Astro 596/496 PC
Lecture 28
March 31, 2010

Announcements:

- PF5 due Friday at noon
- Physics Colloquium tomorrow, 4pm: Stephon Alexander
  "Can Condensed Matter Physics Shed Light on Dark Energy?"

Last time: began inflation
Motivation: cosmic puzzles
  flatness, horizon, monopoles, lumpiness
Inflation: early period of rapid expansion
  scale factor growth $a_{\rm post-inf}/a_{\rm pre-inf} \sim e^{60} \sim 10^{26}$
Simultaneously solves flatness, horizon, monopoles
  horizon: **Plot:** *spacetime, sans and with inflation*
  flatness: acceleration drives $|\Omega - 1| \rightarrow 0$

Q: how does inflation differ from usual cosmic expansion?

Inflating Away the Horizon Problem

quantitative solution to horizon problem:
comoving **particle horizon** $d_{\rm hor,com}$
sets region *ever* causally connected, and
always increases

but: causally connected universe at time t
has size set by comoving **Hubble length** $d_{\rm H,com} = 1/aH = 1/\dot{a}$

▷ if $\dot{a}$ increasing = cosmic acceleration, then $d_{\rm H,com}$ decreasing
causal region of U *shrinks* during inflation

▷ also, shows that horizon, flatness linked
solving one solves the other!

Monopole Problem

qualitatively: solution via dilution: inflate them away!

quantitatively: relic number density $n_{\text{monopole}} \propto a^{-3}$

if a grows by e^{60}, number drops by e^{180}

"dilution is the solution to your pollution!"

some benefits of acceleration appreciated pre-inflation

e.g., Hoyle, Gold, Bondi, Starobinskii, Kazanas...

$\longrightarrow$ by 1980, it was "in the air" that

early U. acceleration is devoutly to be wished for

Yes, But..

but wishing doesn't make it so:

- what causes acceleration?
- how can it be tested?

Inflationary Cosmology

recall from Dark Energy discussion: acceleration demands $P < 0$
can't do this with matter or radiation

But:
★ scalar field ϕ can have $P_\phi < 0$
★ scalar fields *required* for electroweak unification
and appear in all other unification schemes

Alan Guth (1981)
if early Universe
▷ contains a scalar field,
▷ whose potential energy $\rho_\phi \approx V_\phi \approx \rho_{\rm tot}$
then (in 21st century language) $w_\phi \rightarrow -1$
$\rightarrow$ cosmic acceleration and exponential expansion!

Cosmic Scalar Fields: Episode II

recall: cosmic scalar field ϕ, "minimally coupled" – i.e.,

- interacts only to itself via potential $V(\phi)$
- and gravity, via ρ_ϕ

Properties: Note $\hbar = c = 1!$ $\Rightarrow [\phi] = [E] = [\ell^{-1}] = [t^{-1}]$

Equation of motion $$\ddot{\phi} - \nabla^2\phi + 3H\dot{\phi} - \frac{dV}{d\phi} = 0 \quad (1)$$

energy density $$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V \quad (2)$$

pressure $$P_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 - V \quad (3)$$

why? Lagrangian dens $\mathcal{L} = 1/2\ \partial_\mu\phi\partial^\mu\phi - V$ $\Rightarrow$ stress-energy

$$T_{\mu\nu} \equiv \mathrm{diag}(\rho_\phi, p_\phi, p_\phi, p_\phi) \quad (4)$$

$$= \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\mathcal{L} = (\partial_\mu\phi\partial_\nu\phi - \frac{g_{\mu\nu}}{2}\partial_\mu\phi\partial^\mu\phi) + g_{\mu\nu}V \quad (5)$$

Recall: for homogeneous field $\phi(t, \vec{x}) = \phi(t)$, so $\nabla\phi = 0$, and then we have

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0 \tag{6}$$

formally: same as Newtonian ball rolling down hill V
but impeded by friction ("Hubble drag") $3H$

$$\rho_\phi = \dot{\phi}^2/2 + V \tag{7}$$
$$P_\phi = \dot{\phi}^2/2 - V \tag{8}$$

which gives

$$w_\phi = \frac{P_\phi}{\rho_\phi} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \tag{9}$$

and so scalar field can have $w_\phi \in [-1, 1]$!
If $w_\phi < 0$, then ϕ is an *accelerant*, has $P_\phi < 0$!

