Hour Exam 1: Review Questions

Plausible questions: either from old exams,
or at right level for this exam

Telescopes must have "tracking" motors on them which rotate the scope to keep the same celestial objects in the field of view.

- Briefly (1-2 sentences) explain why a tracking motor is needed.

- Find the angular speed at which a telescope must rotate to track a star on the celestial equator.

- The celestial sphere and all objects on it **rotates** once a day realtive to a terrestrial observer. Thus a telescope pointing to a fixed direction relative to the horizon will see stars carried in and out of the field of view; to compenstate for the celestial sphere's rotation, a motor must match its spin rate.

- The motor's spin rate is that of the celestial sphere: covering a full circuit of $\Delta\theta = 360^\circ$ over a period of $\Delta t = P = 1$ day, for an angular speed of

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{360^\circ}{P} = 1^\circ/\text{day} = 15^\circ/\text{hour} = 0.25^\circ/\text{min} \tag{1}$$

Since you were not asked to put your answer in a particular set of units, any of these units would be fine.

- Saturn's moon Titan orbits the planet with a semi-major axis of $a = 1.2 \times 10^9$ m and a period $P = 1.4 \times 10^6$ s. Using these data, calculate the mass of Saturn.

- The Sun is accelerated due to its interactions with the planets. Which is larger: the Sun's acceleration due to its interaction with Earth, or the Sun's acceleration due to its interaction with Jupiter? Show your work. Note that $a_{\mathsf{Jup}} = 5.2$ AU.

- We wish to relate a, P, and M, which calls for the full, generalized version of Kepler III:

$$
\begin{aligned}
a^3 &= \frac{GM}{4\pi^2}P^2 && (2)\\
M &= \frac{4\pi^2 a^3}{GP^2} = 5.2 \times 10^{26}\ \text{kg} && (3)
\end{aligned}
$$

You can verify from the data on the exam front page that this is less than the mass of the Sun, but larger than the mass of the Earth, which means the result is plausible. In fact, this turns out to be about 90 times the mass of the Earth.

- For each planet, the contribution to the Sun's acceleration A is, in magnitude,

$$F_{\odot,\mathrm{planet}} = \frac{GM_\odot m_{\mathrm{planet}}}{r^2} = M_\odot A \tag{4}$$

which gives an acceleration

$$A = \frac{Gm_{\mathrm{planet}}}{r^2} \tag{5}$$

(we use A for acceleration to avoid confusion with semimajor axis a).

One could calculate these separately for the Earth and Jupiter, but note that we are only asked which is larger. A quick way to find this is just to evaluate the ratio of the two accelerations:

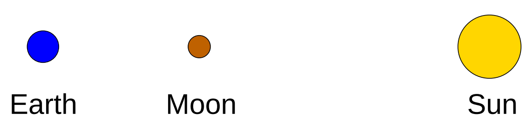
$$\frac{A_{\mathrm{J}}}{A_{\mathrm{E}}} = \frac{M_{\mathrm{J}}}{M_{\mathrm{e}}} \left(\frac{a_{\mathrm{E}}}{a_{\mathrm{J}}}\right)^2 = 12 \tag{6}$$

so that Jupiter dominates, by far, the Sun's acceleration. Note the reduction in number crunching on your calculator when you take the ratio.

During what phase(s) of the moon does a solar eclipse occur? Briefly explain your reasoning.

A solar eclipse occurs when the **Earth, Moon, and Sun** are **aligned**, and in that order. As seen from Earth, the Moon is blocking the Sun, and so it is also in the same direction as the Sun. This corresponds to the $\boxed{\textbf{new moon}}$ phase.

Solar eclipse geometry



At 10am, when Astronomy 210 meets, the moon is observed to be setting. What is the phase of the moon?

If the Moon is on the horizon at 10am (i.e., rising or settgin), it must be either in the waxing crescent or waning gibbious phase. But if it is setting, this uniquely selects the **waning gibbous** phase.

Chambana is at latitude 40° N. For an observer in Chambana, are there any *stars on the celestial sphere which are* never *visible in the night sky at* any *time during the year? Briefly explain your reasoning.*

From our northerm latitude, over the course of the year we can see all stars in the northern hemisphere of the celestial sphere. However, as the diagram shows, there are regions of the southern celestial hemisphere which we *cannot* see–the south celestial pole is one example.

