Astro 596/496 PC
Lecture 31
April 7, 2010

Announcements:

- PS5 due in class **Monday**
  BDF special office hours: Monday 10:30 – 11:30
- No class meeting Friday! woo hoo!

Guest Cosmologist Today: Prof. Paul Ricker

Last time: finished inflation
early U quantum fluctuations $\rightarrow$ cosmic density perturbations
"The Universe is the ultimate free lunch."
– Alan Guth

The rest of the course:
formation of cosmic structure

The Inhomogeneous Universe

Origin and Evolution of Cosmic Structure

The Large-Scale Structure of the Universe
Theoretical and Observational Landscape

On large scales, cosmo principle an excellent approximation
On small scales, fails miserably
Cosmology should explain both: now open our eyes to structure

Theory *Goals? tools? complications?*
Which scales in space, time "easy" to describe? which difficult?

Observations
Goals? observables? complications?
Which scales in space, time "easy" to measure? which difficult?

Arenas for theory–observation comparison
Which well-matched (i.e., clear results from both)?
Which poorly-matched (i.e., one or both ambiguous/difficult)?
What constitutes success? When are we done?

Large-Scale Structure: The Good, the Bad, and the Ugly

Structure Formation Theory

Goal: describe how small density fluctuation "seeds"
grow to form structure today

Tools: baryon-DM-radiation-DE particle & fluid dynamics
in expanding FLRW background
analytic–linearized perturb theory, idealized nonlinear models
numerical–full nonlinear evolution, feedback effects

Complications: nonlinear processes
(virialization, shocks, star feedback)

Degree of Difficulty:

large scales easiest–smoothest, linear perturb theory accurate

smallest scales hardest–very nonlinear

Structure Formation Observations

Goal: measure growth of structures over cosmic history

Tools: CMB anisotropy

surveys (optical, X-ray, IR, radio, γ-ray...): galaxies, quasars,
QSO absorption systems, lensing

Complications: need for statistical completeness
vs sensitivity, resolution

large scales easy in some ways: CMB very clean
galaxy, quasar statistics best over largest volumes

...but difficult in others: sensitivity, resolution lowest
few independent samples of structure at largest scales
"cosmic variance" (e.g., see many 10 Mpc regions,
only one at 4 Gpc)
reshifting, absorption present challenges
only a few epochs accessible

small scales easy in some ways: can probe locally
sample many independent regions
accessible at different epochs

...but difficult in others: hard to measure at large z

Comparing Theory and Observation

Strong Tests

well-matched at large scales:
linear theory accurate, observations (esp CMB) clean

Mismatches

Theory naturally describes density evolution
dominated by dark matter–invisible!
Observations naturally look at light
easiest to look at most nonlinear, baryonic systems
Problem: *mass* vs *light* disconnect
"bias" – rarest=largest structures easiest to see
and baryons collisional, dissipative
$\rightarrow$ more spatially concentrated that DM (think halos!)
Also: most light from stars–but theory of star form
incomplete and uncertain
$\Rightarrow$ *this is the frontier!*

Quantifying Large-Scale Structure

Observed galaxy distribution random

▷ location, form of individually galaxies unpredictable
  but clearly correlations, characteristic scales

▷ reflects randomness of initial conditions

▷ demands a fundamentally statistical treatment

Statistical description of cosmic density fields

consider, e.g., mass density $\rho(t, \vec{x})$

not only *random*, but also *continuous*

yet most observations are of *discrete* objects
  galaxies, clusters, etc.

 how to address this?

Attempt I: Fluctuations of Counts in Cells

fix a lengthscale $L \rightarrow$ volume $V = L^3$
divide patch of U. into cells of this size

then can define avg density $\langle \rho_i \rangle$ in each box i
  or more observationally: galaxy count N_i in box
  then look at statistical properties of N_i distribution

assume: different boxes $\langle \rho_i \rangle$, $\langle \rho_k \rangle$ initially indep
quickly independence lost *Q: why?*

but want a characterization in which different elements
  ("realizations") are independent

 Q: how to do this?

Problem: neighboring cells affect each other
e.g., overdensities drain underdensities next door
$\longrightarrow$ evolution immediately couples cells

Attempt II: Fourier Analysis

Can decompose $\rho(t, \vec{x})$ into plane waves*
in linear theory: different k evolve independently
i.e., small perturbations do not interact
$\longrightarrow$ adopt Fourier analysis

*Experts note here and throughout:
plane-wave expansion implicitly assumes background FRW space
is *flat*, i.e., Euclidean, uncurved, $\kappa = 0$
if global curvature $\kappa \neq 0$ exists: need generalization for curved space
key idea: appropriate modes are eigenfunctions of the Laplacian operator

Quantifying Density Fluctuations

Given $\rho(t, \vec{x})$, define

mean (average) density $\langle \rho \rangle = \langle \rho(t, \vec{x}) \rangle = \rho_{\mathrm{FRW}}(t)$

(suppress t hereafter)

density fluctuation $\delta\rho(\vec{x}) = \rho(\vec{x}) - \langle \rho \rangle$

density contrast

$$\delta(\vec{x}) = \frac{\delta\rho}{\rho} = \frac{\rho(\vec{x}) - \langle \rho \rangle}{\langle \rho \rangle} \tag{1}$$

where $\delta \neq \delta_{\mathrm{Dirac}}$!

Q: possible range of δ values?

Q: what is $\langle \delta \rangle$?

Q: how does cosmic expansion affect δ?

