Announcements:

• Preflight 1 due Friday, 12 noon

Last time:
Observational/Conceptual Foundations of Cosmology
★ Cosmological Principle
★ Observed Cosmic Kinematics: Hubble’s Law
  www: modern Hubble Diagram--HST 2001
★ Implications of Cosmo Principle + Hubble Law

Today: Cosmodynamics I–Newtonian Cosmology
Critiques of Cosmic Egoism

www: sketch of idealized Galaxy distribution, velocity field
We are at the center of the universe?

Philosophically:
• not Copernican ("principle of mediocrity")

Physically:
• haven’t included gravity!

Observationally:
• Milky Way, Local Group don’t look special
  not what expect from center of explosion
  compare supernova → neutron star, black hole

...yet radial $v$ pattern makes us look special...
The Magic of Hubble

consider three arbitrary cosmic points:
\[ \vec{r}_{BC} = \vec{r}_{AC} - \vec{r}_{AB} \]

Assume \( A \) sees Hubble’s law:
• \( \vec{v}_{AB} = H\vec{r}_{AB} \)
• \( \vec{v}_{AC} = H\vec{r}_{AC} \)

Then ask: what does \( B \) see? \( C \)?

find velocities relative to \( B \):
\[ \vec{v}_{BC} = \vec{v}_{AC} - \vec{v}_{AB} = H(\vec{r}_{AC} - \vec{r}_{AB}) = H\vec{r}_{BC} \]

This is huge!

Q: why? What have we proven?
we have shown:
if $A$ sees Hubble’s law, then so do (arbitrary) $B$ and $C$
thus: if any observer measures Hubble’s law
then all observers will measure Hubble’s law!

so: Hubble law implies
→ all galaxies recede according to same law
→ no need for center, space has no special points

Moreover: Hubble law is only motion
which preserves homogeneity and isotropy
i.e., any other motion breaks cosmo principle
...but Hubble law is exactly what’s observed!
Cosmo Principle Constrains Kinematics

consider arbitrary triangle defined by 3 observers at \( t_0 \)
Hubble law \( \rightarrow \) observers in relative motion
\( \rightarrow \) at later time \( t \), larger triangle

the claim:
later \( \Delta \) always similar to original \( \Delta \)

Q: what are similar triangles?
Q: why must similarity hold?

diagram: triangles, two sides \( r(t_0), r(t), s(t_0), s(t) \)
Q: connections among \( r \)'s and \( s \)'s?
similar: triangle sides keep same ratios, so must have

\[ \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \] (1)

but this rule holds for any triangle
so ratios can depend only on time \( t \):

\[ a(t) = \frac{r(t)}{r(t_0)} = \frac{s(t)}{s(t_0)} \] (2)

Q: and so...?
We have shown:
Cosmo Principle demands \textit{any length} \( r(t) \) evolves as

\[ r(t) = a(t) \, r_0 \]  

where we are free to choose \( a(t_0) = 1 \) today, and \( r_0 = r(t_0) \) is present value ("comoving coordinate")

\( a(t) \) must be universal \textbf{scale factor}

can depend only on time

but at any \( t \): \( a \) has same value everywhere in space

This is huge!

\textit{Q: why? What have we proven? What is character of motion?}
Cosmic Expansion

the meaning of Hubble Law: Take 2

2. Einstein interpretation:
will see: General Relativity + Cosmo Principle demand

Universe is expanding
al all galaxies receding from all others
bold, strange idea!
Expansion: Einstein $\rightarrow$ Hubble

transparency demo: photocopy universe

for two arbitrary observers (e.g., “galaxies”) scale factor gives distances
$\vec{r}(t) = r_0 a(t)$
so velocity is: note: “overdot” is time deriv $\dot{x} \equiv dx/dt$

$$\vec{v}(t) = \dot{\vec{r}} = \ddot{r}_0 \dot{a} = \frac{\dot{a}}{a} \ a \vec{r}_0 \equiv H(t) \vec{r}(t)$$  \hspace{1cm} (4)

$\Rightarrow$ Hubble law!
now interpret “Hubble parameter” as expansion rate $H(t) \equiv \dot{a}/a$

Cosmic Scale Factor Revisited

for two “particles” (possibly Galaxies!)
distance evolves according to

\[ \vec{\ell}(t) = a(t) \vec{\ell}_0 \]

scale factor present distance time varying fixed once and for all

(5)

and thus

\[ \vec{v} = H \vec{\ell} \]

\[ H = \frac{\dot{a}}{a} \]

Q: implications—present, past, future values for \( a \)?
present: at $t_0$, $a(t) = 1$
expanding, so

past: $a(t) < 1$
future: $a(t) > 1$

e.g., at some time in past $a = 1/2$
“galaxies twice as close”

