NAME: ____________________________________

Astronomy 210 **Spring 2011**

FINAL EXAM

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1. DO NOT OPEN THIS EXAM UNTIL INSTRUCTED TO DO SO.
2. Write you name and all answers in your test booklet. Turn in your booklet, your study sheet, and this sheet. **You must turn in the exam questions. Answer booklets without questions wil receive a zero for this exam.**
3. Show all of your work in the test booklet, and indicate clearly your final answer! A correct final answer may not receive credit if no work is shown.
4. Budget your time! Don't get stalled on any one question.
5. For your reference there are formulas below. Many of these constants and formulas you will not need for this exam!

Possibly Useful Constants

Astronomical Unit: $1\,\mathrm{AU} = 1.5 \times 10^{11}\,\mathrm{m}$.
parsec: $1\,\mathrm{pc} = 3.1 \times 10^{16}\,\mathrm{m} = 2.1 \times 10^{5}\,\mathrm{AU}$
gravitational constant : $G = 6.7 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$
speed of light: $c = 3.0 \times 10^{8}\ \mathrm{m\,s^{-1}}$
Stefan-Boltzmann constant: $\sigma = 5.7 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$
Planck's constant: $h = 6.6 \times 10^{-34}$ J s $= 4.1 \times 10^{-15}$ eV s
electron Volt: 1 eV $= 1.6 \times 10^{-19}$ J
Boltzmann constant: $k = 1.4 \times 10^{-23}$ J K^{-1} $= 8.6 \times 10^{-5}$eV K^{-1}
mass of the proton: $m_p = 1.7 \times 10^{-27}\,\mathrm{kg}$
Hubble constant (present-day value): $H \simeq 70$ km s^{-1} Mpc^{-1} $= 2.3 \times 10^{-18}$ s^{-1}
age of Universe: $t_0 \simeq 1.4 \times 10^{10}$ yr = 14 billion yrs
present age of Sun $\sim 5 \times 10^{9}$ yr
$M_\odot = 2.0 \times 10^{30}$ kg $M_{\mathrm{Earth}} = 6.0 \times 10^{24}$ kg $M_{\mathrm{Jupiter}} = 1.9 \times 10^{27}$ kg
$L_\odot = 3.8 \times 10^{26}$ Watt $R_\odot = 7 \times 10^{8}$ m $R_{\mathrm{Earth}} = 6.4 \times 10^{6}$ km

Possibly Useful Formulae

$F = ma$	$v_{\rm circ} = 2\pi r/P$	
$F = Gm_1m_2/r^2$	$a_{\rm c} = v^2/r$	$v_{\rm circ}^2 = GM/r$
$a_{\rm AU}^3 = P_{\rm yr}^2$	$a^3 = GMP^2/4\pi^2$	
$r_{\rm p} = (1-e)a$	$r_{\rm a} = (1+e)a$	
$1/P_{\rm inf} = 1/E + 1/S$	$1/P_{\rm sup} = 1/E - 1/S$	
$\lambda f = c$	$E_\gamma = hf = hc/\lambda$	$p = h/\lambda$
$E_n = -13.6\ {\rm eV}/n^2$		
$\lambda_{\rm max}T = 2.9 \times 10^{-3}\ {\rm m \cdot K}$		
$F = \sigma T^4$	$L = 4\pi R^2 F$	
$T_{\rm eq} = [(1-A)/(2 \text{ or } 4)]^{1/4}(R_\odot/d)^{1/2}T_\odot = (332 \text{ or } 279)[(1-A)/d^2]^{1/4}\,{\rm K}$		
$v_{\rm esc}^2 = 2GM/R$	$v_{\rm rms}^2 = 3kT/m$	
$v_{\rm esc} \geq 6v_{\rm rms}$ to retain atmosphere		$n/n_0 = 2^{-t/t_{1/2}}$
$F_{\rm tide} = 2GMmR/d^3$	$R_{\rm Roche} = 2.4(\rho_M/\rho_m)^{1/3}R_M = 2.4(M/m)^{1/3}r_m$	
$L \propto M^4$ for $M < 10M_\odot$		
$E = L\tau$	$\tau = \tau_\odot(M/M_\odot)^{-3}$, $M < 10M_\odot$; $\tau_\odot \simeq 10^{10}$ yr	
$d({\rm pc}) = 1/p({\rm arcsec})$		
$m_2 - m_1 = -2.5\log_{10} F_2/F_1$	$m - M = 5\log_{10}(d/10\ {\rm pc})$	
$PV = NkT$	$dP/dr = -GM(r)\rho(r)/r^2$	
$R_{\rm Sch} = 2GM/c^2$	$\lambda_{\rm obs}/\lambda_{\rm em} = \Delta t_{\rm obs}/\Delta t_{\rm em} = \sqrt{1 - R_{\rm Sch}/r_{\rm obs}}/\sqrt{1 - R_{\rm Sch}/r_{\rm em}}$	
$v = Hd$	$d(t) = a(t)r$	$H = \dot{a}/a$

NOTE: the above symbols may have different meanings in different equations!

Sample Problems

1. Chambana is at latitude 40° N. For an observer in Chambana, are there *any* stars on the celestial sphere which are *never* visible in the night sky at *any* time during the year? Briefly explain your reasoning.

2. The earth's orbit is an ellipse with $e = 0.017$. Calculate the ratio $T_{\text{ap}}/T_{\text{peri}}$ of the earth's average temperature at perihelion to that at aphelion. You may ignore the greenhouse effect.

3. The most abundant elements in the human body are hydrogen, carbon, and oxygen. Ultimately, these elements arise in extreme astrophysical environments.

   (a) When and where were most hydrogen nuclei formed?

   (b) When and where was most carbon nuclei formed?

   (c) When and where were most oxygen nuclei formed?

4. Sketch an H-R diagram (or more than one if the first is too crowded) for a large and unbiased sample of stars. Label and give a scale for each axis, and indicate

   - the position of the Sun
   - the main sequence,
   - the positions of giants, supergiants, and white dwarfs
   - the position of a K-type star with a radius larger than the Sun's; label this A

5. *Main Sequence Stars*

   (a) [10 points] Using the mass-luminosity relation for main-sequence stars, estimate the luminosity of a main sequence star with mass $M = 0.5M_\odot$.

   (b) [10 points] What is the absolute visual magnitude, $\mathcal{M}_V$, of a $0.5M_\odot$ main sequence star, if the absolute visual magnitude of the Sun is $\mathcal{M}_{V,\odot} = +4.77$?

6. Consider a universe in which the scale factor as a function of time t according to

$$a(t) = \sin\left(\pi \frac{t}{\tau_c}\right) \tag{1}$$

   with τ_c a constant, and t is restricted to the range $0 \leq t \leq \tau_c$. Also note that the sine function takes its argument in radians.

   Sketch $a(t)$. Briefly describe the overall behavior of a universe having this scale factor evolution. What is special about each of the three times $t = 0$, $t = \tau_c/2$, and $t = \tau_c$? What is the final fate of such a universe?