Astro 596/496 PC
Lecture 26
March 19, 2010

Announcements:

- PS4 due
- PF5 out, due Friday after break

Last time: began particle dark matter

- must be massive, long-lived, weakly interacting
- BBN: *most* cosmic matter *not* in baryonic form
  $\Omega_{\text{non-baryonic}} = \Omega_{\text{M}} - \Omega_{\text{B}} \approx 0.25$
- known (Standard Model) particle inventory:
  includes excellent DM candidate
  neutrinos: a match made in the heavens?
  no! m_ν too small: $\Omega_\nu < \Omega_{\text{B}} \ll \Omega_{\text{M}}$

⋆ *dark matter must be something else!*

⋆ *dark matter demands physics beyond Standard Model!*

Particle Dark Matter: Thermal Relics

Kolb & Turner, Ch. 5; Dodelson Ch. 3.4

Consider stable particle species χ (& antiparicle $\bar{\chi}$)

- nonrelativistic today: $m_\chi \gg T_0 \sim 3 \times 10^{-4}$ meV
- thermally produced in the early universe

What determines its abundance today?

Q: if χ is still in thermal (chemical) eq?

Q: and so?

Relic Particles

for non-relativistic species:
if *still in (chemical) equilibrium*: number density

$$n_\chi = g_\chi \left(\frac{m_\chi T}{2\pi \hbar^2} \right)^{3/2} e^{-(m_\chi - \mu_\chi)/T} \quad (1)$$

chem potential: $\mu \neq 0$ iff conserved particle number

if χ number *not conserved*–i.e., equal numbers of χ and $\bar{\chi}$
then $\mu_\chi = 0$, and so $n_\chi \sim e^{-m_\chi/T} \to 0$
$\Rightarrow$ no relic particles remain – terrible dark matter candidate!

Lessons: relic dark matter particles should

- *either* have *particle/antiparticle asymmetry*
  this is thought to be origin of baryons
- *or* must have dropped *out of equilibrium*
  Q: how might this happen?

Freezeout and Relic Abundance of a Symmetric Species

a *symmetric* species χ has a cosmic abundance
with *equal* numbers of particle and antiparticle
...or particle = antiparticle

thus $n_\chi = n_{\bar{\chi}}$ exactly: no "net χ number"
$\Rightarrow$ complete annihilation would leave no remaining particles

but: annihilation requires particle interactions!
these must compete successfully with expansion & cooling

in cosmic setting, essentially *gauranteed*
that at some point **annhilations freeze out**:
$\Gamma(\chi\bar{\chi} \to \text{stuff}) < H$
$\Rightarrow$ nonzero relic χ abundance, mass density also *gauranteed*!
Q: so does this gaurantee that χ is the dark matter?

Annihilation Freezeout

Sketch of calculation appears here; more details in extras

Annihilation rate per χ (and $\bar{\chi}$) particle is

$$\Gamma_{\rm ann}(T) \simeq n_{\chi,\rm eq}(T)\ \langle \sigma v \rangle \sim (m_\chi T)^{3/2} e^{-m_\chi/T}\ \langle \sigma v \rangle \qquad (2)$$

where σ is the annihilation cross section,
and $\langle \sigma v \rangle$ is a thermal average

Freezeout temperature $T_{\rm f}$ set by

$$H(T_{\rm f}) \sim \frac{T_{\rm f}^2}{M_{\rm pl}} = \Gamma_{\rm ann}(T_{\rm f}) \sim (m_\chi T_{\rm f})^{3/2} e^{-m_\chi/T_{\rm f}}\ \langle \sigma v \rangle \qquad (3)$$

dominated by exponential: $T_{\rm f} \sim m_\chi$
so freezeout χ density is

$$n_{\chi,\rm f} \simeq \frac{H(T_{\rm f} = m_\chi)}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{M_{\rm pl} \langle \sigma v \rangle} \qquad (4)$$

Relic Abundance and Density

relic χ abundance at freezeout $T_{\rm f} \sim m_\chi$:

$$n_{\chi,{\rm f}} \simeq \frac{H(T_{\rm f}=m_\chi)}{\langle \sigma v \rangle} \sim \frac{m_\chi^2}{M_{\rm pl}\langle \sigma v \rangle} \tag{5}$$

But we want χ abundance and mass density *today*
note that after freeze, χ conserved!
$\rightarrow n_\chi = n_{\chi,{\rm f}}(a_{\rm f}/a)^3 \propto T^3 \propto n_\gamma$
$\rightarrow Y_\chi \equiv n_\chi/n_\gamma$ DM/photon ratio *constant*, set at freeze:

$$Y_\chi = \frac{n_{\chi,{\rm f}}}{n_{\gamma,{\rm f}}} \sim \frac{m_\chi^2/M_{\rm pl}\langle \sigma v \rangle}{m_\chi^3} \sim \frac{1}{M_{\rm pl} m_\chi \langle \sigma v \rangle} \tag{6}$$

