$

$$\frac{1 - X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\pi^{1/2}} \; \eta \; \left(\frac{B}{m_e}\right)^{3/2} \; u^{3/2} e^{u} \equiv A \; u^{3/2} e^{u} \tag{6}$$

where $A = 4\sqrt{2}/\pi^{1/2}\zeta(3) \; \eta \; (B/m_e)^{3/2}$

Q: what is order-of-magnitude of A?
Q: implications for recombination?
Q: physical picture?

in recombination Saha expression $(1 - X_e)/X_e = A(B/T)^{3/2} e^{B/T}$
prefactor is tiny!
$A \sim \eta(B/m_e)^{3/2} \sim 10^{-9}(10^{-5})^{3/2} \sim 10^{-16}$!
why? largely due to *tiny baryon-to-photon ratio*

but when recombine: $1 - X_e \simeq X_e$
so require $1 \sim 10^{-16}(B/T_{\rm rec})^{3/2} e^{B/T_{\rm rec}}$
$\Rightarrow$ so need $B/T_{\rm rec} \gg 1$ to offset prefactor
$\Rightarrow$ and thus $T_{\rm rec} \ll B$!

more carefully define recomb: $X_e = X_{e,\rm rec} = 0.1$
(arbitrary, but not crazy; see PS4)
then solve for $T_{\rm rec}$:

$$\frac{B}{T_{\rm rec}} = \ln\left(\frac{\pi^{1/2}}{4\sqrt{2}\zeta(3)}\right) + \ln\left(\frac{1 - X_{e,\rm rec}}{X_{e,\rm rec}^2}\right) + \ln \eta^{-1} + \frac{3}{2}\ln\frac{m_e}{B} + \frac{3}{2}\ln\frac{B}{T}$$

$$\sim 40 \quad (\gg 1)$$

(ignore or iterate $\ln B/T$ term)

and so

$$
\begin{aligned}
T_{\rm rec} &\approx \frac{B}{40} \simeq 0.3 \text{ eV} \ll B & (7)\\
z_{\rm rec} &\approx 1400 \ll z_{\rm rec,naive} & (8)\\
t_{\rm rec} &\approx \frac{2}{3\sqrt{\Omega_{\rm m}}} H_0^{-1} (1+z_{\rm rec})^{-3/2} = 350{,}000 \text{ yrs} & (9)
\end{aligned}
$$

PS4: try it yourself!

Implications for CMB freqency spectrum:

- at recomb: emission lines created at $h\nu_{\rm rec} \gtrsim B/4$ and thus at $h\nu_{\rm rec} \gtrsim 10kT_{\rm rec}$
- post-recomb: T and ν both redshift the same way, so
- CMB spectrum *distorted* from Planck at high freq: $h\nu \gtrsim 10kT$
- small signal, difficult to observe, but tantalizing `www: predictions`

Q: what physically is responsible for $T_{\rm rec} \ll B$?

Recombination "Delay"

Why is $T_{\text{rec}} \ll B$?

- ▷ because for small X_e, Saha says $X_e \propto 1/\eta \gg 1$
- ▷ many photons per baryon: even if typically $E_\gamma \ll B$, high-E tail of Planck distribution not negligible (at first) lots of ionizing photons with $E_\gamma \geq B$ H dissociated as soon as formed

When does dissociation stop?
can show that fraction of photons with $E_\gamma > B$
is roughly $f_{\text{ionizing}} \sim e^{-B/T}$
so ratio of ionizing photons per baryon is

$$\frac{n_{\gamma,\text{ionizing}}}{n_B} \sim \frac{e^{-B/T}}{\eta} \quad (10)$$

estimate recombination when $n_{\gamma,\text{ionizing}}/n_B \sim 1$
$\rightarrow T \sim B/\ln \eta^{-1} \ll B$ (check!)
$\Rightarrow$ recombination "delayed" to huge photon-to-baryon ratio

