Astronomy 210 Spring 2011
Homework Set #2

Due in class: Friday, Feb. 4
Total Points: 50

1. [**5 points**] Suppose that in a heliocentric model, the stars were located on a true, literal celestial sphere in the form of a "shell" at a radius $r_{\rm shell}$.

   Put the shell at $r_{\rm shell} = 10$ Earth-Sun distances. If two stars, located in the plane of Earth's orbit, are observed to be an angle $\theta_{\rm near} = 2°$ apart when the Earth is closest to them, how far apart $\theta_{\rm far}$ will they appear when the Earth is on the side of its orbit farthest from them? Would the difference $\Delta\theta = \theta_{\rm near} - \theta_{\rm far}$ be observable to the naked eye? Comment on the implications for the heliocentric vs geocentric debate. *Hint*: You may use the small angle approximation for this calculation.

2. *The Receding Moon.* For reasons that will become apparent later in the course, the Moon is slowly moving away from the Earth. This has several implications; here we will look at the impact on solar eclipses. Some data first: the radius of the Earth is 6370 km; the radius of the moon is 1740 km; the radius of the sun is 6.96×10^8 m (careful with units!); the semimajor axis and eccentricity of the Moon's orbit are 384,000 km and 0.055; the semimajor axis and eccentricity of the Earth's orbit are 1.496×10^{11} m and 0.017.

   (a) [**4 points**] What is the angular diameter of the sun, in degrees, as seen from the surface of the Earth at aphelion? at perihelion?

   (b) [**4 points**] What is the angular diameter of the moon when it is directly overhead at apogee (farthest from Earth)? at perigee (closest)?

   (c) [**4 points**] Can a total solar eclipse occur when the moon is at apogee? At perigee? Why?

   (d) [**4 points**] The semimajor axis of the Moon's orbit is increasing by 3.68 cm/yr. How long will it be until total solar eclipses cease (assume that all other orbital elements are fixed)?

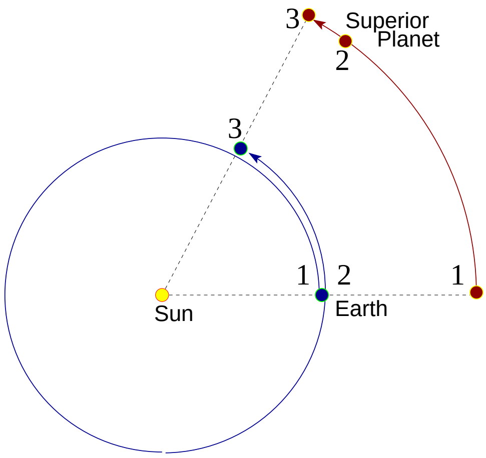
   (e) [**5 points**] Now work backwards. Using the same assumptions as in part (d), find how long ago it was in the past when the first annular eclipse of the Sun occurred. We will see that the age of the Earth is 4.6 billion years = 4.6×10^9 yr. How does your result compare to this–that is, was there a time in the past when no solar eclipses were annular, and all were total?

Annular eclipse of the Sun.

http://antwrp.gsfc.nasa.gov/apod/ap980824.html

3. *Sidereal and Synodic Periods.* There is a subtlety in determining the orbital periods of solar system objects; this stems from the fact that we make our observations from an orbiting earth. One can define two distinct periods (1) The *sidereal* period P of a body is it's orbital period with respect to the fixed stars–i.e., it's orbital period in a non-rotating, inertial frame; Kepler's laws refer to this period. (2) The *synodic* period S of an object is the time it takes to make one complete cycle with respect to the sun–i.e., the time from one opposition to the next, or from one conjunction to the next.

   These two periods, and the earth's sidereal period $E = 1$ yr, are different but related, as you will now show.



(a) **[5 points]** Consider the case of a *superior* planet–i.e., one whose orbit lies outside of ours, and thus has $P > E$ as in the figure at right. Let $\Delta\theta = \theta_{\text{earth}} - \theta_{\text{planet}}$ be the *difference* between the angles the planets sweep as they go around the Sun. At the first opposition (time 1), put $\theta = 0$. Using the sidereal periods of the earth and the planet, find the angular speed ω of each, and use this to determine $\Delta\theta(t)$. For simplicity you may assume a circular orbit in which all ωs are constant in time.

(b) **[5 points]** Find the value of $\Delta\theta$ at the next opposition (time 3). The time elapsed between the two oppositions is by definition $t = S$. Using your expression from part (a), evaluate $\Delta\theta(S)$, and from this show that

$$\frac{1}{S} = \frac{1}{E} - \frac{1}{P}$$

Thus, given the observed quantity S, and knowing E, we can infer P.

(c) **[4 points]** Now consider the case of an *inferior* planet, i.e., one closer to the Sun than us, so that $P < E$. Repeat the analysis of (a) and (b) for this case, and show that

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}$$

(d) **[5 points]** Now consider the Moon. Find the expression relating its sidereal and synodic periods. The cycle of lunar phases repeats every 29.5 days; using this, find the Moon's sidereal and synodic periods. Note that the axis of the Moon's orbit around the Earth is (almost) the same as the Earth's orbit axis around the Sun, i.e., the two orbits are "in the same direction" – both counterclockwise as seen looking down from the north.

(e) **[5 points]** Finally, note that all of the planets in the solar system orbit in the same sense, i.e., the same direction around the Sun and nearly the same plane. Imagine a planet moving in the same plane as the Earth, but in the *opposite* direction as the Earth. If such a planet is in a superior orbit, find the relation between its sidereal and synodic periods. If Mars ($a = 1.5$ AU) were in such a "backwards" orbit, what would be its synodic period?