So present number and mass densities are

$$n_{\chi,0} = Y_\chi n_{\gamma_0} \tag{7}$$

$$\rho_{\chi,0} = m_\chi n_{\chi,0} \sim \frac{1}{M_{\rm pl}\langle \sigma v \rangle} \tag{8}$$

What have we shown?

if a symmetric stable species ever created

(annihilates but not decays)

then annihilations will freeze, and

inevitably have nonzero relic density today, namley

$$\rho_{\chi,0} = m_\chi n_{\chi,0} \sim \frac{1}{M_{\rm pl}\langle\sigma v\rangle} \tag{9}$$

This calculation is of the highest interest to particle physicists

Q: why?

We have calculated a relic density

Q: Notable aspects about what it does, doesn't depend on?

Q: To what should it be compared?

Cold Relics: Present Abundance

⋆ $\rho_{\psi,0}$ indep of m_ψ

⋆ $\rho_{\psi,0} \propto 1/\sigma$: the weak prevail!
Q: what sort of cross section is relevant here?

⋆ To get "interesting" present density:
$\Omega_\psi \sim 1 \rightarrow \rho_\psi \sim \rho_{\rm crit}$ *demands* a *specific* cross section

$$\sigma \sim \frac{n_{\gamma,0}}{\Omega_\psi M_{\rm p} \rho_{\rm crit}} \tag{10}$$

$$\sim 10^{-38}\ {\rm cm}^2 \tag{11}$$

∞ scale of the Weak interaction! $[\sigma_{\rm weak}(E \sim {\rm GeV}) \sim 10^{-38}\ {\rm cm}^2]$

The WIMP Miracle

Dark Matter candidate:
if DM is a cold symmetric relic
needed *annihilation cross section* is at Weak scale!
corresponding energy: if $\sigma \sim \alpha/E^2$
then $\sigma \sim 10^{-36}$ cm^2 $=$ 10 pb $\rightarrow E \sim 1$ TeV

deeper reason for DM as
Weakly Interacting Massive Particle: **WIMP**

that weak-scale annihilations $\rightarrow \Omega_\chi \sim \Omega_{\rm nbdm}$: **"WIMP Miracle"**

How to find them?
What if we do? What if we don't?

WIMP Searches: Accelerators

if WIMPs exist in nature
...and especially if they are supersymmetric particles
likely to be found in $\sim few$ yrs
at CERN Large Hadron Collider (or maybe even Fermilab)
`www: CERN, LHC`
`www: FNAL, CDF`
SUSY/WIMP discovery would revolutionize particle physics
and all but guarantee dark matter = cold relics

Even if nature is not supersymmetric
many particle theories predict new physics at ~ 1 TeV

Note: even if discover supersymmetry,
  maybe not directly see the LSP
but: if dark matter is a WIMP, other ways to find out
Q: namely?

WIMP Searches: Direct Detection

if WIMPs are DM $\rightarrow$ dark halo full of them
local density $\rho = mn \sim 0.3$ GeV cm^{-3}
virial velocities $v_0^2 \sim GM_{\text{halo}}/R_{\text{halo}} \sim (400 \text{ km/s})^2$
$\Rightarrow$ WIMP flux $F_{\text{WIMP}} = nv_0$
$\Rightarrow$ Look for *WIMP-nucleus elastic scattering* – challenging!

Search using sensitive detectors: cryogenic, underground
interaction: *WIMP collision $\rightarrow$ nuclear recoil*
measure: effects of recoiling ($E_{\text{kin}} \sim 1 - 100$keV) nucleus
Q: for example?

WIMP-nucleus recoil signatures

▷ *energy injection*: recoil heats detector
crystal specific heat $C = dE/dT \sim T^3$
$\Delta T = \Delta E/C \propto T^{-3}$
if supercold, can detect ΔT rise

▷ *momentum transfer*: detector lattice (phonons) excited

▷ *scintillation, ionization*: charged recoil nucleus excites me dium
relax via γ, e emission $\rightarrow$ detect these

CDMS Hints at WIMPS?

Cryogenic Dark Matter Experiment `www: CDMS`
most sensitive direct-detection experiment to date

2010 publication of final results from latest run
- **2 events** seen in signal region!
- ...but expected background is $N_{\text{bg}} = 0.9 \pm 0.2$ **events**
  $\simeq 23\%$ chance of an unlucky high background fluctuation

bad news: uncomfortably large chance of false positive!
good news: CDMS & competitors have improved sensitivity
  should know in $1-2$ year timescale if this was real!
  lay your bets now!

that's still not all...
Q: astrophysical means infer WIMP existence and properties?