Last Scattering: Photons Decouple from Matter

"recombination" a smooth transition in X_e, not instantaneous
`www: equilibrium` X_e `plot`
nevertheless, exponential drop in X_e around $z_{\rm rec}$

photons interact with gas via Thompson scattering: $\gamma e \rightarrow \gamma e$
rate per photon of scattering with e:

$$\Gamma_e(\gamma) = n_e \sigma v = n_e \sigma_T c = X_e n_{\rm b} \sigma_T c \qquad (11)$$

drop in $X_e \rightarrow$ abrupt slowdown in scattering

as usual, competition between interaction and expansion
interactions "stop" when

$$\Gamma_e(\gamma) \lesssim H \qquad (12)$$

and solving for $\Gamma_e(T) = H(T)$ gives last scattering :

$$\boxed{z_{\rm ls} \sim 1200} \qquad (13)$$

After last scattering:

- photons "decoupled" from gas
- but $X_e \neq 0$: some free e, p remain

  Q: what is X_e *as* $T \rightarrow 0$*? why?*

Freezing of Recombination

when typical photon has last scattering with e
still some residual ionization: i.e., some free e, p
can they recombine? yes!
do they recombine? yes, for a short while...then no!

Why? recombination rate per p: $\Gamma_{\mathrm{rec},p} \sim n_e \sigma_{\mathrm{rec}} v_{\mathrm{therm}}$
with $\sigma_{\mathrm{rec}} \sim (m_e/T)\sigma_{\mathrm{T}}$ and $v_{\mathrm{therm}} \sim \sqrt{T/m_e}$
recombination stops when $\Gamma_{\mathrm{rec},p} \lesssim H$

after this: cooling does not reduce ionization
fixed value of $X_{e,\mathrm{freeze}} \sim 10^{-4}$: "freeze-in of residual ionization"
at

$$z_{\mathrm{ri}} \simeq 1000 \tag{14}$$

Q: cosmological implications of $X_{e,\mathrm{freeze}} \neq 0$?

Recombination Timeline Summarized

The large drop in free electron density around $z \sim 1000$ leads to three distinct but related events:

(1) recombination U. **ionized → neutral**

$X_e \rightarrow X_{e,\text{rec}} \sim 0.1$: $z_{\text{rec}} \sim 1300$

...but photons still coupled to gas, and vice versa

(2) last scattering typical photons no longer interacts with e

U. **opaque → transparent**

$\Gamma_e(\gamma) \sim H$: $z_{\text{ls}} \sim 1200$

...but gas still coupled to photons *Q: how can this be?*

$T_{\text{gas}} = T_{e,p,\text{H}} = T_\gamma$

(3) residual ionization freeze-in

free e and p diluted until "can't find each other"

But even still: photons scatter off residual ionization
e and thus p, H still exchange energy
with thermal photon bath: $T_{e,p,\mathrm{H}} = T_\gamma$ still!
when does this stop?

(4) gas decoupling

typical residual e no longer has photon interactions
gas decouples from photons
when? Thomps. scattering rate *per* e: $\Gamma_e = n_\gamma \sigma_\mathrm{T} c \lesssim H$
at $z_{\mathrm{dec,gas}} \sim 500$
note: scatter rate *per* e=$\Gamma_e \gg \Gamma_\gamma$=*scatter rate per CMB photon*

Summary of CMB Highlights

CMB Observed

can make precision observations of spectrum, sky distribution thanks to sophisticated radio techniques and instruments

- CMB fantastically isotropic: $\delta T/T \sim few \times 10^{-5}$
- CMB exquisitely thermal

CMB Theory

detailed, precise calculations of recomb, last scattering, thanks well-known atomic physics

- isotropic CMB $\rightarrow$ U. was once very homogeneous
- Planckian CMB spectrum $\rightarrow$ U. was once thermalized $\rightarrow$ plasma hot, dense enough to equilibrate

CMB $\rightarrow$ demands hot big bang in FLRW universe!

Extrapolated current U to $t \sim 400,000$ yr
and $z \sim 1000$ $\rightarrow$ **great success!**
Emboldens us to push earlier!