Q: requirements of workable inflation scenario?

Ingredients of an Inflationary Scenario

Recipe:

1. **inflaton field ϕ must exist** in early U.
2. must have $\rho_\phi \approx V$ so that $w_\phi \to -1$ so that $a \sim e^{Ht}$
3. continue to exponentiate $a \sim e^N a_{\rm init}$
   for at least $N = \int H\, dt \gtrsim 60$ e-folds
4. stop exponentiating eventually (**"graceful exit"**)
5. convert field ρ_ϕ back to radiation, matter (**"reheating"**)
6. then ϕ must "keep a low profile," $\rho_\phi \ll \rho_{\rm tot}$

7 (bonus) what about acceleration and dark energy today?
   is quintessence a rebirth of inflationary ϕ?
      goal of "quintessential inflation" models

Q: to meet 2: $\rho_\phi \approx V \to$ what does this mean?

To inflate, need slow ϕ evolution:

$\ddot{\phi} \ll 3H\dot{\phi}$ $\leftrightarrow$ friction large: achieve "terminal speed"

$$\dot{\phi} \approx -\frac{1}{3H} V' \qquad (10)$$

Slowness conditions $\dot{\phi}^2/2 \ll V$ and $\ddot{\phi} \ll 3H\dot{\phi}$
constrain **"slow-roll parameters"**:

$$\epsilon(\phi) = \frac{m_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad (11)$$

$$\eta(\phi) = m_{\rm pl}^2 \frac{V''}{V} \qquad (12)$$

to be **small**: $\epsilon \ll 1$ and $|\eta| \ll 1$
(you'll get to show this in PS6)

Q: Why is it useful to express slow-roll criteria this way?

Q: What do these imply about the nature of V?

Q: What about magnitude of ϕ during inflation?

The Charms of a Slow Roll

Usefulness of slow roll parameters ϵ, η

$\star$ ϵ, η quantify conditions for *maintaining* inflation
purely in terms of underlying potential V
$\longrightarrow$ an immediate constraint on inflaton physics
i.e., any workable potential most satisfy slow roll
want derivatives small $\longrightarrow$ need flat potential

$\star$ ϵ, η quantify inflaton energy scale
• typically expect $V'/V \sim 1/\phi$
• but slow roll $(V'/V)^2 \sim \epsilon \, m_{\mathrm{pl}}^2$
together these give $\phi \gtrsim m_{\mathrm{pl}}$ during inflation

generically expect $\phi \gtrsim m_{\mathrm{pl}}$

$\Rightarrow$ for successful inflation, pushed to Planck scale (?)

;-) a good thing?

hints at quantum gravity

if $\Omega_{\mathrm{init}} \gtrsim 1$, inflation prevents U. collapse $\rightarrow$ black hole

=:-o a bad thing?

quantum gravity a prerequisite for inflation models?

moves away Guth's original idea, GUT physics?

$\star$ ϵ, η also can quantify conditions for *ending* inflation

Amount of Inflation

during inflation scale factor grows exponentially
(in most models); in any case
quantify "amount" of inflation as
$N = \ln(a_{\rm fin}/a_{\rm init})$: number of "$e$-foldings"

What is needed?

to solve horizon, flatness, monopoles back to GUT scale:
$N \gtrsim N_{\rm min} \sim 60$ (PS6)

What is predicted?

