Astro 596/496 PC
Lecture 4
Jan 25, 2008

Announcements:

- Preflight 1 due Friday, 12 noon

Last time:
Observational/Conceptual Foundations of Cosmology

⋆ Cosmological Principle
⋆ Observed Cosmic Kinematics: Hubble's Law
  `www: modern Hubble Diagram--HST 2001`
⋆ Implications of Cosmo Principle + Hubble Law

Today: Cosmodynamics I–Newtonian Cosmology

Critiques of Cosmic Egoism

`www: sketch of idealized Galaxy distribution, velocity field`
We are at the center of the universe?

Philosophically:
• not Copernican ("principle of mediocrity")

Physically:
• haven't included gravity!

Observationally:
• Milky Way, Local Group don't look special
not what expect from center of explosion
compare supernova $\rightarrow$ neutron star, black hole

...yet radial v pattern makes us look special...

The Magic of Hubble

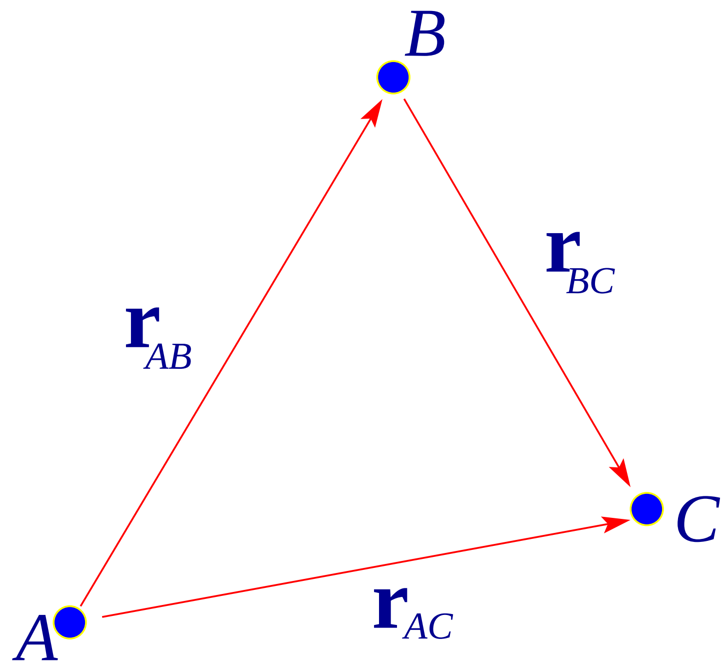
consider three arbitrary cosmic points:

$\vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB}$

Assume A sees Hubble's law:

- $\vec{v}_{AB} = H\vec{r}_{AB}$
- $\vec{v}_{AC} = H\vec{r}_{AC}$

Then ask: *what does B see? C?*



find velocities relative to B:

$\vec{v}_{BC} = \vec{v}_{AC} - \vec{v}_{AB} = H(\vec{r}_{AC} - \vec{r}_{AB}) = H\vec{r}_{BC}$

ω This is huge!
Q: why? What have we proven?

we have shown:

if A sees Hubble's law, then so do (arbitrary) B and C

thus: if *any* observer measures Hubble's law
then *all* observers will measure Hubble's law!