Spectrum of Density Fluctuations

In (large) volume V write $\delta(\vec{x})$ as Fourier series

$$\delta(\vec{x}) = \sum_{\vec{k}} \delta_{\vec{k}} \; e^{-i\vec{k}\cdot\vec{x}} \rightarrow \frac{V}{(2\pi)^3} \int \delta_{\vec{k}} \; e^{-i\vec{k}\cdot\vec{x}} \; d^3\vec{k} \tag{2}$$

(last expression is continuum limit as $V \rightarrow \infty$)
where Fourier coefficients are

$$\delta_{\vec{k}} = \frac{1}{V} \int \delta(\vec{x}) e^{i\vec{k}\cdot\vec{x}} \; d^3\vec{x} \tag{3}$$

reality: $\delta(\vec{x})^* = \delta(\vec{x}) \rightarrow \delta^*_{\vec{k}} = \delta_{-\vec{k}}$

Beware!
conventions differ on factors of V, sign of exponential
$\rightarrow$ affects dimensions of δ_k

Fourier mode described by amplitude $|\delta_k|$
and **comoving wavenumbers** $k \equiv k_{\text{comov}} = 2\pi/\lambda_{\text{comov}}$
and $\vec{x}$ is comoving as well
physical values are $d\vec{x}_{\text{phys}} = a(t)d\vec{x}$, $\vec{k}_{\text{phys}} = \vec{k}/a(t)$

Q: what is $\delta_{\vec{k}=0}$*?*
Q: what is connection between $\delta_{\vec{k}}$, $\delta_{\vec{k}'}$ *if* $|\vec{k}| = |\vec{k}'| = k$*?*

Q: how compute a typical value of $\delta\rho/\rho$*?*
what is it for scale k*?*

Fun Fourier Facts

$$\delta_{\vec{k}=0} = \int d^3\vec{x}\ \delta(\vec{x}) = \langle \delta \rangle = 0 \qquad (4)$$

by definition!

but deeper reason: small $k \leftrightarrow$ large λ

$k \to 0$ is $\lambda \to \infty$ = whole universe

on largest scales, U better be homogeneous!

so $\delta_{\text{large}\,k} \to 0$

For $|\vec{k}| = |\vec{k}'| = k$, i.e., same mag, different direction

must find same amplitude fluctuations

...else have a preferred direction*

so cosmo principle $\to$ $\delta_{\vec{k}} = \delta_k$

i.e., wavelength is all that counts k magnitude

* In fact, would $\vec{k}$ anisotropy would manifest not as preferred direction in structure distribution in real space, but rather as preferred *orientation* of structures! (thanks to Z. Lukic for pointing this out)

The Power Spectrum

Want a measure of "typical" fluctuation size"

$\langle \delta\rho/\rho \rangle = \langle \delta \rangle = 0$ by definition, but $\langle (\delta\rho/\rho)^2 \rangle = \langle \delta^2 \rangle \neq 0$

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \int d^3\vec{x}\ \delta(\vec{x})^2 \tag{5}$$

$$= \frac{V^2}{(2\pi)^6} \int d^3\vec{x}\ d^3\vec{k}\ d^3\vec{q}\ \ \delta_{\vec{k}} \delta_{\vec{q}}\ e^{-i(\vec{k}+\vec{q})\cdot\vec{x}} \tag{6}$$

$$= \frac{V}{(2\pi)^3} \int d^3\vec{k}\ d^3\vec{q}\ \ \delta_{\vec{k}} \delta_{\vec{q}}\ \delta_{\mathrm{Dirac}}(\vec{k}+\vec{q}) \tag{7}$$

$$= \frac{V}{(2\pi)^3} \int d^3\vec{k}\ \ \delta_{\vec{k}} \delta_{-\vec{k}} \tag{8}$$

$$= \frac{V}{(2\pi)^3} \int d^3\vec{k}\ |\delta_{\vec{k}}|^2 = \frac{V}{(2\pi)^3} \int d^3\vec{k}\ P(k) \tag{9}$$

where $P(k) = |\delta_k|^2$ is the power spectrum

Rewrite in terms of fluctuations per log interval in wavenumber dk/k:

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \frac{V}{(2\pi)^3}\int d^3\vec{k}\; P(k) = \frac{4\pi V}{(2\pi)^3}\int dk\; k^2\; P(k) \tag{10}$$

$$= \int \frac{4\pi k^3 P(k) V}{(2\pi)^3}\frac{dk}{k} \tag{11}$$

$$\equiv \int \frac{dk}{k}\left(\frac{\delta\rho}{\rho}\right)_k^2 \tag{12}$$

where the variance over $dk/k = d\ln k \sim 1$ is

$$\left(\frac{\delta\rho}{\rho}\right)_k^2 \approx \Delta^2(k) \equiv 4\pi k^3 P(k)\frac{V}{(2\pi)^3} \tag{13}$$

dimensionless measure of fluctuations on scale k

power spectrum $P(k) \Leftrightarrow \Delta^2(k)$
central object in structure formation
`www: Observed power spectrum` *Q: what stands out?*

Observed Power Spectrum

Gross features of observed $P(k)$:

⋆ fairly simple shape: roughly, broken power law
roughly, $P(k) \sim k^1$ at low k,
then steepening negative slope, approaching k^{-3}
we will want to understand why

⋆ break at peak: $k_{\rm peak} \sim 0.02\ h^{-1}{\rm Mpc}^{-1}$
$\rightarrow$ characteristic scale $\lambda_{\rm peak} = 2\pi/k_{\rm peak} \sim 300$ Mpc
we will want to understand what sets this scale!

Features of $\Delta(k) = \sqrt{\Delta^2(k)}$:

⋆ $\Delta \gtrsim 1$ at $k \gtrsim 0.03$ Mpc $\rightarrow \lambda \lesssim 20$ Mpc
characteristic scale of nonlinearities today

⋆ $\Delta \ll 1$ at small k: U $\rightarrow$ homogeneous on large scales
cosmo principle vindicated! Good guess, Al!