Astronomy 210
Homework Set #10: Data Analysis Option
Due in class: Friday, April 22
Total Points: 50 + 10 bonus

Choosing a Computational Project

This week's homework uses computer-based analysis to get important astrophysics results. There are *two* options, this one involving collection and analysis to derive properties of galaxies in the local and distant universe, the other involving basic programming to simulate a star.

You should choose *one* of these projects to do.

This Option: Surveying the Universe Online

1. *Some Basic Cosmology.* This project encompasses the largest scope we will take in the course, looking beyond our own Galaxy to other galaxies throughout the universe. Because we will discuss these topics last, a "preview of coming attractions" is needed to help you understand and analyze the data we will be gathering. Here we will simply summarize some basic results without much justification; in class we'll see more about where these come from.

   (a) *Hubble's Law: The Result and its Meaning.* In 1929, Edwin Hubble (raised in Wheaton, Illinois!) studied the motion of galaxies of known distance. He found that essentially *all* galaxies are moving away from us. Moreover, he found that the motion is highly ordered: Hubble showed that a galaxy's distance r is *directly proportional to* to the speed v at which it moves away from us. That is, $v \propto r$ in magnitude, and moreover the directions are the same. Thus we have

   $$\vec{v} \propto \vec{r} \tag{1}$$

   for galaxies.

   To help you visualize what this means, sketch a set of galaxies randomly scattered around us. For convenience, place the Milky Way at the center of the sketch. On the sketch, draw a velocity vector on each galaxy, with the lengths proportional to the velocity magnitude in a way that follows eq. (1). Comment on the nature of the motion you see, and give a possible interpretation of this motion.

   (b) *Hubble's Law Quantitatively.* Hubble not only found that galaxy velocities are proportional to distance, but he also was the first to measure the constant of proportionality linking the two. This is known as the "Hubble constant," and in honor of Hubble is known as H (often it is also called H_0, for reasons which will become apparent next week).

   Thus we can fully state *Hubble's Law* of galaxy motion as:

   $$\vec{v} = H\vec{r} \tag{2}$$

Show that the physical dimensions of H are [time^{-1}].

In practice, astronomers typically quote galaxy speeds in units of [km/sec], and galaxy distances in millions of parsecs, i.e., [Mpc] (where 1 Mpc = 10^6 pc). Thus H is often quoted in units of [km s^{-1} Mpc^{-1}]. In particular, the current best value for Hubble's constant is about

$$H = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \tag{3}$$

To show that this has the right units, convert it to units of yr^{-1}, i.e., inverse years.

(c) *Cosmological Redshifts.* What one actually measures is not a galaxy's speed, but a redshift, i.e., a shift in galaxy emission/absorption wavelengths. This is usually quantified by the dimensionless number, called the *redshift*, which compares emitted (i.e., rest-frame or laboratory) wavelength $\lambda_{\rm em}$ and observed wavelength $\lambda_{\rm obs}$:

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}} \tag{4}$$

Show that, if we interpret the redshift as a Doppler shift, then a galaxy's speed is just

$$v = cz \tag{5}$$

and then in terms of a galaxy's redshift and distance, Hubble's law becomes

$$cz = Hr \tag{6}$$

(d) *Estimating the Age of the Universe* In class, we will discuss the age of the universe in the context of our full (Einsteinian) cosmology. But already we can get a rough estimate, just using Hubble's Law.

Imagine some initial time $t = 0$ when all galaxies started at the same point $r = 0$ (in fact, here!). They are then launched with different speeds–the same speeds they have now (momentum conservation). If the universe has lived for a time t_0, then a galaxy with speed v will have moved some distance r: find this in terms of v and t_0 (not a hard problem!). But if the galaxy also obeys Hubble's Law, show that in this "coasting galaxies" approximation, the age of the universe must be related to the Hubble constant by

$$t_0 = t_H \equiv \frac{1}{H} \quad \text{coasting galaxies approx} \tag{7}$$

where the characteristic timescale $t_H \equiv 1/H$ is sometimes called the "Hubble time." For the above value of H, find t_0. Express your result in billions of years (Gyr). Compare your result to the age of the solar system, and comment.

(e) *Estimating the Size of Observable Universe.* If the universe has a finite age, then it is also true that only a finite region of it is visible to us. We can estimate the radius of the observable universe by finding the distance light can travel in time t_0. Show that this distance is

$$r_{\rm observable} = d_H \equiv \frac{c}{H} \quad \text{coasting galaxies approx} \tag{8}$$

where the characteristic lengthscale $d_H \equiv c/H$ is sometimes called the "Hubble length."

Calculate the Hubble length d_H in units of Mpc. Compare your result to the size of the Milky Way (about 15,000 pc in radius).

2. *The Digital Universe Online.* The Sloan Digital Sky Survey (SDSS) is an ongoing project to map a large fraction of the sky with digital images and spectra. The Sloan survey contains a tremendous amount of cosmological data, most of it publicly available online. The SDSS website and some key pages within it are accessible from the ASTR210 `links` page. Gathering a bit of data, you can quickly arrive at some interesting cosmological conclusions. (*Note*: recall that while you should consult discussions on the website, the wording of your answers must of course be your own! To help avoid even accidental problems, I strongly suggest you close your browser while writing your responses.)

   (a) Go to the SDSS main page. Briefly summarize some of the goals and/or main results of the survey?

