Hour Exam 2: Review Questions

Plausible questions: either from old exams,
or at right level for this exam

As discussed in class, planets in the solar system have many characteristics in common and also many differences. These shed light on the process of planet formation.

- Give *one* example of a feature that is common to both terrestrial and jovian planets, and reflects information about the formation of the solar system.

Briefly describe how our current theories for the formation of the solar system account for this similarity.

Exoplanets of roughly Jupiter-size masses have been found with surprisingly short orbital periods.

Consider a planet with Jupiter's mass $M = 2 \times 10^{27}$ kg, radius $R = 7 \times 10^{7}$ m, albedo $A = 0.5$, and composition. The planet orbits a star with the same mass, radius, and temperature as the Sun; the orbital period is $P = 3.65$ days $= 0.01$ yr

(a) Assume the planet is slowly rotating, and find the daytime temperature of the planet. Express your answer in Kelvin.

(b) Assume your answer to part (a) was $T = 1600$ K (which would be wrong!). For a Jovian-type atmosphere, would the most abundant species be evaporated? Justify your answer with the appropriate calculation.

(a) Semi-major axis is given by $a_{\text{AU}}^3 = P^2$, or $a = 10^{-2/3}$ AU = 0.046 AU. The usual temperature formula for this a and $A = 0.5$ gives

$$T = 332[(1 - A)/a_{\text{AU}}^2]^{1/4} \text{ K} = 1300 \text{ K} \tag{1}$$

(b) We need to compare escape speed and thermal speed for the dominant atmospheric species, molecular hydrogen: $m(\text{H}_2) = 2m_p$. We have

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} = 62 \text{ km/s} \tag{2}$$

and

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{2m_p}} = 4.4 \text{ km/s} \tag{3}$$

and thus we have

$$\frac{v_{\text{esc}}}{v_{\text{rms}}} = 14 > 6 \tag{4}$$

and so the hydrogen atmosphere survives.

Not only the Moon, but also the Sun contributes to the tides on Earth. For reference, the Moon's mass is $M_{\text{Moon}} = 3.7 \times 10^{-8} M_{\odot}$ and its distance from Earth is $d = 2.6 \times 10^{-3}$ AU.

- Which exerts stronger tidal forces on material at the Earth's surface $R_{\oplus}$: the Sun or the Moon? Justify your answer with a calculation. *Hint:* all you are asked is to find which is larger.

- The ocean tides on the Earth respond to both the Sun and the Moon. During which phase(s) of the Moon are the tides the strongest? (These are known as the *spring* tides.)

(a) A test mass m on the surface $R_\oplus$ of the Earth feels a tidal force (relative to the center of the Earth) of magnitude

$$F_{\text{moon}} = \frac{2GM_\oplus M_{\text{Moon}} R_\oplus}{d_{\text{Moon}}^3} \tag{5}$$

and a tidal force due to the Sun of

$$F_\odot = \frac{2GM_\oplus M_\odot R_\oplus}{d_\odot^3} \tag{6}$$

Thus we have

$$\frac{F_{\text{moon}}}{F_\odot} = \frac{M_{\text{Moon}}}{M_\odot} \left(\frac{d_\odot}{d_{\text{Moon}}} \right)^3 = 2 \tag{7}$$

where the mass and radius of the Earth drop out. and so the Moon has tidal forces which are larger in magnitude, by a factor of about 2, than those of the Sun.

(b) Each perturbing object, Sun or Moon, raises tides which are high on *both* the near *and* the far side of the perturbing object. The tides are *low* on the points on the Earth halfway between the near and far points.

This means that the solar and lunar tides partially cancel when the Sun and Moon are at right angles, which corresponds to first quarter and third quarter phases ("neap tides"). However, we are asked to find when they are maximum ("spring tides"), which occurs when the Earth, Sun, and Moon are aligned–but *either* configuration will do: new and full phases.

Billions of year from now, the Sun will become a giant star which will be both cooler ($T = 3000$ K $= T_\odot/2$) and much larger ($R = 100R_\odot \simeq 0.5$ AU) than the Sun is today.

1. *Find the peak wavelength in the spectrum of the giant Sun. Will the Sun appear redder or bluer than it does today?*

2. *Find the new equilibrium temperature T_{E} of the Earth. You may assume the Earth still retains an atmosphere, and has an albedo $A = 0.4$, and you may ignore the greenhouse effect.*

3. *Assume your answer to (b) was $T_{\mathrm{E}} = 1600$ K (which is wrong!). Will the Earth's atmosphere be evaporated? You may take the atmosphere to be pure N_2, each particle of which has molecular weight of 28 and thus has mass $m = 28m_p$.*

4. Bonus–*Will earth's atmosphere be tidally stripped (torn away from the Earth) by the bloated giant Sun? Show your work.*

1. Wien's law tells us that for a blackbody, the peak wavelength and temperature are related by

$$\lambda_{\mathrm{max}} T = 3 \times 10^{-3} \text{ m K} \tag{8}$$

Using $T = 3000$ K, we find

$$\lambda_{\mathrm{max}} = \frac{3 \times 10^{-3} \text{ m K}}{T} = \frac{3 \times 10^{-3} \text{ m K}}{3 \times 10^{3} \text{ K}} = 10^{-6} \text{ m} = 1\mu \text{ m} \tag{9}$$

This turns out to be in the infrared part of the spectrum. But in any case, since $\lambda_{\mathrm{max}} \propto 1/T$, a cooler Sun will have a larger peak wavelength and thus a redder color.