A large space boulder, asteroid Yikes, is discovered to orbit the Sun with period $P = 8$ years.

1. *Find semimajor axis a of asteroid Yikes, in AU.*

   Since the motion is around the Sun, we can use the simple version of Keplers third law,

   $$P_{\text{yr}}^2 = a_{\text{AU}}^3 \tag{7}$$

   We are given $P_{\text{yr}} = 8 = 2^3$, and so we infer

   $$a_{\text{AU}} = P_{\text{yr}}^{2/3} = (2^3)^{2/3} = 2^2 = 4 \tag{8}$$

   and so $\boxed{a = 4 \text{ AU}}$. Note that the numbers were given so that you didn't need a calculator!

2. *Find* minimum *eccentricity e asteroid Yikes must have so that its orbit intersects that of the Earth. You may assume the Earth's orbit to be circular.*

Since $a = 8$ AU $\neq$ $\mathrm{a}_\oplus = 1$ AU , the asteroid will only intersect the Earth's orbit if it has nonzero eccentricity. The orbit's closest point to the Sun is the perihelion, which is at radius

$$r_{\text{min}} = r_{\text{peri}} = (1-e)a \tag{9}$$

To intersect the Earth, we require $r_{\text{peri}} \leq a_\oplus = 1$ AU. This is the same as requiring a focal length $c = a - r_{\text{peri}} = ea \geq 3$ AU. Using $r_{\text{peri}} \leq a_\oplus = 1$ AU, we have

$$(1-e)a \leq a_\oplus \tag{10}$$

ans solving for the eccentricity we have

$$\begin{aligned} -ea &\leq a_\oplus - a && (11)\\ ea &\geq a - a_\oplus && (12)\\ e &\geq 1 - \frac{a_\oplus}{a} = 1 - \frac{1}{4} = \frac{3}{4} && (13) \end{aligned}$$

So the minimum eccentricity is $\boxed{e_{\text{min}} = 3/4}$.

In a future industrial accident ("mistakes were made"), the mass of the Sun is reduced to exactly 1/2 of its present value: $M_{\text{Sun,new}} = M_{\text{Sun,today}}/2$. If the Earth is to be kept in an orbit with the same *semi-major axis as today ($a_{\text{new}} = a_{\text{today}}$), find the new orbital period P_{new} that is required. Express your answer in (present-day) years.*

Since the Sun's mass changes, we cannot use the simple version of Kepler's third law, but rather the generalized version due to Newton:

$$a^3 = \frac{GM}{4\pi^2} P^2 \tag{14}$$

Useing this expression, and solving

$$P = 2\pi \sqrt{\frac{a^3}{GM}} \tag{15}$$

But that equation (14) holds for *any* Keplerian orbit, and so it must apply to both the pre-accident and post-accident orbits:

$$a_{\rm old}^3 = \frac{GM_{\rm old}}{4\pi^2}P_{\rm old}^2 \qquad (16)$$

$$a_{\rm new}^3 = \frac{GM_{\rm new}}{4\pi^2}P_{\rm new}^2 \qquad (17)$$

Dividing one equation by the other, we have

$$\left(\frac{a_{\rm new}}{a_{\rm old}}\right)^3 = \frac{M_{\rm new}}{M_{\rm old}} \left(\frac{P_{\rm new}}{P_{\rm old}}\right)^2 \qquad (18)$$

but since we have $a_{\rm new} = a_{\rm old}$ and $M_{\rm new} = M_{\rm old}/2$, we find

$$\left(\frac{P_{\rm new}}{P_{\rm old}}\right)^2 = 2 \qquad (19)$$

and thus

$$P_{\rm new} = \sqrt{2}\ P_{\rm old} = 1.4 \text{ years} \qquad (20)$$

Give one *example of an observed naked-eye phenomenon that fits within a geocentric cosmology, and briefly explain how the geocentric model accounts for the phenomenon.*

Many possible answers are correct. Simple examples are the daily rising and setting of stars, due to the spin of the celestial sphere. More complicated examples are epicycles, or the non-observation of parallax.