Announcements:
• PF5 was due at noon
• PS5 out, due in class Monday April 12

PF5:
Q: horizon solution without inflation?
Q: inflation/dark energy compare/contrast?
Q: what does it mean for the U to expand faster than light?

Last time: Inflation
• scalar field dynamics in an expanding universe
  slow roll conditions constrain inflaton potential
Q: what’s rolling? why must roll be slow?
  what is required to make it slow?
Ingredients of an Inflationary Scenario

Recipe:
1. **inflaton field** \( \phi \) **must exist** in early U.
2. must have \( \rho_\phi \approx V \) so that \( w_\phi \rightarrow -1 \) so that \( a \sim e^{Ht} \)
3. continue to exponentiate \( a \sim e^{N a_{\text{init}}} \)
   for at least \( N = \int H \, dt \gtrsim 60 \) e-folds
4. stop exponentiating eventually (**graceful exit**)
5. convert field \( \rho_\phi \) back to radiation, matter (**reheating**)
6. then \( \phi \) must “keep a low profile,” \( \rho_\phi \ll \rho_{\text{tot}} \)
7. (bonus) what about acceleration and dark energy today?

Q: **what can we say about how inflation fits**
   **in the sequence of cosmic events**, e.g. **monopole production, baryon genesis, BBN, CMB?**
Cosmic Choreography: The Inflationary Tango

Inflation must occur such that it solves various cosmological problems, then allows for the universe of today, which **must**

- contain the known particles, e.g., a net baryon number
- pass thru a radiation-dominated phase (BBN) and a matter-dominated phase (CMB, structure formation)

⇒ this forces an ordering of events

**Cosmic Choreography: Required time-ordering**

1. monopole production (if any)
2. inflation
3. baryogenesis (origin of $\eta \neq 0$)
4. radiation $\rightarrow$ matter $\rightarrow$ dark energy eras

Electroweak woes: hard to arrange baryogenesis afterwards!
Models for Inflation

Inflation model: specifies $V(\phi)$

[+ initial conditions, reheat prescription]

Polynomial Potentials

e.g., Klein-Gordon $V = m^2 \phi^2/2$, quartic $V = \lambda \phi^4$

• simplest models giving inflation
• require Planck-scale initial conditions for $\phi$
• but to achieve sufficient inflation (enough $e$-foldings $N$) and perturbations at observed (CMB) scale demands tiny coupling $\lambda \sim 10^{-13}$ (!)

$\rightarrow$ potential energy scale $V \ll m^4_{\text{pl}}$

but why is coupling so small?

Illustrates characteristics of (successful) inflation models:

▷ large initial $\phi \gtrsim m_{\text{pl}}$ value
▷ small coupling in $V \rightarrow$ scale $V^{1/4} \sim 10^{15-16}$ GeV (GUT?)
Exponential Potentials: Power-Law Inflation if

\[ V = V_0 \exp \left( -\sqrt{\frac{2}{p m_{pl}}} \phi \right) \]  

(1)

then can solve equations of motion exactly:

\[ a \sim t^p; \] if \( p > 1 \), U. accelerates, but not exponentially

Designer Potentials

can customize \( V \) to give desired \( a(t) \), e.g.,

\[ a \sim \exp(At^f), \] 0 < \( f \) < 1 if

\[ V(\phi) \sim \left( \frac{\phi}{m_{pl}} \right)^{-\beta} \left[ 1 - \frac{\beta^2}{6} \left( \frac{m^2_{pl}}{\phi^2} \right) \right] \]  

(2)
How about the Higgs?

from electroweak unification, we “know” of one scalar → Higgs field $H^0$, $M_H \gtrsim 100$ GeV?
same symbol as Hubble, right kind of field → is it $\phi$? i.e., what about inflation at the electroweak scale? not a bad idea—possibly correct!—but nontrivial at best problem not with inflation, but its place in the cosmic dance
Inflation, Inhomogeneities, and Quantum Mechanics

Thus far: classical treatment of inflaton field
(except for inflaton decays during reheating)
• $\phi$ described by classical equations of motion
• taken to hold for arbitrarily small $\phi$

In this picture:
when exit inflation, universe essentially
▷ perfectly flat, and
▷ perfectly smooth—i.e., density spatially uniform
regardless of initial conditions (as long as they allowed inflation)
Q: why?
Classical Inflation and Smoothness

expect initial spatial inhomogeneities in $\phi(\vec{x})$
but evolves classically as

$$\ddot{\phi} - \nabla^2 \phi + 3H\dot{\phi} - V' = 0$$

(3)

where

$$\nabla^2 = \sum \frac{\partial^2}{\partial x^2_{\text{phys}}} = \frac{1}{a^2} \sum \frac{\partial^2}{\partial x^2_{\text{com}}}$$

(4)

inhomogeneities $\delta\phi(\vec{x})$ measured by nonzero gradients
but since $\nabla^2 \propto 1/a^2 \rightarrow 0$ exponentially, classically: $\delta\phi(\vec{x}) \rightarrow 0$

$\Rightarrow$ after inflation $\phi$ and $\rho = V(\phi)$ exponentially smooth in space

good news: solved flatness, smoothness problems

bad news: we have done too much! too smooth!

can’t form structures if density perfectly uniform
Quantum Mechanics to the Rescue

but quantum mechanics exists and is mandatory
classical $\phi$ field $\rightarrow$ quantized as inflatons
think $\vec{E}, \vec{B}$ vs photons

inflaton field **must** contain quantum fluctuations
before, during inflation

What we want: **statistical** properties of fluctuations
- typical magnitude of fluctuations $\delta \phi$
- how $\delta \phi$ depends on lengthscales
- corresponding fluctuations in $\rho_\phi$
- correlations at different length scales