2. For a fast-rotator like the Earth, we have

$$T = \left(\frac{1 - A}{4}\right)^{1/4} \sqrt{\frac{R_{\odot}}{d}} T_{\odot} \tag{10}$$

using $A = 0.4$ and $d = 1$ AU, we find

$$T = 1300 \text{ K} \tag{11}$$

3. To evaluate whether a gas will be evaporated, we must compare the thermal speed v_{rms} with the escape speed v_{esc}.

In gas of molecules having mass m and at temperature T, the typical molecular speed is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{28m_p}} = 1.2 \times 10^3 \text{ m/s} \tag{12}$$

where for the last two expressions we used $m = 28m_p$ and $T = 1600$ K.

The escape speed for a distance r from an object of mass M is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2GM_\oplus}{R_\oplus}} = 1.2 \times 10^4 \text{ m/s} \tag{13}$$

where the last two expressions are for escape from the Earth.

Comparing these two, we find

$$\frac{v_{\text{esc}}}{v_{\text{rms}}} = 10 \tag{14}$$

To retain a gas in the atmosphere, we require $v_{\text{esc}}/v_{\text{rms}} > 6$. Thus we conclude that the atmosphere will be saved.

4. The magnitude of the tidal force due to an object of mass M and distance r, and felt by particles separated by distance d is (there was a factor of 2 typo on the first page equations)

$$F_{\text{tide}} = \frac{2GMmd}{r^3} \tag{15}$$

Since we want to know the tidal force of the Sun on the Earth's atmosphere, we need to use $M = M_\odot$, $r = 1$ AU is the distance to the Sun, m the mass of an atmospheric molecule (it ultimately drops out of the problem), $d = R_{\rm Earth}$ for the distance between the atmosphere and the center of the Earth.

The tidal force pulls atmospheric gas away, but we also need to compare this to the Earth's attractive gravitational force on the gas:

$$F_{\rm Earth} = \frac{GM_{\rm Earth}m}{R_{\rm Earth}^2} \tag{16}$$

The ratio of the forces is

$$\frac{F_{\rm tide}}{F_{\rm Earth}} = \frac{2M_\odot}{M_{\rm Earth}} = \left(\frac{R_{\rm Earth}}{1 \text{ AU}}\right)^2 \tag{17}$$

If we were to evaluate this, we would find that $F_{\rm tide}/F_{\rm Earth} \ll 1$ and the atmosphere is safe.

But we don't even need to evaluate the numbers. Note that the tidal force only depends the *mass* M and distance r from the object exerting the tides; there is no dependence on the size of this object. Thus, the tidal forces due to the bloated Sun are the same as those due to the Sun today, and these are obviously not enough to strip away the atmosphere!

Note that if you used the simplified version of the Roche limit, $R_{\text{Roche}} = 2.4 R_M$, you would conclude the opposite, but in fact the full expression has

$$R_{\text{Roche}} = 2.4 \left(\frac{M}{m} \right)^{1/3} R_m \tag{18}$$

which again doesn't change from its value today.

After walking on the Moon 41 years ago, Apollo 11 astronauts left their lander, Eagle, *on the Moon's surface. Assume* Eagle *is an object 4 meters across, resting on the surface of the Moon at a distance* 4×10^8 *meters away.*

Now imagine the Hubble Telescope (diameter $D = 2.5$ *meters) looks at the Moon in green light (wavelength* $\lambda = 500$ nm $= 5 \times 10^{-7}$ *m). Will Hubble resolve the* Eagle *lander? Justify your answer with a calculation.*

The angular resolution of a telescope with diameter D, operating at wavelength λ, is

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 2.44 \times 10^{-7} \text{ radian} \tag{19}$$

Note that the formula gives the diffraction limit in radians!

We need to compare this to the angular size of *Eagle*, which is

$$\theta_{\text{Eagle}} = \frac{\ell}{r} = 10^{-8} \text{ radian} \tag{20}$$

where $\ell = 4$ meters and $r = 4 \times 10^8$ m.

Thus we have $\theta_{\text{Eagle}} \ll \theta_{\text{min}}$, and we conclude that *Eagle* is much smaller than the smallest angle *Hubble* can resolve. And thus *Hubble* cannot resolve *Eagle*.