Astro 596/496 PC
Lecture 5
Jan 29, 2010

Announcements:

- PS1 due next Friday, Feb. 5
  - ▷ Director's Cut Extras today: magnitude scale
- Office Hours: 3–4 pm Thursday, or by appointment
  note phase correlation with Friday due date
- Preflight 1 was due today–thanks!

Last time: Cosmodynamics I–Newtonian Cosmology
result: the right answer–Dr. Friedmann's famous equation
Suitable for framing, T-shirts, tattoos...
Q: what's the Friedmann eq? who cares–i.e., why is it useful?
Q: in Friedmann–what's a parameter? what's a variable?

Friedmann (Energy) Equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\kappa c^2}{R^2 a^2} \tag{1}$$

variables change with time

a: cosmic scale factor

ρ: total cosmic mass-energy density

parameters constant, fixed for all time

$\kappa = \pm 1$ or 0: sign of "energy" (curvature) term

R: characteristic lengthscale, GR $\rightarrow$ curvature scale

Q: how does expansion of U depend on contents of U?

Q: how does expansion of U not *depend on contents of U?*

Q: what about acceleration–$\ddot{a}$?

Friedmann Acceleration Equation

Newtonian analysis gives $\ddot{a}$ for $P \to 0$
In full GR: with $P \neq 0$, get Friedmann **acceleration** eq.

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G(\rho + 3P/c^2) \qquad (2)$$

Pressure and Friedmann

⋆ in "energy" ($\dot{a}$) eq.: P *absent*, even in full GR
⋆ in acceleration eq., GR $\rightarrow$ P present, *same* sign as ρ
adds to "active gravitational mass"
Q: why? Q: contrast with hydrostatic equilibrium?

Friedmann energy eq is "equation of motion" for scale factor
i.e., governs evolution of $a(t)$.
To solve, need to know how ρ depends on a
Q: how figure this out?
Q: hint: what is $\rho(a)$ for non-rel matter?

Density Evolution: Matter

if cosmic matter is non-relativistic:

- particle speeds $v \ll c$, and/or $kT \ll mc^2$ (particle rest energy)
- mass is conserved

in comoving sphere with volume $V \propto a^3$, mass conservation gives:

$$dM = d(\rho V) \propto d(\rho a^3) = 0 \quad (3)$$

gives density

$$\rho_{\text{non-rel}} \propto \frac{1}{V} \propto a^{-3} \quad (4)$$

definition: to cosmologist, **matter** $\equiv$ *non-relativistic* matter
today: $\rho_{\text{matter}}(t_0) \equiv \rho_{\text{m},0}$
at other epochs (while still non-relativistic):

$$\rho_{\text{m}} = \rho_{\text{m},0}\ a^{-3} \quad (5)$$

Alternative Derivation: Fluid Picture

in fluid picture: mass conservation $\rightarrow$ continuity equation

$$\partial\rho/\partial t + \nabla \cdot (\rho\vec{v}) = 0 \tag{6}$$

put $\rho = \rho(t)$ and $\vec{v} = H\vec{r}$:

$$\dot{\rho} + H\rho\nabla \cdot \vec{r} = \dot{\rho} + 3\frac{\dot{a}}{a} \tag{7}$$

$$\frac{d\rho}{\rho} = -3\frac{da}{a}\rho \tag{8}$$

$$\rho \propto a^{-3} \tag{9}$$

A Matter-Only Universe

consider a universe containing *only* non-relativistic matter

Friedmann:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R^2}\frac{1}{a^2} \quad (10)$$

$$= \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{\kappa c^2}{R^2} a^{-2} \quad (11)$$

For $\kappa = 0$: "Einstein-de Sitter"

$$(\dot{a}/a)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} \quad (12)$$

evaluate today: $H_0^2 = 8\pi G \rho_0 / 3$

$$a^{1/2} da = H_0 dt \quad (13)$$

$$2/3 \; a^{3/2} = H_0 t \quad (14)$$

Q: implicit assumptions in solution?

Einstein-de Sitter:

$$t = \frac{2}{3} a^{3/2} H_0^{-1} \qquad (15)$$

$$a = \left(\frac{3}{2} H_0 t\right)^{2/3} = \left(\frac{t}{t_0}\right)^{2/3} \qquad (16)$$

Now unpack the physics:

- boundary condition: $a = 0$ at $t = 0 \rightarrow$ "big bang"
- $a \propto t^{2/3}$ *Q: interpretation?*
- $H = 2/3\ 1/t \neq const$ "Hubble parameter" *Q: interp?*
- present age: $t_0 = 2/3\ H_0^{-1} = 2/3\ t_H < t_H$ *Q: interp?*
- U. half its present age at $a = 2^{-2/3} = 0.63$
- objects half present separation (and 8× more compressed) at $t = 2^{-3/2} t_0 = 0.35 t_0$
- using measured value of H_0, calculate $t_0 = 8.9$ Gyr but know globular clusters have ages $t_{\rm gc} \gtrsim 12$ Gyr *Q: huh?*

Matter and Curvature

What if $\kappa = 1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} - \frac{c^2}{R^2}a^{-2}$$

a cannot grow without bound *Q: why?*
Q: what is a_{max}*?*
Q: why are we sure that U recollapses after $t(a_{\text{max}})$*?*
fate: collapse continues back to $a = 0$: "big crunch!"

What if $\kappa = -1$?

