Astro 596/496 PC
Lecture 38
April 26, 2010

Announcements:

- PS6 out, due in class next time
  Updated version posted April 22!
  office hours 3–4pm tomorrow
- **ICES**! please don't skip written comments
- *No class this Friday* – woo hoo!

Last time: finished CMB

- goldmine of cosmological parameters
- inventory of density perturbation at $z_{\rm rec}$

From here on out:

- how do those tiny perturbations grow
  to make present-day universe

Structure and Horizons

Particle horizons set range for causal physics
including growth of structure
so two requirements for perturbation growth
⋆ perturbation must be inside "horizon," i.e., $\lambda \leq d_H = H^{-1}$
⋆ U. must be matter-dominated: $z < z_{\rm eq}$

Choreography:
inflation lays down perturbations at z enormous
all frozen in until matter domination , then
• on scales already inside Hubble length at $z_{\rm eq}$
 δ_m growth stalled until matter-domination
• on superhorizon scales at $z_{\rm eq}$, δ_m growth begins immiedately after $d_H > \lambda$

Today: observe scales in both regimes
Q: What should be the difference?
 What characteristic scale divides these regimes?

Key scale in cosmic structure distribution:
comoving Hubble length at matter-rad equality

$$d_{\mathrm{H,com}}(z_{\mathrm{eq}}) = \frac{1}{a_{\mathrm{eq}} H_{\mathrm{eq}}} = \frac{a_{\mathrm{eq}}^{1/2} d_{\mathrm{H,0}}}{\sqrt{2\Omega_{\mathrm{m}}}} \sim 60\ h^{-1}\ \mathrm{Mpc} \tag{1}$$

corresponding to $k_{\mathrm{eq}} = 1/d_{\mathrm{H,com}} = 0.02\ h\ \mathrm{Mpc}^{-1}$
Q: sound familiar?

How do does perturbation growth differ
on scales sub/super horizon at at z_{eq}?

in linear regime ($\delta \ll 1$)
linear growth factor: $D(t) = \delta_k(t)/\delta_k(t_{\mathrm{init}})$; k-indepedent

- large scales have linear growth factor D_0/D_{enter}
- small scales have grown more in absolute terms
  but less than linear extrap from horizon entry
  only grown by $D_0/D_{\mathrm{eq}} < D_0/D_{\mathrm{enter}}$

Dividing scale at equality horizon:

$\lambda_{\rm eq} = d_{\rm com,hor}(z_{\rm eq}) \sim \eta_{\rm eq}$ and corresponding $k_{\rm eq}$

if smaller scale, horizon entry at pre-eq redshift $z_{\rm enter}$

such that $d_{\rm hor,com}(z_{\rm enter}) = \eta_{\rm enter} = \lambda$

$\rightarrow$ small scales have growth "stunted" by factor

$$\frac{D_{\rm small}}{D_{\rm large}} = \frac{a_{\rm enter}}{a_{\rm eq}} = \left(\frac{\eta_{\rm enter}}{\eta_{\rm eq}}\right)^2 = \left(\frac{\lambda}{\lambda_{\rm eq}}\right)^2 = \left(\frac{k_{\rm eq}}{k}\right)^2 < 1 \tag{2}$$

where we used $D \propto a \propto \eta^2$ in matter-dom

Different scales have not grown by same amount!

$\rightarrow$ to recover initial power spectrum need to account for this

Transfer Function

Theory (initial power spectrum) connected with
Observation (power spectrum processed by growth)
via transfer function–measures "stunting correction"

$$T_k(z) = \frac{\text{present density spectrum}}{\text{extrapolated initial spectrum}} = \frac{\delta_{k,\text{today}}}{D(z)\delta_k(z)} \tag{3}$$

$$\rightarrow \begin{cases} 1 & k < k_{\text{eq}} \\ (k_{\text{eq}}/k)^2 & k > k_{\text{eq}} \end{cases} \tag{4}$$

Note: since $\delta_{k,\text{init}} \sim \delta_{k,0}/T_k$
power spectrum goes as $P_{k,\text{init}} \sim P_{k,0}/T_k^2$

Now apply to observations

Recovering the Initial Power Spectrum

Apply transfer function to invert observed spectrum

Observed power spectrum

- peak at ~ 30 Mpc $\simeq \lambda_{\rm eq}$ (check!)