WIMP Searches: Indirect Detection

if WIMPs are DM $\longrightarrow$ Galactic dark halo full of them
but Galactic halo density $\gg$ cosmic mean
$\longrightarrow$ annihilation rate $q \propto \langle \sigma v \rangle \rho^2_{\rm wimp}$ can be large
$\longrightarrow$ annihilation products potentially observable

Local annihilations

Q: how see if $\psi\bar{\psi} \rightarrow \gamma\gamma$ *only?*
Q: how see if $\psi\bar{\psi} \rightarrow$ *other Standard Model particles?*
e.g., $\psi\bar{\psi} \rightarrow e^+ e^-$ *or* $q\bar{q}$*?*

Galactic center annihilations

Q: how see if $\psi\bar{\psi} \rightarrow \gamma\gamma$ *only?*
Q: how see if $\psi\bar{\psi} \rightarrow$ *other Standard Model particles?*
e.g., $\psi\bar{\psi} \rightarrow e^+ e^-$ *or* $q\bar{q}$*?*

Indirect Detection: Local Annihilation Signatures

if $\psi\bar{\psi} \to \gamma\gamma$ only: line emission $E_\gamma \sim m_\psi$
$\Rightarrow$ local contribution to diffuse γ signature
but: two-photon annihilation $\psi\bar{\psi} \to \gamma\gamma$ must be *suppressed*
else χ has direct EM coupling $\to$ electric charge $\to$ DM not dark!
but *can and often do* have things like $\psi\bar{\psi} \to \pi's \to \gamma's$

if $\psi\bar{\psi} \to q\bar{q}$: hadronize, sometimes to nucleons $N\bar{N}$
source of $\bar{n}, \bar{p}$, and $\bar{d} = \boxed{\bar{n}\bar{p}}$
$\Rightarrow$ can look for these in cosmic rays!
but *"foreground"*: "normal" antimatter
from cosmic ray propagation
e.g., $p_{\rm cr} + p_{\rm ism} \to ppp\bar{p}$

if $\psi\bar{\psi} \to e^+ e^-$: local source of high-energy e^+

Indirect Detection: Galactic Center Annihilation

Galactic center is $\rho_{\rm DM}$ peak $\rightarrow$ annihilation goldmine!?!

Direct Photon Production

⋆ $\psi\bar{\psi} \rightarrow \gamma\gamma$ line: $E_\gamma = m_\psi$, and

⋆ $\psi\bar{\psi} \rightarrow q\bar{q} \rightarrow \pi^0 \rightarrow \gamma\gamma$ continuum $E_\gamma < m_\psi$

Galactic center seen in GeV range (1990's: EGRET)
with poorly understood "GeV excess"!?!
But new γ-ray observatory `www: Fermi`
launched 2008, finds no GeV excess

Galactic center seen in TeV range
`www: HESS`
but point source too localized(?), energy spectrum a power-law

Dark Matter: Where Do We stand?

Obviously, no clear detections thus far

Current status:
accelerator and astrophysical constraints are:
competitive: both place strong constraints
on particle models for WIMPS
complementary: different methods strong in different parts
of parameter space

Upgrades coming soon on all fronts
$\longrightarrow$ the race is on!
$\longrightarrow$ an answer will emerge in the non-distant future!

If confirmed WIMP detection:

- DM found
- need particle physics beyond Standard Model

★ payoff big!

If no WIMP signature

- SUSY much less attractive
- dark matter not a cold relic $\rightarrow$ what is it?
  an asymmetric relic? but why asymmetrical?
  modified gravity?
  hidden in braneworld?

Director's Cut Extras

Freezeout and Relic Abundanceof a Symmetric Species

for *conserved* species ψ (chem. pot. $\mu_\psi \neq 0$)
constant comoving number: $d(na^3) = 0$
$\Rightarrow \dot{n}_\psi + 3\frac{\dot{a}}{a} n_\psi = 0$

for *non-conserved* species: $d(n_\psi a^3) = qa^3 \; dt \neq 0$, where
q = *source/sink* rate = creation/destruction rate per unit vol
$\Rightarrow \boxed{\dot{n}_\psi + 3\frac{\dot{a}}{a} n_\psi = q}$
assume annihilation $\psi\bar{\psi} \rightarrow X\bar{X}$ product X thermal,
with chem. pot. $\mu_X \ll T \;\; \Rightarrow \;\; n_X = n_{\bar{X}}$

$$q = q_{\rm net} = q_{\rm prod} - q_{\rm ann} \tag{12}$$
$$= \langle\sigma v\rangle_{\rm prod} n_X n_{\bar{X}} - \langle\sigma v\rangle_{\rm ann} n_\psi n_{\bar{\psi}} \tag{13}$$
$$= \langle\sigma v\rangle_{\rm prod} n_X^2 - \langle\sigma v\rangle_{\rm ann} n_\psi^2 \tag{14}$$

