Astro 596/496 PC

Lecture 36

April 21, 2010

Announcements:

- PS6 out, due in class *Wednesday* next week
- **ICES** available online – please do it!

Last time: began CMB anisotropies

*Q: dark matter perturbations before matter-rad eq? after?*

*Q: baryonic perturbations before recomb? after?*

*Q: physical origin of CMB* $\Delta T$ *on scales* $\gg d_{\rm hor,recomb}$*?*

**dark matter**: pressureless

$\longrightarrow$ all $k$ modes unstable if inside Hubble length

but: perturbations grow verry slooooowly during radiation era

$\longrightarrow$ dark matter structures begin formation at matter-radiation equality

**baryons**: until recomb, tightly coupled to photons

$\longrightarrow$ feel huge photon pressure $P_\gamma \propto T^4$

$\longrightarrow$ sound speed $c_s \sim c/\sqrt{3}$ huge!

so all sub-horizon modes stable! just oscillate

$\longrightarrow$ relativistic pressure-mediated (i.e., acoustic) standing waves!

oscillation frequency $\nu = c_s/\lambda$:

small-scale modes oscillate many times

largest-scale modes $\lambda = c_s \eta_{\text{hor}}$ oscillates only once

# Inflation and Sachs-Wolfe

Inflation: quantum fluctuations $\longrightarrow$ density fluctuations

- adiabatic (all species)
- Gaussian
- scale invariant–what does this mean?

In detail: inflation predicts that the dimensionless fluctuations in the *gravitational potential* $\leftrightarrow$ *local curvature* are independent of scale
$\longrightarrow$ this was what we really calculated in Inflation discussion

inflationary scale-invariance is for grav potential:
i.e., Fourier mode contribution $\Delta_\Phi^2 \sim k^3 |\Phi_k|^2 \sim const$ indep of $k$
$\longrightarrow$ scale invariant: $|\Phi_k|^2 \sim k^{-3}$

need to connect gravitational potential/curvature perturbations to density perturbations

But in Newtonian regime, know how to do this:
Poisson relates potential and density:
$\nabla^2 \delta\Phi = 4\pi G \delta\rho \to \Phi_k \sim \delta_k / k^2$
and so $P(k) = |\delta_k|^2 \sim k^4 |\Phi_k|^2$

thus scale invariant gravitational potential gives power spectrum:

$$P(k) \sim k^4 |\Phi_k|^2 \sim k \quad (1)$$

i.e., scale invariance: $P(k) \sim k^n$, $n_{\rm scale-inv} = 1$

# Angular vs Linear Scales

So far: decomposed fluctuations in (3-D) $\vec{k}$-space
but observe on sky: 2-D angular distribution

Transformation: projection of plane waves
at fixed $k$: see intersection of wave with last scattering shell
**www: Wayne Hu animation**

appears on a range of angular scales
but typical angular size is $\theta \sim \lambda / d_{\rm rec,com} \sim (k d_{\rm rec,com})^{-1}$

large angles $\rightarrow$ large $\lambda$ (check!)
for large angular scales $\theta > \theta_{\rm hor,diam} \sim 1^\circ$, superhorizon perturbations not affected by oscillation
for small angular scales, see standing waves

- peaks at extrema, harmonics of sound horizon $k$ are in ratios 1:2:3:...

# The CMB Observed

- Observe 2-D sky distribution of $\frac{\Delta T}{T}(\hat{n}) \equiv \Theta(\hat{n})$ in direction $\hat{n}$
- Decompose into spherical harmonics

$$\Theta(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \qquad (2)$$

with $Y_{\mathrm{lm}}$ spherical harmonics *Q: why not* $\ell = 0, 1$*?*
*Q: angular size vs* $\ell$*?* $\lambda$ *vs* $\ell$*?*

Form angular correlation function *Q: what is this physically?*

$$\langle \Theta(\hat{n}_1)\Theta(\hat{n}_2) \rangle = \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) \langle |a_{\ell m}|^2 \rangle P_\ell(\hat{n}_1 \cdot \hat{n}_1) \qquad (3)$$

$$= \frac{1}{4\pi} \sum_{\ell=2}^{\infty} (2\ell+1) C_\ell P_\ell(\cos\vartheta) \qquad (4)$$


where $\cos\vartheta = \hat{n}_1 \cdot \hat{n}_1$
*Q: averaged over the* $m$ *azimuthal modes–why?*

all interesting anisotropy information encoded in

$$C_\ell = \langle |a_{\ell m}|^2 \rangle \quad (5)$$

isotropy $\rightarrow$ azimuthal dependence averages to zero

Note: analog of $\Delta^2$ (variance per log scale) is
$\mathcal{T}^2(\ell) = \ell(\ell+1)C_\ell$: usually what is plotted

Since $P_\ell(\cos\theta) \sim (\cos\theta)^\ell \sim \cos(\ell\theta)$
at fixed $\ell$, angular size $\theta \sim 2\pi/\ell = 180^\circ/\ell$
e.g., $\ell = 2$ quadrupole $\rightarrow \theta \sim 90^\circ$
and horizon size $\theta \sim 1^\circ$ is at $\ell \sim 200$

and since $\theta \sim \lambda/d_{\rm rec} \sim 1/dk$:
$\neg$ multipoles scale as $\ell \sim 1/\theta \sim k \sim 1/\lambda$
low $\ell \rightarrow$ big angular, physical scales $\rightarrow$ small $k$

# CMB Anisotropy Observations: Strategy

- achieve high sensitivity, remove systematics
  make a "difference experiment"
  i.e., measure $\delta T$ directly, don't subtract
- observe as much of the sky as possible (or as needed!)
  balloons/ground: limited coverage
  satellites (COBE, WMAP, Planck): all-sky
- remove Galactic contamination: "mask" plane
- recover $\Theta$ for observed region

- decompose into spherical harmonics $Y_{\ell m}$
- construct power spectrum $\ell(\ell+1)C_\ell$
- report results
- collect thousands of citations, prominent Prizes

# CMB Temperature Anisotropies: Results

## COBE (1993)

- first detection of $\delta T/T \neq 0$
- receiver horn angular opening $\sim 8^\circ$
  $\rightarrow$ only sensitive to large angular scales
  i.e., superhorizon size
- found $(\delta T/T)_{\rm rms} \sim 10^{-5}$
- power $\ell(\ell+1)C_\ell$ flat $\rightarrow$ implies $P(k) \sim k$!
  $n = 1$ spectrum: scale invariant!

## Interregnum (late 90's, early 00's)

- ground-based, balloons confirmed anisotropy
- acoustic peaks discovered
  strong indication of first peak

## WMAP (2003-)

- first all-sky survey of small angular scales
- $n = 1$ confirmed, indication of small tilt $n - 1 \neq 0$?
  consistent with inflation! and non-trivially so!
- acoustic peaks mapped: good measurement of 1st, 2nd
  detection of third
- first peak: $\ell \sim 200$ horizon at recomb!
- power dropoff seen at large $\ell$
  $\longrightarrow$ nonzero thickness of last scattering
  due to photon diffusion, non-instantaneous decoupling