- for $k < k_{\rm eq}$, $P_{\rm obs}(k) \sim k^1 = P_{\rm init}(k)$
  $\rightarrow$ scale invariant! (check!)

- for $k > k_{\rm eq}$, turnover in power spectrum (check!)
  quantitatively: $P_{\rm obs}(k) \rightarrow k^{-3}$
  so $P_{\rm init} \sim P_{\rm obs}/T^2 \sim k^4 P_{\rm obs} \sim k$
  also scale invariant (check!)

observed power spectrum consistent with
gravitational growth of scale-invariant spectrum!

Dark Matter–Cold and Hot

Perturbation *growth* & clustering depends on dark matter internal motions–i.e., "temperature" or *velocity dispersion*

key idea: velocity dispersion (spread) is like pressure

$\longrightarrow$ stability criterion is Jeans-like

Cold Dark Matter (CDM)

slow velocity dispersion–trapped by gravitational potentials

no lower (well, very small) limit to structure sizes

perturbation growth only limited by onset of matter dom

$\longrightarrow$ small, subhorizon objects form first, then larger

$\longrightarrow$ **hierarchical structure formation**: "bottom-up"

Hot Dark Matter (HDM)

high velocity dispersion–escape small potentials

small objects can't form–large must come first then fragment to form smaller: "top down"

Q: particle candidate for HDM?

Q: physical implications for HDM structure formation?

Q: how can this be tested?

Q: how does HDM alter the power spectrum (transfer function)?

Hot Dark Matter: Neutrino Cocktail

HDM classic candidate: massive ($m_\nu \sim 1$ eV) neutrinos
if light enough, relativistic before $z_{\rm eq}$
$\rightarrow$ "free streaming" motion out of high-density regions
$\rightarrow$ characteristic streaming scale: horizon size when $\nu \rightarrow$ nonrel

$$\lambda_{{\rm FS},\nu} \sim 40\ \Omega_m^{-1/2} \sqrt{1\ {\rm eV}/m_\nu}\ {\rm Mpc} \tag{5}$$

⋆ perturbations on scales $\lambda < \lambda_{\rm FS}$ suppressed
⋆ $\lambda_{{\rm FS},\nu}$ sensitive to absolute ν masses!

If HDM is dominant DM: expect *no* structure below $\lambda_{\rm FS}$
$\rightarrow$ a pure HDM universe already ruled out!

If "mixed dark matter," dominant CDM, with "sprinkle" of HDM
HDM reduces structure below $\lambda_{\rm FS}$
$\rightarrow$ $\lambda_{\rm FS}$ written onto power spectrum (transfer function)
$\rightarrow$ accurate measurements of, e.g., $P(k)$ sensitive to m_ν
cosmic structure can weigh neutrinos! (goal of DES, et al)

ΛCDM

"Standard" Cosmology today: ΛCDM ...namely:

- FLRW universe
- today dominated by cosmological constant $\Lambda \neq 0$
- with cold dark matter
  $\Rightarrow$ hierarchical, bottom-up structure formation
- ...and usually also inflation: scale invariant, gaussian, adiabatic

This is the "standard" model but not the only one
Q: arguments in favor?
Q: arguments for other possibilties?
Q: which pieces most solid? which shakiest?

At minimum: ΛCDM is *fiducial* / *benchmark* model
standard of comparison for alternatives

...and so we will adopt ΛCDM the rest of the way

Nonlinear Reality

So far: much success in understanding structures in the linear regime $\delta \ll 1$

But the real universe is nonlinear!
What happens when perturbations become large?
$\Rightarrow$ both theory and observations become challenging!

