Astro 596/496 PC
Lecture 35
April 19, 2010

Announcements:

- PF6 was due at noon–congrats! *Q: adiabatic? gaussian?*
- PS7 out, due in class *Wednesday* next week
- **ICES** available online – please do it!

Last time: Newtonian perturbation theory in expanding U
again find characteristic Jeans length $\lambda_J \sim c_s/\sqrt{G\rho}$
but in expanding case: $\lambda_J \simeq a c_s t$ comoving "sound horizon"
when matter-dom, density contrast δ of unstable modes grow as

$$\delta_k(t) \equiv D(t)\delta_k(t_i) = \left(\frac{t}{t_i}\right)^{2/3} \delta_k(t_i) = \frac{a(t)}{a(t_i)}\delta_k(t_i) \quad (1)$$

$\rightarrow$ RMS fluctuation at scale k or in sphere R
grow as $\sigma(R) \sim \Delta(k = 1/R) \propto D(t) \propto a(t)$

Q: at which δ should perturbation analysis break down?

Applications to CMB: Naïve Inferences

before decoupling: pressure dominated by photons
$\rightarrow$ expect oscillations – and see them!
after decoupling: growing mode

CMB anisotropies are a snapshot
of perturbations at last scattering
can quantify level: $(\delta T/T)_{\rm ls} \sim 10^{-5}$ at $z_{\rm ls} \sim 1100$

But matter has $\rho \propto a^{-3} \propto T^3$, so $\delta\rho/\rho = 3\delta T/T$
$\rightarrow \delta_{\rm init} = \delta(z = 1100) \sim 3 \times 10^{-5}$ at last scattering
So today, expect fluctuations of size

$$\delta_0 = \frac{D_0}{D_{\rm ls}}\delta_{\rm ls} = \frac{a_0}{a_{\rm ls}}\delta_{\rm ls} = (1 + z_{\rm ls})\delta_{\rm ls} \sim 0.03 \ll 1 \qquad (2)$$

Should still be very small–no nonlinear structures, such as us!
Q: obviously wrong–egregiously naïve! What's the flaw? What's the fix?

Cosmic Diversity: Evolution of Multiple Components

Thus far: implicitly assumed a baryons-only universe: not ours!

Cosmic "fluid" contains many different species
with different densities, interactions
baryons, photons, neutrinos, dark matter, dark energy

Each component i has its own equations of motion, e.g.:

$$\ddot{\delta}_i + 2H\dot{\delta}_i = -\frac{c_{s,i}^2 k^2}{a^2}\delta_i + 4\pi G\rho_0 \sum_j \Omega_j \delta_j \quad (3)$$

species interact via pressure, gravity: evolution eqs *coupled*

- ▷ gravity from dominant Ω drives the other components
- ω ▷ each species' (pressure) response depends on
  microphysics of its interactions, encoded in sound speed $c_{s,i}$

Matter Instability in the Radiation Era

(dark) matter perturbation δ_m during radiation domination

- pick subhorizon scale: growth possible
- focus on $k < k_J$: Jeans unstable (can ignore pressure) and high-k modes just oscillate anyway
- treat radiation perturbations as *smooth*: $\delta_{\rm rad} \approx 0$
  $P_r = \rho_r/3$: huge, fast $c_s \sim c$
  any perturbations will be oscillatory anyway
- dark matter: weak interactions $\rightarrow$ pressureless $\rightarrow$ $c_s = 0$!

Evolution simple – to rough approximation, for these k:

$$\ddot{\delta}_m + 2\frac{\dot{a}}{a}\dot{\delta}_m \stackrel{\rm rad-dom}{=} \ddot{\delta}_m + \frac{1}{t}\dot{\delta}_m \approx 0 \tag{4}$$

Simple solutions: growing mode plus decaying mode

$$\delta_m(t) = D(t)\delta_m(t_i) = \left(D_1 \log t + \frac{D_2}{t}\right)\delta_m(t_i) \tag{5}$$

Q: implications? what about baryons?

Found $D(t) \sim D_1 \log t$: "growing" mode hardly grows!

$\star$ dark matter perturbations *frozen* during rad dom
dark matter growth quenched by
$\rightarrow$ non-growth of radiation perturbations
$\rightarrow$ extra expansion due to radiation

$\star$ *dark matter perturbation growth stalled*
until end of radiation era: **matter-radiation equality**
i.e., $\rho_{\rm matter} = \rho_{\rm radiation}$ when $z_{\rm eq} \sim 3 \times 10^4$
Q: is before or after BBN? recomb?
$\Rightarrow$ this marks onset of structure formation

Q: how does this update our naive CMB calculation?
Hint: then, correct reasoning for $\delta = \delta_b$ only

baryons tightly coupled to photons till recombination
$\longrightarrow$ so dark matter perturbations begin growth earlier

And so: DM has grown more! update earlier estimate
and focus on dark matter

$$\delta_{m,0} = \frac{D_{\rm ls}}{D_{\rm eq}} \delta_{b,0} \sim \frac{1+z_{\rm eq}}{1+z_{\rm ls}} \delta_b \sim 30 \times 0.05 \sim 1 \tag{6}$$

DM can grow to nonlinearity today!