so: Hubble law implies

$\rightarrow$ *all* galaxies recede according to same law

$\rightarrow$ no need for center, space has no special points

Moreover: Hubble law is *only* motion
which preserves homogeneity and isotropy

i.e., *any* other motion breaks cosmo principle

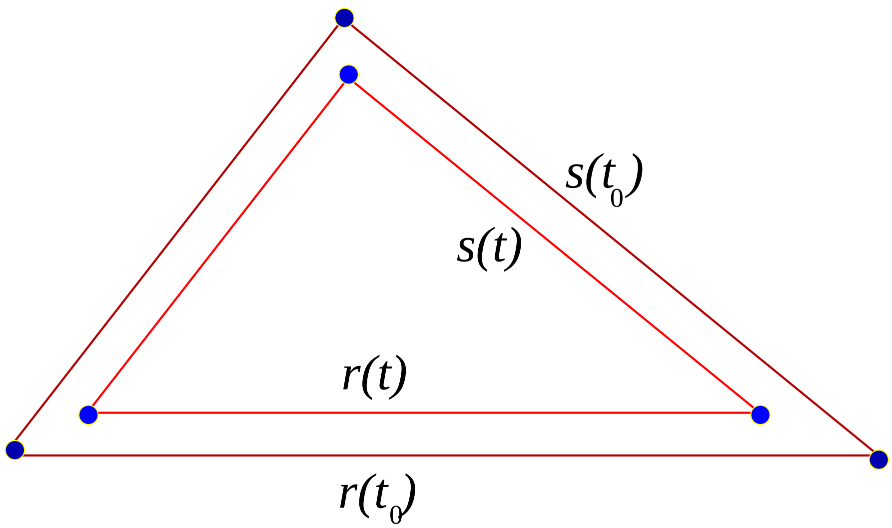
...but Hubble law is exactly what's observed!

Cosmo Principle Constrains Kinematics

consider arbitrary triangle defined by 3 observers at t_0
Hubble law $\longrightarrow$ observers in relative motion
$\longrightarrow$ at later time t, larger triangle

the claim:
later Δ always similar to original Δ
Q: what are similar triangles?
Q: why must similarity hold?

diagram: triangles, two sides $r(t_0)$, $r(t)$, $s(t_0)$, $s(t)$
Q: connections among r*'s and* s*'s?*



similar: triangle sides keep same ratios, so must have

$$\frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \tag{1}$$

but this rule holds for *any triangle*
so ratios can depend *only on time t*:

$$a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \tag{2}$$

Q: and so...?

We have shown:

Cosmo Principle demands *any length* $r(t)$ evolves as

$$r(t) = a(t)\ r_0 \tag{3}$$

where we are free to choose $\boxed{a(t_0) = 1}$ today, and
$r_0 = r(t_0)$ is present value ("comoving coordinate")

$a(t)$ must be universal **scale factor**
can depend only on time
but at any t: a has same value everywhere in space

This is huge!
Q: why? What have we proven? What is character of motion?

Cosmic Expansion

the meaning of Hubble Law: Take 2

2. Einstein interpretation:
will see: General Relativity + Cosmo Principle demand

Universe is expanding

all galaxies receding from all others
bold, strange idea!

Expansion: Einstein $\rightarrow$ Hubble

transparency demo: photocopy universe

for two arbitrary observers (e.g., "galaxies")
scale factor gives distances
$\vec{r}(t) = \vec{r}_0 a(t)$
so velocity is: note: "overdot" is time deriv $\dot{x} \equiv dx/dt$

$$\vec{v}(t) = \dot{\vec{r}} = \vec{r}_0 \dot{a} = \frac{\dot{a}}{a} a\vec{r}_0 \equiv H(t)\,\vec{r}(t) \tag{4}$$

$\Rightarrow$ Hubble law!
now interpret "Hubble parameter"
as **expansion rate** $H(t) \equiv \dot{a}/a$

Cosmic Scale Factor Revisited

for two "particles" (possibly Galaxies!)
distance evolves according to

$$\vec{\ell}(t) = \underbrace{a(t)}_{\substack{\text{scale factor} \\ \textit{time varying}}} \underbrace{\vec{\ell}_0}_{\substack{\text{present distance} \\ \textit{fixed once and for all}}} \quad (5)$$

and thus

$$\vec{v} = H\vec{\ell} \quad (6)$$

with $H = \dot{a}/a$

Q: implications–present, past, future values for a?

present: at t_0, $a(t) = 1$
expanding, so

past: $a(t) < 1$
future: $a(t) > 1$

e.g., at some time in past $a = 1/2$
"galaxies twice as close"

Q: how do cosmic volumes depend on a*?*
e.g., *Q: when* $a = 1/2$*?*

Expansion and Areas, Volumes

consider a cube, galaxies at corners

present side length L_0

diagram: cube, label L_0, expansion arrows

$\longrightarrow$ cube is "comoving" w/ expansion

draw arrows

volume $V \propto a^3$

$\longrightarrow V = L^3 = L_0^3 a^3 = V_0 a^3$

side area $A = A_0 a^2$

`www: raisin cake analogy`

`www: balloon analogy`

Q: what is tricky, imperfect about each analogy?

