Astro 596/496 PC
Lecture 6
Feb 1, 2010

Announcements:
- PS1 due Friday
- Physics Colloquium this week: S. James Gates, Jr. (UMD)
- "Is Physical Reality a Matrix?"
  supersymmetry, string theory, information theory

Last time:
Friedmann equations, simple solutions
Ω and cosmic fate/curvature
Q: What's Ω? What's "critical" about $\rho_{\rm crit}$?

Today:
expand cosmic inventory, examine implications

To Be or Not to Be Relativistic

for a particle ("species") of mass m
relativistic status set by comparison: typical v vs c
equivalent to comparing: typical E_{kin} vs mc^2
but if thermal, $E_{\text{kin}} \sim kT$
$\rightarrow$ relativistic: $kT \gg mc^2$ $\rightarrow$ non-relativistic: $kT \ll mc^2$

massless particles
if $m = 0$: always have $v = c$ $\rightarrow$ forever relativistic
e.g., photons! also gravitons (if they exist...)

massive particles
if $m > 0$: *always* a time in Early U when $kT \gg mc^2$
$\rightarrow$ massive particles born relativistic, become non-rel!
$\rightarrow$ relativistic status is time-dependent!

Q: are there species which are always *relativistic? non?*
Q: what is relativistic, non-rel today?

Today: $kT_{\mathrm{CMB},0} \sim 10^{-4}$ eV
always: photons relativistic
clearly: $m_e c^2, m_p c^2 \gg kT_0 \rightarrow$ non-relativistic today!
but *were* relativistic in early U

but what about neutrinos?
we know: 3 massive species exist
do not (yet!) know mass of any species
but we *do* know a laboratory-based *lower limit*:
heaviest neutrino must have $m_\nu > 0.04$ eV
$\rightarrow$ at least one ν species non-relativistic today!
$\rightarrow$ contributes to Ω_{matter}

Redshifts I

quick-n-dirty: wavelengths are lengths! ..it's right there in the name!
$\longrightarrow$ expansion stretches photon $\lambda \Rightarrow \lambda \propto a$

if emit photon at $t_{\rm em}$, then at later times

$$\lambda(t) = \lambda_{\rm emit} \frac{a(t)}{a(t_{\rm em})} \tag{1}$$

if observe later, $\lambda_{\rm obs} = \lambda_{\rm em} \; a_{\rm obs}/a_{\rm em}$
measure redshift today:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} = \frac{1 - a_{\rm em}}{a_{\rm em}} \Rightarrow \quad a_{\rm em} = a(z) = \frac{1}{1+z}$$

Newtonian Derivation of Redshift: Hubble & Doppler

slower-n-cleaner: non-relativistic Doppler
non-rel Doppler sez:

$$\frac{\delta\lambda}{\lambda} \equiv z = \frac{v}{c} \tag{2}$$

Hubble sez:

$$cz = Hr \tag{3}$$

Together

$$\frac{\delta\lambda}{\lambda} = \frac{Hr}{c} \tag{4}$$

But light travels distance r in time $\delta t = r/c$, so

$$\frac{\delta\lambda}{\lambda} = H\delta t = \frac{\dot{a}\delta t}{a} = \frac{\delta a}{a} \tag{5}$$

for arriving light, fractional λ change = fractional a change!

Scale Factor and Redshift

$$a = \frac{1}{1+z}$$
$$z = \frac{1}{a} - 1$$

most distant quasar: $z = 6.4$

`www: SDSS QSO recordholder`

most distant gamma-ray burst: $z \approx 8.2$!

`www: GRB recordholder`

When GRB exploded:

$\rightarrow$ scale factor was $a = 1/(1 + 8.2) = 0.11$

interparticle (intergalactic) distances 11% of today!

$\rightarrow$ galaxies were $1 + z = 9.2$ times closer

squeezed into volumes $(9.2)^3 = 780$ times smaller!

$\rightarrow$ age at $z = 8.2$: concordance U gives 650 Myr

Redshifts and Photon Energies

in photon picture of light: $E_\gamma = hc/\lambda$

so in cosmological context photons have

$$E_\gamma \propto \frac{1}{a} \tag{6}$$

$\longrightarrow$ γ energy redshifts

Consequences:

- ▷ *Q: photon energy density $\varepsilon(a)$?*
- ▷ if thermal radiation,

  Q: $T \leftrightarrow \lambda$ connection?

  Q: expansion effect on T?