in equilib, *Q: what condition holds for q?*

chem equilib: $q = 0 \Rightarrow \boxed{q_{\text{prod}} = q_{\text{ann}}}$
so in general

$$\dot{n}_\psi + 3Hn_\psi = q = -\langle\sigma v\rangle_{\text{ann}} \left[n_\psi^2 - (n_\psi^{\text{eq}})^2\right] \tag{15}$$

and a similar expression for $\bar{\psi}$

Change variables:
(1) use comoving coords:
photon density $n_\gamma \propto T^3 \propto a^{-3}$,
so put $Y = n_\psi/n_\gamma$ to remove volume dilution
then $\dot{n_\psi} + 3\dot{a}/a \; n_\psi = n_\gamma \dot{Y}$
(2) put $x = m_\psi/T \propto a$
since $t \propto 1/T^2 \propto x^2$,
$dY/dt = dY/dx \; \dot{x} = H \; x \; dY/dx$

Then:

$$Hx\frac{dY}{dx} \;=\; -n_\gamma\langle\sigma v\rangle_{\text{ann}} \left(Y^2 - Y_{\text{eq}}^2\right) \tag{16}$$

(17)

finally

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_A}{H}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \qquad (18)$$

where $\Gamma_A = n_\psi^{\rm eq}\langle\sigma v\rangle_{\rm ann}$: annihil. rate

So: change in comoving ψ controlled by
(1) annihil. effectiveness Γ/H
(2) deviation from equil

when $\Gamma/H \gg 1$
Q: what if $Y < Y_{\rm eq}$? $Y > Y_{\rm eq}$?

when $\Gamma/H < 1$
Q: how does Y change?

Q: how you you expect Y to evolve?

when $\Gamma/H \gg 1$, Y driven to $Y_{\rm eq}$
when $\Gamma/H < 1$, Y change is small $\rightarrow$ freezeout!

relic abundance at $T \rightarrow 0$ or $x \rightarrow \infty$ is
$Y_\infty \simeq Y_{\rm eq}(x_f)$: value at freezeout

Step back:
How can a symmetric species have
$n_\psi = n_{\bar{\psi}} \neq 0$ at $T \ll m$?

physically: expansion is key

if $H = 0$, $Y_\infty = Y_{\rm eq}(\infty) = 0$:

$\rightarrow$ all ψ find $\bar{\psi}$ partner, annihilate

but $H \neq 0$: when U dilute enough,

ψ never finds $\bar{\psi}$: i.e., $\Gamma \ll H$

nonzero relic abundance

hot relics: $x_f \ll 1$ ($T_f \gg m$)

cold relics: $x_f \gg 1$

Note: hot/cold *relics* refers to freezeout conditions

But: hot/cold *dark matter* refers to structure formation criteria

(namely, m vs temp $T_{\rm eq}$ at matter-rad equality)

Cold Relics: WIMPs

cold relic: non-relativistic at freezeout
so $x_f = m/T_f \gg 1 \rightarrow T_f \ll m$
$\Rightarrow n_{\rm eq} \sim e^{-m/T}(mT)^{3/2}$
$\Rightarrow Y_{\rm eq} \sim e^{-x}x^{3/2}$

Freezeout:
$\Gamma_{\rm ann} = H$ at $T = T_f$
$\Rightarrow n_{\rm eq}\langle\sigma v\rangle_{\rm ann} \sim \sqrt{G}T^2$

what needed to find value of T_f?

To solve:

- need annihilation cross section
  for many models, $\langle \sigma v \rangle \propto v^n$ (S-wave: $n = 0$)
  $\Rightarrow \langle \sigma v \rangle(x) = \sigma_1 c x^{n/2}$, where $\sigma_1 = \sigma(E = m)$
- convenient rewrite $1/\sqrt{G} = M_{\mathrm{Pl}} \simeq 10^{19}$ GeV
  (Planck Mass)

set $\Gamma_{\mathrm{ann}}(T_f) = H(T_f)$, and solve for T_f

Find: $x_f \sim \ln(m M_{\mathrm{Pl}} \sigma_1) \Rightarrow T_f = m/x_f$

So

$$Y_\infty \simeq Y_{\mathrm{eq}}(x_f) \quad (19)$$

$$\sim \frac{x_f^{3/2}}{m M_{\mathrm{Pl}} \sigma_1} \quad (20)$$

$\longrightarrow$ present relic number density

$$n_{\psi,0} = Y_\infty n_{\gamma,0} = 400\ Y_\infty\ \mathrm{cm}^{-3} \qquad (21)$$

present relic mass density

$$\rho_{\psi,0} = m n_{\psi,0} \simeq \frac{x_f^{3/2} n_{\gamma,0}}{M_{\mathrm{Pl}} \sigma_1} \qquad (22)$$

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