Cosmodynamics II

$a(t)$ gives expansion history of the Universe
which in turn tells how densities, temperatures change
$\longrightarrow$ given $a(t)$ can recover all of cosmic history!

but...

How do we know $a(t)$?
What controls how scale factor $a(t)$ grow with time?
Q: what force(s) are at work microscopically? between galaxies?
Q: how are the force(s) properly described?

Cosmic Forces

- on microscale: particles scatter, collide
  via electromagnetic forces (also strong and weak forces)
  but no net charges or currents
  $\rightarrow$ no EM, strong, or weak forces on cosmo scales
- pressure forces: manifestation of random velocities
  but pressure spatially uniform $\rightarrow$ no net pressure forces!*
  Q: why uniform? why no net P force? (recall hydrostat eq)
- at large scales: only force is **gravity**

Q: what theoretical tools needed to describe this?

*Fine print for experts:
since $P \propto$ KE density, *does* contribute to net mass-energy and thus to *gravity*,
this is a real effect and can be important for relativistic species with $v \approx c$
...but even in this case, no pressure *forces* in the usual sense

Cosmodynamics Computed

cosmic dynamics is evolution of a system which is

- gravitating
- homogeneous
- isotropic

Complete, correct treatment: General Relativity
⇒ we will sketch this starting next week

quick ‘n dirty:
Non-relativistic (Newtonian) cosmology
pro: gives intuition, and right answer
con: involves some ad hoc assumptions only justified by GR

Inputs:

- arbitrary cosmic time t
- cosmic mass density $\rho(t)$, spatially uniform
- cosmic pressure $P(t)$: in general, comes with matter
  but for non-relativistic matter, P not important source of
  energy and thus mass ($E = mc^2$) and thus gravity
  so ignore: take $P = 0$ for now (really: $P \ll \rho c^2$)

Construction:

pick arbitrary point $\vec{r}_{\rm center} = 0$,
center of "comoving" sphere of some radius $r(t)$
which always encloses some arbitrary but fixed mass

$$M(r) = \frac{4\pi}{3}\, r^3 \rho = const \tag{7}$$

a point on the sphere feels acceleration *Q: what?*

Newtonian Cosmodynamics

a point on the sphere feels acceleration

$$\ddot{\vec{r}} = \vec{g} = -\frac{GM}{r^2}\hat{r} \tag{8}$$

with pressure $P = 0$

multiply by $\dot{\vec{r}}$ and integrate:

$$\dot{\vec{r}} \cdot \frac{d}{dt}\dot{\vec{r}} = -GM\frac{\hat{r} \cdot d\vec{r}/dt}{r^2} \tag{9}$$

$$\frac{1}{2}\dot{r}^2 = \frac{GM}{r} + K = \frac{4\pi}{3}G\rho r^2 + K \tag{10}$$

Q: physical significance of K? of it's sign?

Friedmann (Energy) Equation

introduce scale factor: $\vec{r}(t) = a(t)\vec{r}_0$

"energy" eqn: **Friedmann equation**

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\rho - \frac{\kappa c^2}{R^2 a^2} \tag{11}$$

we will see: full GR gives $K = -r_0^2(\kappa c^2/R^2)$

with parameters

- $\kappa = \pm 1, 0$, and
- const R is lengthscale: "curvature" of U.

In full GR:

▷ Friedmann eq. holds even for relativistic matter, but

▷ where $\rho = \sum_{\text{species},i} \varepsilon_i/c^2$: mass-energy density

Q: $a(t)$ behavior if $K = \kappa = 0$? if $\kappa \neq 0$?