Relativistic Species

Photon energy density: $\varepsilon = E_\gamma \, n_\gamma$
avg photon energy: $E_\gamma \propto a^{-1}$
photon number density: conserved $n_\gamma \propto a^{-3}$ (if no emission/absorption)
$\Rightarrow \varepsilon_\gamma \propto a^{-4}$

Thermal (blackbody) radiation:
Wien's law: $T \propto 1/\lambda_{\rm max}$
but since $\lambda \propto a \rightarrow$ then $T \propto 1/a$

Consequences:

- $\varepsilon \propto T^4$: Boltzmann/Planck!
- T decreases $\rightarrow$ U cools!
  today: CMB $T_0 = 2.725 \pm 0.001$ K
  distant but "garden variety" quasar: $z = 3$
  "feels" $T = 8$ K (effect observed!)

Radiation and Friedmann

definition: to cosmologist, **radiation** $\equiv$ *relativistic* matter
photons or *any* particle with $v \sim c$, $E \sim T \gg mc^2$
energy density $\varepsilon_{\rm rad} \propto a^{-4}$
equivalent gravitational mass density: $\varepsilon = \rho c^2 \rightarrow \rho_{\rm rad} \propto a^{-4}$

Add radiation to Friedmann:

$$\rho = \rho_{\rm total} = \rho_{\rm m} + \rho_{\rm rad} = \rho_0(\Omega_{\rm m,0}a^{-3} + \Omega_{\rm r,0}a^{-4})$$

note: today, $\Omega_{\rm r,0} = 4.15 \times 10^{-5} h^{-2} \ll 1$

Also: Maxwell says pressure $P_{\rm EM} = \varepsilon_{\rm EM}/3$

- include this in Friedmann acceleration
- put $V = a^3$, so $\varepsilon \propto V^{-4/3}$, and

$$d(\varepsilon_{\rm rad} V) = -1/3\, \varepsilon\, dV = -p_{\rm rad}\, dV$$

Q: physical interpretation?

1st Law and Equation of State

Generalize: Cosmological "1st Law of Thermodynamics"

$$d(\rho c^2 a^3) = -p d(a^3) \tag{7}$$

GR verifies this is correct!
$\Rightarrow$ reconciles Friedmann energy, accel eqs:
ensures that $\ddot{a} = d\dot{a}/dt$ (try it!)

to solve, need to relate p to ρc^2 $\rightarrow$ **equation of state**

- non-rel matter: $p_{\rm m} \ll \rho_{\rm m} c^2 \approx 0$ *Q: why? e.g., ideal gas?*
- radiation: $p_{\rm rad} = \rho_{\rm rad} = \rho_{\rm rad} c^2/3$
- generalize: $\boxed{p = w\rho c^2}$ defines "*state parameter*" w
  Q: $w_{\rm matter}$*?*, $w_{\rm rad}$*?*

Can solve 1st Law eq for matter with **constant** w:

$$\rho_w \propto a^{-3(1+w)} \tag{8}$$

Q: what if $w = 0, +1/3, -1$*?*

Cosmological Constant

Consider substance ("Λ") with $w = -1$

- $p_\Lambda = -\rho_\Lambda c^2 < 0$!?
  negative pressure !?!

- $\rho_\Lambda \propto a^0 = const$
  constant energy density (and pressure) !?!
  i.e., expansion does not change ρ_Λ, p_Λ!

Einstein (1917) "**cosmological constant**" Λ

Cosmodynamics in a Minimally Realistic(?) Universe

For sure, the universe contains:

- Matter *Q: evidence?*
  $\rho_{\mathrm{m}} \propto a^{-3}$
- Radiation *Q: evidence?*
  $\rho_{\mathrm{r}} \propto a^{-4}$

Quite possibly, the universe could contain:

- Curvature
  curvature term $\propto a^{-2}$
- Cosmo Const (or worse!)
  $\rho_{\Lambda} \propto a^{0} = const$

So: “minimal” but also “realistic” account of U must include these pieces:

$$\rho = \rho_{\mathrm{tot}} = \sum_i \rho_i \tag{9}$$

then Friedmann sez:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_{\mathrm{r},0}a^{-4} + \rho_{\mathrm{m},0}a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2}a^{-2}$$
$$= H_0^2\left[\Omega_\mathrm{r}a^{-4} + \Omega_\mathrm{m}a^{-3} + \Omega_\Lambda + (1 - \Omega_\mathrm{tot})a^{-2}\right]$$