$\star$ existence of collapsed cosmic structures
requires collisionless dark matter!
$\star$ independent argument for large amounts of
weakly interacting matter throughout universe!

CMB Anisotropies

CMB Anisotropies

Between matter-radiation equality and recombination:

- dark matter perturbations grow
  form deepening potential wells
- baryons, electrons tightly coupled to photons (plasma)
  undergo acoustic oscillations

Q: what is the largest scale which can oscillate?
Q: for each mode k, what sets oscillation frequency?
Q: at fixed t, which scales have oscillated the most? the least?
Q: how is this written on the CMB?

Pre-Recombination: Acoustic Oscillations

Baryons in DM-dominated background

$$\ddot{\delta}_b + 2\frac{\dot{a}}{a}\dot{\delta}_b \simeq 4\pi G\rho\delta_{dm} - \frac{k^2c_s^2}{a^2}\delta_b \sim \frac{\delta_{dm}}{t^2} - \frac{k^2c_s^2}{a^2}\delta_b \qquad (7)$$

key comparison: mode scale $\lambda \sim k^{-1}$
vs **comoving sound horizon** $c_s t/a = d_{s,com}$

for large scales $kc_s t/a \ll 1$: baryons follow DM
for small scales $kc_s t/a \gg 1$: baryons oscillate, as

$$\delta_b \sim \frac{1}{\sqrt{ac_s k}} e^{i\int kc_s d\eta} \qquad (8)$$

(PS 6) where $d\eta = dt/a$ is conformal time

for constant c_s, $\delta_b \sim e^{ikc_s\eta}$ sinusoidal at each k
phase counts number of cycles $N = kc_s\eta/2\pi = c_s\eta/\lambda$
oscillation frequency: $\omega \sim kc_s \sim c_s/\lambda \propto 1/\lambda$
small $\lambda \to$ rapid oscillations

Recombination: Snapshot Taken

At recombination, free e^- abundance drops
baryons quickly decouple from photons
  huge drop in pressure $\to$ $c_s \to 0$
  begin to collapse onto DM potentials
photons travel freely (typically) afterwards
  fluctuation pattern at recomb is "frozen in"
  δ vs scale records different # of cycles at recomb

$$P(k) = \|\delta_k\|^2 \sim \frac{\sin(2kc_s\eta_{\rm rec})}{2kc_s\eta_{\rm rec}} P_{\rm init}(k) \tag{9}$$

written onto temperature pattern ("say cheese!")
`www: simulation of overdensity evolution thru recombination`

Recomb fast $\to$ CMB is image of last scattering surface

Q: on small scales, is an overdensity a hot spot or cold spot? why?

Spots Cold and Hot: Small Scales

Define temperature fluctuation $\Theta = \delta T/T$

On Small Scales: Adiabatic
standing waves lead to fluctuations in $\rho_b \sim T^3$, so

$$\Theta \equiv \frac{\delta T}{T} = \frac{1}{3}\left(\frac{\delta\rho}{\rho}\right)_b \tag{10}$$

$\Rightarrow$ extrema in density $\rightarrow$ extrema in $\Theta \propto \delta_\gamma$

$\star$ photon T contrast reflects T distribution at source
$\rightarrow$ hot is hot and cold is cold

- but both high *and* low density give *large* $(\delta T/T)^2$!
  photon climb out of potential doesn't change $\delta T/T$ much
  $\rightarrow$ CMB hot spots are high density, cold are low

Q: what about on large scales?

Very Large Scales: Sachs-Wolfe

beyond horizon: no oscillations, main effects gravitational (GR):

- gravitational redshift: photon climbs out of potential $\delta\Phi < 0$
  redshift $\delta\lambda/\lambda = \Phi_0 - \Phi_{\rm ls} = -\delta\Phi$
  and since $T \sim 1/\lambda$, $(\delta T/T)_{\rm redshift} = \delta\Phi$: photons cooled!
- time dilation: takes longer to climb out of overdensity
  looking at younger, hotter universe
  $\delta t/t = \delta\Phi$, and since $a \sim t^{2/3}$ and $T \sim 1/a$
  then $T \sim t^{-2/3}$, and $(\delta T/T)_{\rm dilation} = -2/3\ \delta\Phi$

net effect: Sachs - Wolfe

$$\left(\frac{\delta T}{T}\right)_{SW} = \left(\frac{\delta T}{T}\right)_{\rm redshift} + \left(\frac{\delta T}{T}\right)_{\rm dilation} = \frac{1}{3}\delta\Phi \qquad (11)$$

★ overdensities are cold spots, underdensities hot

Note: this regime is what tests inflation

Q: what predicted?