Astro 596/496 PC

Lecture 22

March 10, 2010

Announcements:

- PF4 due noon Friday

Last time: at last–cosmic recombination

★ $T_{\rm rec} \simeq B_{\rm H}/40 \ll B_{\rm H}$ *Q: why?*

★ after recomb: $X_e \neq 0$: some free e, p remain *Q: why?*

Today: spurred by successes at $z_{\rm rec} \sim 1500$, $t \sim 350,000$ yr
push to $t \sim 1$ sec

Primordial Nucleosynthesis

Prelude to Nucleosynthesis

Q: what sets T scale for element (nuclei) synthesis?

Q: what component dominates cosmic density, expansion then?

Q: what is the particle content of the universe then?

Q: what form(s) do the baryons take then? mesons?

Nucleosynthesis: Setting the Stage

⋆ light elements formed in nuclear reactions
relevant scale: nuclear binding energies $\sim$ MeV

⋆ $T \sim$ MeV at redshift $z_{\rm bbn} = T/T_0 - 1 \sim 10^{10}$!
since $z_{\rm bbn} \gg z_{\rm eq}$ (matter-rad equality)
well into radiation dominated era: $\rho \approx \rho_{\rm rad}$
`www:` Ω `vs` a `plot`
will see: $t(1 \text{ MeV}) \sim 1$ sec

⋆ particle content at BBN
relativistic species: photons, neutrinos, $e^{\pm}$ when $T \gtrsim m_e$
non-relativistic species: baryons, e^{-} when $T \ll m_e$
what about dark matter? energy?

DM presumably non-rel, weakly interacting: inert during BBN
DE: also assume not important for dynamics, microphyiscs
...but can later relax these assumptions and test them!

Who Feels What? Particles and Forces

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{array}{l} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \ \end{array} \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{array}{l} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \ \end{array} \quad (1)$$

quarks: feel all fundamental forces (strong, EM, weak, gravity)
carry conserved quantum number: **baryon number**
leptons: do *not* feel strong force
but also carry conserved quantum number: **lepton number**
▷ *charged* leptons: feel EM, weak, gravity
▷ neutrinos: only feel weak, gravity

More bragging rights:
in BBN, *all four* fundamental forces play a crucial role!

Neutrinos: Essential Ingredient yet Barely There

antineutrinos: $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$
since electric charge $Q(\nu) = 0$, possible that ν is own antiparticle
Q: is it?

masses: known that m_ν are nonzero (oscillations observed)
mass values not known (but for sure $\lesssim few \times 10$ eV $\ll m_e$)
Q: implications for BBN?
for quarks and charged leptons, masses increase with each family
$\rightarrow$ same for νs??

weak interaction: qualitative characteristics
(1) “signature” is transformation of quark, lepton flavor
e.g., $n \rightarrow p + e^- + \bar{\nu}_e$ decay
really a quark change $d(ud) \rightarrow u(ud) + e^- + \bar{\nu}_e$
(2) for $E \lesssim 100$ GeV ($= M_W, M_Z$), rxn strength is weak (duh!)
e.g., $\nu_e e \rightarrow \nu_e e$ scattering ~ 1 MeV: $\sigma_{\nu_e e} \sim 10^{-44}$ cm^2 $\sim 10^{-20} \sigma_T$

Nucleosynthesis: Particle Content Revisited

relativistic species:
γ, $\nu_i\bar{\nu}_i$ ($i \in e\mu\tau$), $e^{\pm}$ (for $T \gtrsim m_e$)

non-relativistic species:
baryons in BBN: when $T \gtrsim$ MeV: p, n only
when $T \leq m_e \rightarrow e$ non-rel too

⋆ neutrinos in BBN
Q: what sets n_ν, ρ_ν, T_ν? how do they evolve?
Q: assumptions needed?