Limiting cases: one term $\gg$ all others

- radiation-dominated: $\rho_\mathrm{tot} \approx \rho_\mathrm{r} \gg \rho_\mathrm{other}$ $\Rightarrow$ $a \sim t^{1/2}$
- matter-dominated: $a \sim t^{2/3}$
- curvature-dominated ($\kappa = -1$; *Q: why?*): $a \propto t^1$
- Λ-dominated: $a \propto e^{+H_\Lambda t}$

The Cosmic Past

Friedmann sez

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_{\mathrm{r}} + \rho_{\mathrm{m}} + \rho_\Lambda) - \frac{\kappa c^2}{R^2} a^{-2}$$
$$= \frac{8\pi G}{3}\left(\rho_{\mathrm{r},0} a^{-4} + \rho_{\mathrm{m},0} a^{-3} + \rho_\Lambda\right) - \frac{\kappa c^2}{R^2} a^{-2}$$

Plot: H^2 vs a

Mix-n-match:

Q: evolution if only matter & rad? Ω?

Q: ... if matter, rad, and curv($\pm$)? Ω?

Q: ... if matter, rad, and Λ? Ω?

Q: ... if matter, rad, curv, and Λ? Ω?

Menu at Al Friedmann's Cosmo Café

Possible Histories of the Universe

Matter + Radiation only: $(\Omega = 1)$

rad-dom $\rightarrow$ matter-dom; expand forever

Matter + Radiation + Curvature($-$): $(\Omega < 1)$

RD $\rightarrow$ MD $\rightarrow$ CD; expand forever

Matter + Radiation + Curvature($+$): $(\Omega > 1)$

RD $\rightarrow$ MD $\rightarrow$ CD $\rightarrow$ reverse; recollapse

Matter + Radiation + Λ: $(\Omega = 1)$

RD $\rightarrow$ MD $\rightarrow$ ΛD: expand forever *exponentially!*

Matter + Radiation + Λ + curv: $(\Omega \neq 1)$

many possibilities! fate depends on detailed composition

Radiation and the Early Universe

note: radiation *always wins out* at early times
$\Rightarrow$ Early U is radiation-dominated
Q: why?

later evolution depends on cosmic ingredients
and their relative amounts

Density and Destiny

Fate (and geometry) of U. depend on
current values of $\Omega_{i,0} = \rho_{i,0}/\rho_{\text{crit},0}$
and $\Omega_0 = \sum \Omega_i$ where

$$\begin{aligned}
\rho_{\text{crit},0} &= 3H_0^2/8\pi G \\
&= 1.9 \times 10^{-29}\ h^2\ \text{g/cm}^{-3} \approx 10^{-29}\ \text{g/cm}^{-3} \\
&= 2.78 \times 10^{11}\ h^2\ M_\odot\ \text{Mpc}^{-3} \approx 1.4 \times 10^{11}\ M_\odot\ \text{Mpc}^{-3} \\
&\approx 6\ \text{H atoms m}^{-3}
\end{aligned}$$

Empirical question:

- is $\rho_{\text{tot},0}$ bigger or smaller than this number?
- *density is destiny!* *weight is fate!*

Cosmic Fate & Geometry: Theory Prejudice

Consider a universe with $\Omega \neq 1$

Friedmann says

$$\Omega - 1 = \frac{\kappa c^2}{R^2 a^2 H^2} = \frac{\kappa c^2}{R^2 \dot{a}^2} \propto \frac{1}{\dot{a}^2} \tag{10}$$

i.e., Ω *changes* with time

Q: is $|\Omega - 1|$ increasing or decreasing?

Q: limiting values of Ω at large t?

Q: physical interpretation of these limits?

Q: timescale for Ω to change?

Q: implications for Ω_0?

Time change of $|\Omega - 1| \propto 1/\dot{a}^2$ is

$$\frac{d}{dt}|\Omega - 1| \propto \frac{d}{dt}\frac{1}{\dot{a}^2} = -\frac{\ddot{a}}{\dot{a}^3} \tag{11}$$

Two possibilities

- if $\ddot{a} > 0$: expansion of U *accelerating*
  then $|\Omega - 1|$ *decreasing* with time
  $\rightarrow$ evolution drives $\Omega \rightarrow 1$

but recall Friedmann acceleration: $\ddot{a}/a \propto -(\rho + 3P)$
matter, radiation lead to *deceleration*
i.e., all known cosmic ingredients have attractive gravity...

- if $\ddot{a} < 0$: ordinary attractive gravity, *decelerating* U
  then $|\Omega - 1|$ *increasing* with time
  $\rightarrow$ Ω driven increasingly away from 1