Since $H = \dot{a}/a = d\ln a/dt = \dot{N}$, and $dt = d\phi/\dot{\phi}$, we have

$$N = \int_{t_{\rm init}}^{t_{\rm fin}} H\,dt = \int_{\phi_{\rm init}}^{\phi_{\rm fin}} \frac{H\,d\phi}{\dot{\phi}} \tag{13}$$

slow roll: $\dot{\phi} \simeq -V'/3H$, so

$$N = \int_{\phi_{\rm fin}}^{\phi_{\rm init}} \frac{3H^2\, d\phi}{V'} = m_{\rm pl}^2 \int_{\phi_{\rm fin}}^{\phi_{\rm init}} \frac{V}{V'} d\phi \tag{14}$$

typically expect $V'/V \sim 1/\phi$, which gives

$$N \sim \frac{\Delta\phi^2}{m_{\rm pl}^2} \tag{15}$$

amount of inflation set by:

- nature of potential V
- change in ϕ

note also that need $N \gg 1$ and thus
typically expect $\phi_{\rm init} \gtrsim m_{\rm pl}$
...but already required by slow roll

Q: what determines inflation end physically? mathematically?

A Graceful Exit from Inflation

inflaton continues until acceleration stops ($w_\phi > 0$)
$\rightarrow$ potential energy no longer dominates cosmic ρ
all matter and radiation inflated away, so "rescue" comes from
kinetic energy $\dot{\phi}^2/2$ (by itself, has $w = +1$!)

in terms of potential, exit when slow roll stops
quantified by slow-roll parameters
i.e., ϕ evolves until $\epsilon(\phi) \sim 1$

inflaton requirements:

- to achieve slow roll $\rightarrow$ need flat V far from minimum
- to end slow roll $\rightarrow$ need non-flat
  $V' \gtrsim V/m_{\rm pl}$ approaching minimum

Q: and then...? What's the Universe like? What happens next?

Reheating: Back to the Hot Big Bang

After $e^{60} \sim 10^{26}$ expansion
radiation, matter particles diluted to negligibility as a^{-3}
temperature drop $T \sim 1/a \to 0$: "supercooling"

But since $V(\phi) \sim const$ during inflation
inflaton energy density still large afterwards
must convert to hot, radiation-dominated early U: **reheating**

Details complicated, model-dependent; basic idea:
★ ϕ evolves in non-inflationary way
★ quantum effects drive energy conversion

Reheating I: Inflaton Oscillations

near minimum $V \simeq \frac{1}{2} V'' \phi^2 \equiv \frac{1}{2} m_\phi^2 \phi^2$

$$\ddot{\phi} + 3H\dot{\phi} + V' \approx \ddot{\phi} + m_\phi^2 \phi = 0 \qquad (16)$$

simple harmonic oscillator!

▷ classically field oscillates around zero rapidly
and coherently: within particle horizon, same oscillation phase
so $\langle \dot{\phi}^2/2 \rangle = \langle V \rangle = \langle m_\phi^2 \phi^2/2 \rangle$

- which means $\langle P_\phi \rangle = 0$ *Q: why?*
- which means $w_\phi = 0$, and so

$\langle \rho_\phi \rangle \sim a^{-3(1+w_\phi)} = a^{-3}$ like NR matter!

$\Rightarrow$ so ρ_ϕ drops $\to$ oscillation amplitude decays

Reheating II: Downfall of the Inflaton

▷ quantum mechanically field excitations → quanta
inflaton particles (mass m_ϕ) created

But the inflaton must be unstable *Q: why?*
→ decays to particles with Standard Model interactions
- if ϕ only decays to fermions
  does so slowly, products made thermally
- if ϕ can decay to bosons, resonances likely
  rapid decay far from equilibrium

In either case: decay products interact, exchange energy
thermalize: $\rho_\phi \rightarrow \rho_{\rm rad} \sim T^4$

$T_{\rm reheat} \sim \rho_{\phi,{\rm fin}}^{1/4}$

Q: what is rock-bottom minimum $T_{\rm reheat}$?

Inflation and the Rest of Cosmology

Reheating Temperature

⋆ All of 'usual" hot big bang begins *after* reheat

⋆ Must reheat enough for U to undergo
any and all known hot big phases
e.g., have to *at least* heat up to have nucleosynthesis
i.e., successful nuke requires $T_{\text{reheat}} > 1$ MeV
earlier phases (if any) demand hotter reheat

What about other physics, cosmology?

Q: How must inflation fit in with, e.g., monopole production, baryon genesis, BBN, CMB?