BBN Initial Conditions: Ingredients of Primordial Soup

Begin above nuke binding: $T > 1$ MeV

EM reactions fast: typical rate $\Gamma_{\text{EM}} \sim n_\gamma \sigma_{\text{T}} c \gg H$
$\Rightarrow$ baryon, photon, $e^\pm$ pair plasma in thermal equilib:
$T_B = T_e = T_\gamma \equiv T$

weak int fast too (for now)! $\Gamma_{\text{weak}} \sim n_\nu \sigma_{\text{weak}} c \gg H$
all ν species coupled to each other, and plasma
$\rightarrow T_\nu = T_\gamma$

What sets densities n_ν, ρ_ν?
not only T_ν, but also dreaded chem potential μ_ν
physics issue: is there a net neutrino excess: $n_\nu \neq n_{\bar{\nu}}$?
c.f. net baryon excess $\rightarrow$ exists: $n_B \neq n_{\bar{B}}$, but small: $n_B/n_\gamma \ll 1$
if net lepton number $n_L \sim n_B$, turns out $\mu_\nu/T \sim \eta$ negligible
we will assume $\mu_\nu \ll T \Leftrightarrow$ no large lepton/baryon excess
if otherwise, changes story!

BBN Initial Conditions: Radiation Domination

Neutrino densities: for sure $m_\nu \ll T$
assume $\mu_\nu \ll T$ $\rightarrow$ absolute n_ν, ρ_ν, P_ν set by T_ν
$\rightarrow$ each ν species i has $n_{\nu_i} = n_{\bar{\nu}_i}$ and

$$n_{\nu\bar{\nu},i} \propto T^3 = \frac{3}{4} n_\gamma \quad \rho_{\nu\bar{\nu},i} \propto T^4 = \frac{7}{8} \rho_\gamma \tag{2}$$

total relativistic energy density:

$$\rho_{\text{rel}} = \rho_\gamma + \rho_{e^\pm} + N_\nu \rho_{1\nu\bar{\nu}} \equiv g_* \frac{\pi^2}{30} T^4 \tag{3}$$

where g_* counts "effective # of relativistic degrees of freedom"
at $T \gtrsim 1$ MeV, $g_* = 43/4 = 10.75$, and Friedmann:

$$\frac{t}{1\ \text{sec}} \approx \left(\frac{1\ \text{MeV}}{T} \right)^2 \tag{4}$$

Q: simple way to see $t \sim 1/T^2$ scaling is right?

now focus on baryons *Q: what sets n_B? n/p?*

BBN Initial Conditions: The Baryons

Cosmic baryon density n_B, and thus $\eta = n_B/n_\gamma$
not changed by reactions with $T \lesssim E_{\text{Fermilab}} \sim 1$ TeV $= 10^6$ MeV
i.e., baryon non-conservation not observed to date

▷ n_B set somehow in early universe ("cosmic baryogenesis")

▷ don't *a priori* know n_B, treat as free parameter (η)

neutron-to-proton ratio n/p *can* and *does* change at ~ 1 MeV
weak int fast: $n \leftrightarrow p$ interconversion

$$n + \nu_e \leftrightarrow p + e^- \tag{5}$$
$$p + \bar{\nu}_e \leftrightarrow n + e^+ \tag{6}$$

also recall $m_n - m_p = 1.29$ MeV: close in mass but not same!

Q: implications for n/p?

n/p ratio "thermal"

think of as *2-state* system: the "nucleon,"

- nucleon "ground state" is the *proton*: $E_1 = m_p c^2$
- nucleon "excited state" is the *neutron*: $E_2 = m_n c^2$

when in equilibrium, Boltzmann sez:

$$\left(\frac{n}{p}\right)_{\text{equilib}} = \frac{g_n}{g_p} e^{-(E_2 - E_1)/T} = e^{-(m_n - m_n)/T} \tag{7}$$

with $\Delta m = m_n - m_p = 1.293318 \pm 0.000009$ MeV

at $T \gg \Delta m$: $n/p \simeq 1$
at $T \ll \Delta m$: $n/p \simeq 0$

Equilibrium maintained until weak interactions freeze out
i.e., competition between weak physics, gravity physics

Q: how will weak freezeout scale compare to nuclear binding energy scale ~ 1 MeV?

