Astro 596/496 PC
Lecture 29
April 2, 2010

Announcements:
- PF5 was due at noon
- PS5 out, due in class **Monday** **April 12**

PF5:
Q: horizon solution without inflation?
Q: inflation/dark energy compare/contrast?
Q: what does it mean for the U to expand faster than light?

Last time: Inflation
- scalar field dynamics in an expanding universe
  slow roll conditions constrain inflaton potential

Q: what's rolling? why must roll be slow?
what is required to make it slow?

Ingredients of an Inflationary Scenario

Recipe:

1. **inflaton field ϕ must exist** in early U.
2. must have $\rho_\phi \approx V$ so that $w_\phi \rightarrow -1$ so that $a \sim e^{Ht}$
3. continue to exponentiate $a \sim e^{N} a_{\rm init}$
   for at least $N = \int H\, dt \gtrsim 60$ e-folds
4. stop exponentiating eventually (**"graceful exit"**)
5. convert field ρ_ϕ back to radiation, matter (**"reheating"**)
6. then ϕ must "keep a low profile," $\rho_\phi \ll \rho_{\rm tot}$

7 (bonus) what about acceleration and dark energy today?

Q: what can we say about how inflation fits
in the sequence *of cosmic events, e.g.*
monopole production, baryon genesis, BBN, CMB?

Cosmic Choreography: The Inflationary Tango

Inflation must occur such that it
solves various cosmological problems, then
allows for the universe of today, which *must*

- ▷ contain the known particles, e.g., a net baryon number
- ▷ pass thru a radiation-dominated phase (BBN)
  and a matter-dominated phase (CMB, structure formation)

$\Rightarrow$ this forces an ordering of events

Cosmic Choreography: Required *time-ordering*

1. monopole production (if any)
2. inflation
3. baryogenesis (origin of $\eta \neq 0$)
4. radiation $\rightarrow$ matter $\rightarrow$ dark energy eras

Electroweak woes: hard to arrange baryogenesis afterwards!

Models for Inflation

Inflation model: specifies $V(\phi)$
[+ initial conditions, reheat prescription]

Polynomial Potentials
e.g., Klein-Gordon $V = m^2\phi^2/2$, quartic $V = \lambda\phi^4$
- simplest models giving inflation
- require *Planck-scale* initial conditions for ϕ
- but to achieve sufficient inflation (enough e-foldings N) and perturbations at observed (CMB) scale demands *tiny coupling* $\lambda \sim 10^{-13}$ (!)
  $\rightarrow$ potential energy scale $V \ll m_{\rm pl}^4$
  but why is coupling so small?

Illustrates characteristics of (successful) inflation models:
▷ large initial $\phi \gtrsim m_{\rm pl}$ value
▷ small coupling in V $\rightarrow$ scale $V^{1/4} \sim 10^{15-16}$ GeV (GUT?)

Exponential Potentials: Power-Law Inflation if

$$V = V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{m_{\rm pl}}\right) \tag{1}$$

then can solve equations of motion exactly:
$a \sim t^p$; if $p > 1$, U. accelerates, but not exponentially

Designer Potentials

can customize V to give desired $a(t)$, e.g.,
$a \sim \exp(At^f)$, $0 < f < 1$ if

$$V(\phi) \sim \left(\frac{\phi}{m_{\rm pl}}\right)^{-\beta}\left[1 - \frac{\beta^2}{6}\left(\frac{m_{\rm pl}^2}{\phi^2}\right)\right] \tag{2}$$

How about the Higgs?

from electroweak unification, we “know” of one scalar

$\rightarrow$ Higgs field H^0, $M_H \gtrsim 100$ GeV?

same symbol as Hubble, right kind of field $\rightarrow$ is it ϕ?

i.e., what about inflation at the electroweak scale?

not a bad idea–possibly correct!–but nontrivial at best

problem not with inflation, but its place in the cosmic dance

Inflation, Inhomogeneities, and Quantum Mechanics

Thus far: classical treatment of inflaton field

(except for inflaton decays during reheating)

- ϕ described by classical equations of motion
- taken to hold for arbitrarily small ϕ

In this picture:

when exit inflation, universe essentially

▷ perfectly flat, and

▷ perfectly smooth–i.e., density spatially uniform

regardless of initial conditions (as long as they allowed inflation)

Q: why?

Classical Inflation and Smoothness

expect initial spatial inhomogeneities in $\phi(\vec{x})$
but evolves classically as

$$\ddot{\phi} - \nabla^2 \phi + 3H\dot{\phi} - V' = 0 \qquad (3)$$

where

$$\nabla^2 = \sum \frac{\partial^2}{\partial x_{\rm phys}^2} = \frac{1}{a^2} \sum \frac{\partial^2}{\partial x_{\rm com}^2} \qquad (4)$$

inhomogeneities $\delta\phi(\vec{x})$ measured by nonzero gradients
but since $\nabla^2 \propto 1/a^2 \rightarrow 0$ exponentially, classically: $\delta\phi(\vec{x}) \rightarrow 0$
$\Rightarrow$ after inflation ϕ and $\rho = V(\phi)$ exponentially smooth in space

good news: solved flatness, smoothness problems
bad news: we have done too much! too smooth!
can't form structures if density perfectly uniform

Quantum Mechanics to the Rescue

but quantum mechanics exists and is mandatory
classical ϕ field $\rightarrow$ quantized as inflatons
think $\vec{E}, \vec{B}$ vs photons

inflaton field must contain quantum fluctuations
before, during inflation

What we want: statistical properties of fluctuations

- typical magnitude of fluctuations $\delta\phi$
- how $\delta\phi$ depends on lengthscales
- corresponding fluctuations in ρ_ϕ
- correlations at different length scales