Astronomy 210
Homework Set #11

Due in class: Friday, April 29

Total Points: 50

1. *The energetics of a supernova explosion.*

   (a) **[5 points]** In a typical supernova explosion, most of the mass of the pre-explosion star is ejected a high speed. For a typical ejected mass of $M_{\rm eject} \approx 15 M_\odot$ and velocity $v \approx 3000$ km/s, find the kinetic energy of the ejecta. Express your answer in Joules and in "foe", where 1 foe $= 10^{51}$ erg $= 10^{44}$ Joules (that is, ten to the *f*ifty-*o*ne *e*rgs–an actual unit used in the supernova community).

   (b) **[5 points]** Supernova explosions are enormously bright, with absolute magnitudes which peak at $M_{\rm peak} \approx -19$. Using $M_\odot = +4.83$, find the ratio of peak luminosity to that of the Sun, $L_{\rm peak}/L_\odot$.

   Typically, a supernova's brightness is near the peak for about 10 days. Using this, estimate the total photon energy radiated in the supernova explosion, expressing your answer in Joules and in foe.

   Also calculate the total photon energy emitted by the Sun over its entire main sequence lifetime of 10^{10} yr, express this is the same units, and compare this to your result for the supernova.

   (c) **[5 points]** Neutrinos were measured from supernova 1987A, with typical neutrino energies of about 20 MeV, with 1 MeV $= 1.60 \times 10^{-13}$ Joule. These neutrinos traveled almost at the speed of light and thus in many ways behave like photons. Using the relations we found for photons, find the wavelength of a neutrino with energy 20 MeV. Then assuming this is the peak wavelength of thermal neutrino emission, find the neutrino *temperature* of SN 1987A, in Kelvin (don't be surprised if this is a big number!).

   Neutrinos come from the center of the explosion, emitted from the surface of the newborn neutron star with radius of about 10 km. Using this and the temperature you found, find the neutrino luminosity of the explosion. In fact, it turns out there are six varieties of neutrinos (three kinds, each of which also has antineutrinos), and the supernova radiates all of them. Using this, find the total neutrino luminosity in all neutrino species. Finally, in SN 1987A the neutrinos were measured for about 20 sec. Using this, find the total neutrino energy emitted from 1987A, in Joules and in foe.

   What is the dominant form of energy release in a supernova?

2. *The Black Hole at the Center of our Galaxy.* As discussed in class, our Galaxy contains an optically invisible (so far!) radio source at its center, called Sagittarius A* or just SgrA*. This object does not move, but is the center of motion for a swarm of stars whose orbits have been tracked for years now.

   (a) **[5 points]** The star whose orbit comes the closest to SgrA* has been found to have an orbit period $P = 15.6$ yr, and semimajor axis $a = 960$ AU. Use these to find the mass of SgrA*.

   One might first imagine SgrA* is just a star. Why doesn't this make sense?

   (b) **[5 points]** Based on your result from part (a), find the Schwarzschild radius $R_{\rm Sch}$ of SgrA*. Express your answer in AU and in pc. If you were 1 AU from SgrA*, what might you notice?

   (c) **[5 points]** SgrA* was first detected at radio wavelengths, and now has been seen in X-rays as well. The emission is not from the black hole itself, but rather surrounding material which is being dragged in. In fact, circular orbits around a black hole are only stable at distances $> 3R_{\rm Sch}$; objects moving closer rapidly fall in. Thus the emission we see comes from distances $> 3R_{\rm Sch}$.

i. Find the ratio $\lambda_{\rm obs}/\lambda_{\rm emit}$ of wavelengths for photons which we observe at Earth and which were emitted at $3R_{\rm Sch}$. Comment on how the emitted and observed spectra of SgrA* compare.

ii. Also, if we observe an event at $3R_{\rm Sch}$ which appears to last for $\Delta t_{\rm obs} = 10$ sec, how long did the event last from the point of view of an unlucky observer at $3R_{\rm Sch}$?

(d) **[5 points]** The mass density of stars around SgrA* is much higher than in the rest of our Galaxy, but let's adopt a more typical value of $\rho \approx 1 M_\odot/{\rm pc}^3$. Find distance from SgrA* at which the enclosed mass due to stars is equal the black hole mass; express your answer in pc. Explain why this amounts to the gravitational "sphere of influence" of the black hole. Compare your result to the size of our Galaxy, and comment.

3. *Dark Matter and You.* As discussed in class, our Galaxy contains large amounts of mass which emits no light.

(a) **[5 points]** The evidence for this dark matter is the "flat rotation curve" of our Galaxy. What is a rotation curve–what is plotted? What about it is flat? How does this provide evidence for dark matter?

(b) **[5 points]** At the Sun's distance $R_\odot = 8$ kpc from the Galactic center, the rotation speed is about 200 km/s. Using the usual formula for circular orbit speed around a gravitating mass, find the mass needed to maintain our orbit around the Galaxy; express your answer in units of $M_\odot$. This measures the Galaxy's mass inside $R_\odot$. Also find the average Galactic mass density ρ, in ${\rm kg/m^3}$.

(c) **[5 points]** Many people (including your instructor!) believe that dark matter consists if exotic, weakly-interacting particles left over from the big bang. Popular theories for dark matter suggest that such a particle might have a mass around $m_{\rm dm} \approx 100 m_p$, i.e., about 100 times the proton mass. If dark matter is a particle, the mass density ρ and the particle number density n are related by $\rho = mn$. Using this, estimate the number density of dark matter particles at Earth, if about half of the Galaxy's mass inside $R_\odot$ is due to dark matter.

Finally, estimate the volume of your body, and using this calculate the number of dark matter particles inside of yourself. Briefly comment.