Q: how do cosmic volumes depend on $a$?
e.g., Q: when $a = 1/2$?
Expansion and Areas, Volumes

consider a cube, galaxies at corners
present side length \( L_0 \)
\( \text{diagram: cube, label } L_0, \text{ expansion arrows} \)
\( \rightarrow \) cube is “comoving” w/ expansion
\( \text{draw arrows} \)
volume \( V \propto a^3 \)
\( \rightarrow V = L^3 = L_0^3 a^3 = V_0 a^3 \)
side area \( A = A_0 a^2 \)

www: raisin cake analogy
www: balloon analogy

\( Q: \text{what is tricky, imperfect about each analogy?} \)
Cosmodynamics II

\( a(t) \) gives expansion history of the Universe
which in turn tells how densities, temperatures change
\( \rightarrow \) given \( a(t) \) can recover all of cosmic history!

but...

How do we know \( a(t) \)?
What controls how scale factor \( a(t) \) grow with time?
\( Q: \) what force(s) are at work microscopically? between galaxies?
\( Q: \) how are the force(s) properly described?
Cosmic Forces

- on microscale: particles scatter, collide via electromagnetic forces (also strong and weak forces) but no net charges or currents → no EM, strong, or weak forces on cosmo scales
- pressure forces: manifestation of random velocities but pressure spatially uniform → no net pressure forces!*
  \[ Q: \text{why uniform? why no net } P \text{ force? (recall hydrostat eq)} \]
- at large scales: only force is gravity
  \[ Q: \text{what theoretical tools needed to describe this?} \]

*Fine print for experts:
  since \( P \propto \text{KE density}, \) does contribute to net mass-energy and thus to gravity, this is a real effect and can be important for relativistic species with \( v \approx c \)
  ...but even in this case, no pressure forces in the usual sense
Cosmodynamics Computed

Cosmic dynamics is evolution of a system which is
• gravitating
• homogeneous
• isotropic

Complete, correct treatment: General Relativity
⇒ we will sketch this starting next week

Quick ‘n dirty:
Non-relativistic (Newtonian) cosmology
pro: gives intuition, and right answer
con: involves some ad hoc assumptions only justified by GR
Inputs:
- arbitrary cosmic time \( t \)
- cosmic mass density \( \rho(t) \), spatially uniform
- cosmic pressure \( P(t) \): in general, comes with matter
  but for non-relativistic matter, \( P \) not important source of
  energy and thus mass \( (E = mc^2) \) and thus gravity
  so ignore: take \( P = 0 \) for now (really: \( P \ll \rho c^2 \))

Construction:
pick arbitrary point \( \hat{r}_{\text{center}} = 0 \),
center of “comoving” sphere of some radius \( r(t) \)
which always encloses some arbitrary but fixed mass
\[
M(r) = \frac{4\pi}{3} r^3 \rho = \text{const}
\]  
(7)
a point on the sphere feels acceleration \( Q \): what?
Newtonian Cosmodynamics

a point on the sphere feels acceleration

\[ \ddot{\mathbf{r}} = \mathbf{g} = -\frac{GM}{r^2} \hat{\mathbf{r}} \] (8)

with pressure \( P = 0 \)

multiply by \( \dot{\mathbf{r}} \) and integrate:

\[ \dot{\mathbf{r}} \cdot \frac{d}{dt} \dot{\mathbf{r}} = -GM \frac{\dot{\mathbf{r}} \cdot d\mathbf{r}/dt}{r^2} \] (9)

\[ \frac{1}{2} \dot{r}^2 = \frac{GM}{r} + K = \frac{4\pi}{3} G \rho r^2 + K \] (10)

Q: physical significance of \( K \)? of it's sign?
Friedmann (Energy) Equation

introduce scale factor: \( \vec{r}'(t) = a(t)\vec{r}_0 \)

“energy” eqn: Friedmann equation

\[
H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3} G\rho - \frac{\kappa c^2}{R^2 a^2}
\]  \hspace{1cm} (11)

we will see: full GR gives \( K = -r_0^2(\kappa c^2/R^2) \)

with parameters

\begin{itemize}
  \item \( \kappa = \pm 1, 0 \), and
  \item const \( R \) is lengthscale: “curvature” of \( \mathbf{U} \).
\end{itemize}

In full GR:

\begin{itemize}
  \item \( \triangleright \) Friedmann eq. holds even for relativistic matter, but
  \item \( \triangleright \) where \( \rho = \sum_{\text{species, } i} \varepsilon_i/c^2 \): mass-energy density
\end{itemize}

\( Q: a(t) \text{ behavior if } K = \kappa = 0? \text{ if } \kappa \neq 0? \)