Theory: nonlinear dynamics rich = interesting = hard
some ingenious analytical approximations, special cases
but serious calculations require numerical solution

Observation: collapsed objects can be easy to find
e.g., bright galaxies–but more to the picture than meets the eye

- can't see the DM halos (usually!); mass doesn't trace light
- how to define a halo? measure its mass?

Q: why would this be ambiguous?

Spherical Collapse

consider idealized initial conditions ("top hat"):
- spherical overdensity, uniform density
- embedded in flat, matter-dom universe
  (and so "compensated" by surrounding underdense shell)

spherical collapse model a cosmological workhorse
a nonlinear problem with analytic solution!

Given: initial density contrast $\delta_i \ll 1$ at some t_i
Want to calculate: density contrast $\delta(t)$
lucky break–Newton's "iron sphere"/Gauss' law/Birkhoff's:
in spherical matter distribution, interior ignorant of exterior
$\Rightarrow$ overdense region evolves exactly as closed universe!

PS6: solution is parametric (cycloid)

$$a(\theta) = \frac{a_{\text{max}}}{2}(1 - \cos\theta) \quad (6)$$

$$t(\theta) = \frac{t_{\text{max}}}{\pi}(\theta - \sin\theta) \quad (7)$$

$$(8)$$

- "development angle" $\theta \propto \eta$ conformal time!
- formally, collapse (to a point!) at $t_{\text{coll}} = 2t_{\text{max}}$

Q: describe overdensity evolution qualitatively?
Q: what really happens when $t \gtrsim t_{\text{coll}}$?

Spherical Collapse: Qualitative Lessons

Formal solution

$$a(\theta) = \frac{a_{\max}}{2}(1 - \cos\theta) \quad ; \quad t(\theta) = \frac{t_{\max}}{\pi}(\theta - \sin\theta) \tag{9}$$

- initially expand with Universe
- but extra gravity in overdensity slows expansion
- reach max expansion at $t_{\max}$, then begin collapse
  "turnaround" epoch
- in reality: after turnaround, infalling matter virializes
  marks birth of halo as collapsed object
- Note: Brooklyn is not expanding! Nor is SS, MW, LG
  Q: what is criterion not to expand?

Beyond the formal solution:

- halo still overdense $\rightarrow$ neighboring shells fall in
  $\rightarrow$ mass continues to grow by accretion!
- in real life: mergers too

Spherical Collapse: Quantitative Lessons

want overdensity: since $\rho \propto 1/a^3$

$$\delta(t) = \frac{\rho(t)}{\rho_{\rm bg}(t)} - 1 = \left(\frac{a_{\rm bg}}{a}\right)^3 - 1 \quad (10)$$

with $a_{\rm bg} \propto t^{2/3}$ the matter-dom background
$\rightarrow$ exact nonlinear solution (pre-virial)

For small t, to first order $a(t) \sim t^{2/3} = a_{\rm bg}(t)$:
background result; $\delta(t) = 0$
to second order: $a(t) = a_{\rm bg}(t)[1 - (12\pi t/t_{\rm coll})^{2/3}/20]$

$$\delta(t) \approx \frac{3}{20}\left(\frac{12\pi t}{t_{\rm coll}}\right)^{2/3} = \delta_{\rm lin}(t) \quad (11)$$

$\delta(t) \propto D(t) \propto t^{2/3} \propto a_{\rm bg}$ same as linear result!

Very useful result:

$$\delta_{\rm nonlin}(t) = \left(\frac{a_{\rm bg}}{a_{\rm nonlin}}\right)^3 - 1 \quad (12)$$

$$\delta_{\rm lin}(t) \approx \frac{3}{20}\left(\frac{12\pi t}{t_{\rm coll}}\right)^{2/3} \quad (13)$$

connects full nonlinear result with linear counterpart
$\rightarrow$ maps between the two

E.g., at turnaround
$\delta_{\rm nonlin} = (6\pi)^2/4^3 = 5.6$, but $\delta_{\rm lin} = 1.06$
at virialization (PS6):
$\delta_{\rm nonlin} \approx 180$, but $\delta_{\rm lin} = 1.69$
$\rightarrow$ defines a critical linear overdensity

Q: why useful?