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_0 a^{-3} + \frac{c^2}{R^2}a^{-2}$$

a grows without bound *Q: why?*
fate: expand forever–"big chill"
at large t, "curvature-dominated": $a(t) \to ct/R$ *Q: why?*

Q: how can we tell what our κ *value is?*

Geometry, Density, and Dynamics

rewrite Friedmann

$$1 = \frac{8\pi G\rho}{3H^2} - \frac{\kappa c^2}{R^2}(aH)^{-2} = \Omega - \frac{\kappa c^2}{R^2}(aH)^{-2} \tag{17}$$

where the density parameter is

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} \tag{18}$$

and the critical density is

$$\rho_{\text{crit}} = \frac{3H^2}{8\pi G} \tag{19}$$

Note: for a particular density component ρ_i
corresponding density parameter is $\Omega_i = \rho_i/\rho_{\text{crit}}$
and thus total sums all species: $\Omega \equiv \Omega_{\text{tot}} = \sum_i \Omega_i$

Note that

$$\kappa = \left(\frac{aHR}{c}\right)^2 (\Omega - 1) = (\text{pos def}) \times (\Omega - 1)$$

fate* (and geometry) of Universe $\Leftrightarrow \kappa \Leftrightarrow \Omega - 1$

if $\Omega = 1$ ever:

- $\Omega = 1$ always; $\kappa = 0 \rightarrow$ expand forever

if $\Omega < 1$ ever:

- $\Omega < 1$ always; $\kappa = -1 \rightarrow$ expand forever

if $\Omega > 1$ ever:

- $\Omega > 1$ always; $\kappa = +1 \rightarrow$ recollapse

Q: but if Ω just a stand-in for κ, why useful?

*κ always gives geometry, but κ and fate decoupled if $\Lambda \neq 0$

can determine $\Omega \propto \rho/H^2$
from *locally measurable quantities* ρ and H:
$\rightarrow$ cosmic fate & geometry knowable!
...and become *experimental* questions!

But recall:
so far, only have considered non-relativistic matter
definitely an incomplete picture
$\rightarrow$ at minimum, must include photons!

Director's Cut Extras: The Magnitude Scale

Star Brightness: Magnitudes

star brightness (flux) measured in **magnitude** scale
magnitude = "rank" : smaller $m \rightarrow$ **brighter**, *more* flux
Sorry.

Magnitudes use a logarithmic scale:

- difference of 5 mag is factor of 100 in flux:

$m_2 - m_1 = -2.5 \log_{10} F_2/F_1$ (definition of mag scale!)

- mag units: dimensionless! (but usually say "mag") since always a log of ratio of two dimensionful fluxes with physical units like $\mathrm{W/m^2}$

What is mag difference $m_2 - m_1$:
Q: if $F_2 = F_1$*?*
Q: what is sign of difference if $F_2 > F_1$*?*
Q: for equidistant light bulbs, $L_1 = 100\mathrm{Watt}$, $L_2 = 50\mathrm{Watt}$*?*

Apparent Magnitude

a measure of star flux = (apparent) brightness

- no distance needed
- arbitrary mag zero point set for convenience:
  historically: use bright star Vega: $m(\text{Vega}) \equiv 0$
  then all other mags fixed by ratio to Vega flux

- ex: Sun has **apparent** magnitude $m_\odot = -26.74$
  i.e., $-2.5 \log_{10}(F_\odot / F_{\text{Vega}}) = -26.74$
  so $F_{\text{Vega}} = 10^{-26.74/2.5} F_\odot = 2 \times 10^{-11} F_\odot$
- ex: Sirius has $m_{\text{Sirius}} = -1.45 \rightarrow$ brighter than Vega

so: $F_{\text{Sirius}} = 3.8 F_{\text{Vega}} = 8 \times 10^{-11} F_\odot$

- ex: $m_{\text{Polaris}} = 2.02$ *Q: rank Polaris, Sirius, Vega?*

⋆ if distance to a star is known
can also compute Absolute Magnitude

abs mag M = apparent mag if star placed at $d_0 = 10$ pc

Q: what does this measure, effectively?

Absolute Magnitude

absolute magnitude M = apparent mag at $d_0 = 10$ pc

places all stars at constant fixed distance
$\rightarrow$ a stellar "police lineup"
$\rightarrow$ then differences in F only due to diff in L
$\rightarrow$ absolute mag effectively measure luminosity

Sun: abs mag $M_\odot = 4.76$ mag
Sirius: $M_{\rm Sirius} = +1.43$ mag
Vega: $M_{\rm Vega} = +0.58$ mag
Polaris: $M_{\rm Polaris} = -3.58$ mag
ϵ Eridani: $M_{\epsilon \rm Eri} = +6.19$ mag (nearest exoplanet host; $d = 3.2$ pc)
Q: rank them in order of descending L?

Immediately see that Sun neither most nor least luminous star around

Distance Modulus

take ratio of actual star flux vs "lineup" flux
at abs mag distance $d_0 = 10$ pc:

$$\frac{F}{F_0} = \frac{L/4\pi d^2}{L/4\pi d_0^2} \tag{20}$$

which, after simplification, leads to

$$m - M = 5 \log \left(\frac{d}{10 \text{ pc}} \right) \tag{21}$$

- depends only on distance d, not on luminosity!
  can use as measure of distance
- $m - M \equiv$ "distance modulus", sometimes denoted μ