Weak Freezeout Temperature

Weak interactions freeze when $H = \Gamma_{\mathrm{weak}}$, i.e.,

$$\sqrt{G_{\mathrm{N}}}T^2 \sim \sigma_0 m_e^{-2} T^5 \tag{8}$$

$$\Rightarrow T_{\mathrm{weak\ freeze}} \sim \frac{(G_{\mathrm{N}})^{1/6}}{(\sigma_0/m_e^2)^{1/3}} \sim 1\ \mathrm{MeV} \tag{9}$$

gravity & weak interactions conspire to give $T_{\mathrm{f}} \sim m_e \sim B_{\mathrm{nuke}}$!

for experts: note that $G_{\mathrm{N}} = 1/M_{\mathrm{Planck}}^2$, so

$$\frac{T^2}{M_{\mathrm{Pl}}} \sim \alpha_{\mathrm{weak}} \frac{T^5}{M_W^2} \tag{10}$$

$$\Rightarrow T_{\mathrm{freeze}} \sim \left(\frac{M_W}{M_{\mathrm{Pl}}}\right)^{1/3} M_W \sim 1\ \mathrm{MeV} \tag{11}$$

freeze at nuclear scale, but by accident!

Q: what happens to n, p then? what else is going on?

Directors' Cut Extras

Elementary Particles for the Minimalist
Antimatter

fundamental result of Relativistic QM
every particle has an antiparticle
e.g., $\overline{e^-} = e^+$ positron
e.g., $\bar{p} =$ antiproton; Fermilab: $p\bar{p}$ collisions

note: mass $m(\bar{x}) = m(x)$
decay lifetime $\tau(\bar{x}) = \tau(x)$
spin $S(\bar{x}) = S(x)$
electric charge $Q(\bar{x}) = -Q(x)$

sometimes particle = own antiparticle (must have charge 0)
e.g., $\bar{\gamma} = \gamma$, but note: $\bar{n} \neq n$

Cosmic Antimatter: rule of thumb
$x, \bar{x}$ abundant when thermally produced: $T > m_x$

Baryons

n and p not fundamental particles
made of 3 pointlike particles: "quarks"
two types ("flavors") in n, p: u "up," d "down"
$p = uud$, $n = udd \rightarrow$ quark electric charge $Q_u = +2/3$, $Q_d = -1/3$
spin $S(u) = 1/2 = S(d)$

baryon $\equiv$ made of 3 quarks

baryon conservation:
assign "baryon number" $A(q) = +1/3$, $A(\bar{q}) = -1/3$
$\rightarrow A(n) = A(p) = +1$
in all known interactions: baryon number conserved:
$\sum A_{\text{init}} = \sum A_{\text{fin}}$
$\rightarrow$ guarantees stability of the proton *Q: why?*
but free n unstable, decay to p *Q: why not n decay in nuclei?*

Periodic Table of Elementary Particles

known fundamental particles (& antipartners): 3 **families**

$$\begin{pmatrix} u \\ d \\ e \\ \nu_e \end{pmatrix} \begin{pmatrix} c \\ s \\ \mu \\ \nu_\mu \end{pmatrix} \begin{array}{l} \text{charm quark} \\ \text{strange quark} \\ \text{mu lepton (muon)} \\ \end{array} \begin{pmatrix} t \\ b \\ \tau \\ \nu_\tau \end{pmatrix} \begin{array}{l} \text{top quark} \\ \text{bottom quark} \\ \text{tau lepton} \\ \end{array} \quad (12)$$

all of these are spin-1/2: matter made of fermions!

Family Resemblances

1st family: quarks, charged lepton (e) comprise ordinary matter

2nd, 3rd family particles

- same electric charges, same spins, (mostly) same interactions as corresponding 1st family cousins
- but 2nd, 3rd family quarks, charged leptons more massive and & *unstable* $\rightarrow$ decay into 1st family cousins

lifetimes very short, e.g., longest is $\tau(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = 2 \times 10^{-6}$ s

Q: implications for BBN epoch?

Weak $n \leftrightarrow p$ Rates

example: want rate Γ_n per n of $\nu + n \rightarrow e^- + p$
as func. of T

Generally,

$$\Gamma_n = n_\nu \langle \sigma v \rangle \sim T^3 \langle \sigma \rangle \tag{13}$$

since $v_\nu \simeq c$

can show: cross section $\sigma \sim \sigma_0 (E_e/m_e)^2$
where $\sigma_0 \sim 10^{-44}$ cm^2 very small!
so thermal avg: $\langle \sigma \rangle \sim \sigma_0 (T/m_e)^2$

for experts: $\sigma \sim G_F^2 T^2 \sim \alpha_{\text{weak}} T^2 / M_W^4$
so $\Gamma_{\text{weak}} \sim \alpha_{\text{weak}} T^5 / M_W^4$