   (b) Briefly summarize the essentials of the survey telescope. Where is the survey telescope located? How large is the telescope, and what kind of telescope is it? How are the images taken? How are the spectra taken in later follow-up observations?

   (c) Follow the *navigate* link to go to the SDSS `Navigate Tool`, which will start you on a nearby (and thus large and bright) spiral galaxy; this will be the starting point of your journey. Note that you can use the tool to wander around the sky. As you do this, you can click on objects and an automated code will give information, including a (usually but not always correct) classification of the object as a star in our own Galaxy, or an external galaxy. Estimate the fraction or percentage of the SDSS objects that are stars, and the fraction that are galaxies. A rough estimate is fine, though you are welcome to do something more detailed; in any case, explain how you made your estimate. On the basis of appearance, how are these distinguished?

   (d) Now go to a random location in the survey, and from the Explore window launch the `Finding Chart` window. From the help window, see how to label all galaxies in the field of view (to do this, type the letter G into the text box below "Use query to mark objects"; typing S instead gives stars). Using this feature, and drawing a grid, estimate the number of galaxies in SDSS per square arc minute (one arc minute is denoted 1 arcmin $= 1'$, while one arc second is 1 arcsec $= 1'' = \frac{1}{60}$ arcmin). You will want to zoom to get a field of view that contains enough galaxies to give a good estimate, but not so many you can't count them.

   (e) Assuming isotropy, use your estimate from (d) to calculate the total number $N_{\rm gal,SDSS}$ of galaxies the SDSS survey would find over the entire sky. Note that in the whole sky the angular "area" (technically, "solid angle") is 4π steradians $= 1.5 \times 10^8$ arcmin^2. Explain why your result is an *underestimate* of the number of galaxies in the observable universe. Also, compare your result to the population of the United States, now almost exactly 300 million $= 3 \times 10^8$ people.

(f) Much more information is available for the SDSS galaxies with measured spectra. Not all SDSS galaxies have spectra, but using the `SpecObjs` option you can identify those that do. Find at least 5 such galaxies randomly (be sure they are galaxies and not stars!). For each, click on the galaxy in the image, then click the `Explore` button to find data and a spectrum for the galaxy. For each of these galaxies, record the redshift z (the number between `fiberid` and `zErr`, and *not* the last number in the `ugriz` list; the correct value is alo written at the bottom of the image of the spectrum). Also record the observed apparent r-band magnitude m_r, which is the r entry in the `ugriz` list.

Use the redshift and the Doppler relation $v = cz$ to compute the speed of each of your galaxies.

(g) Use your results from part (f), and Hubble's Law, to compute the distance to each galaxy; express your answer in Mpc. Take the average of these, and use this as an estimate of the average distance $d_{\rm sdss}$ to an SDSS galaxy. This is sometimes also called the "depth" of the survey. Show that your result obeys $d_{\rm sdss} < d_{\rm H}$, where $d_{\rm H}$ is the Hubble length.

(h) The flux from each galaxy (in different wavelength bands–i.e., colors) are given as the values of $ugriz$, which are expressed as apparent magnitudes. For each of your five SDSS galaxies, the r-band magnitude m_r, you recorded measures the flux centered at 625 nm and thus "red." Using your distance to each galaxy, also calculate its distance modulus $m_r - M_r = 5\log_{10}(r/10\text{pc})$. Use this to find the absolute magnitude M_r for each galaxy.

Together with the absolute r-band magnitude of the Sun $M_{\odot,r} = 4.4$ mag, compute each galaxy's r-band luminosity in units of $L_{\odot,r}$. Compare your galaxies' luminosities to the Milky Way luminosity, $L_{{\rm MW},r} \sim 2 \times 10^{10} L_\odot$, and comment.

(i) Using your value for $d_{\rm sdss}$, compute the volume $V_{\rm sdss}$ of the sphere around us that is accessible to the survey. Then combine your result with that of part (e) to compute the number density $n_{\rm gal}$, i.e., the number of galaxies per volume, in the local universe today. Express your answer in galaxies/Mpc^3.

(j) Imagine galaxies are evenly spread in space (e.g., in a cubic lattice). In this case, show that distance between nearest neighbors is $\ell_{\rm gal} = n_{\rm gal}^{-1/3}$, where $n_{\rm gal}$ is the number density of galaxies.

Use your value for the galaxy number density to estimate the typical spacing between galaxies in the universe today. Express your answer in Mpc. Compare your result to the size of the Milky Way, and to the distance to our nearest sibling galaxy, Andromeda (a.k.a. M31) which lies at $d_{\rm M31} = 0.7$ Mpc.

(k) Finally, use your number density from part (2i) to compute the number of galaxies within the "Hubble volume" today. That is, find the number of galaxies within the a spherical volume of radius $d_{\rm H}$.

Congratulations! You have now calculated–on the basis of real modern data–the number of galaxies in the observable universe today. Bravo! Compare your result with the current US population and with the global population of about 6.6 billion people.

(l) Finally, comment on any complications that would be involved in firming up some of the estimates we have made in this problem.

3. **Bonus [10 points]**
Come up with your own original mnemonic for the stellar spectral types OBAFGKM, to replace the standard "O Be A Fine Girl/Guy Kiss Me." Submissions should be no worse than PG-13 rated to receive credit. For style points, extend this to include the new spectral types L and T (i.e., create something that can also work for OBAFGKML or even OBAFGKMLT). Prizeworthy entries will be shown in class.

   To receive credit, submit your entry on Compass by the start of class on Friday April 22.