Astronomy 210 Spring 2011
Homework Set #6

Due in class: Friday, March 11 Total Points: 50 + 10 bonus

1. *On the Lookout for the Asteroid of Doom.* While most asteroids have orbits that confine them to the asteroid belt between Mars and Jupiter, there are still many that are found in the inner solar system, to include some with orbits which cross the Earth's. Just one asteroid impact can ruin your entire day, as the dinosaurs discovered 65 million years ago.

   (a) [**5 points**] Consider a near-Earth asteroid which collides with the Earth. The asteroid begins its fall at a great distance $r \gg R_\oplus$ and with an initial speed v_0. Calculate the speed $v_{\rm hit}$ with which the asteroid impacts the Earth (i.e., arrives at the Earths' surface $r = R_\oplus$). Energy conservation will be useful here. You should find that $v_{\rm hit} \geq v_{\rm esc}$, that is, the impact speed is always at least the escape speed.

   (b) [**5 points**] Find an expression for the kinetic energy of an asteroid impacting the Earth at the escape speed. You may assume the asteroid is spherical with radius s and with uniform density ρ.

   Evaluate this impact energy for a medium-sized $s = 1$ km asteroid with density $\rho = \rho_{\rm rock} = 3000\ {\rm kg/m^3}$. Compare your answer to the energy released from a "small" nuclear explosion (equivalent to 1 kTon of TNT, 4×10^{12} Joules), and a large explosion (1 MTon of TNT, 4×10^{15} Joules), and comment on possible effects for the impact region and for global civilization. For comparison, the global nuclear arsenal represents about 10^4 MTon.

   (c) [**5 points**] It is obviously of interest to identify and track near-Earth asteroids. The upcoming Large Synoptic Survey Telescope (LSST) will, throughout 10 years of operation, repeatedly scan the entire southern sky every 2 to 4 days. The resulting "movie" will reveal anything that changes in the sky over timescales from days to years.

   Consider a spherical asteroid of radius s and albedo A, located a distance d from the Sun. Find the luminosity L of the sunlight *reflected* by the asteroid. Then show that, if the asteroid is a distance r from the Earth, then the flux of this reflected sunlight is

   $$F = \frac{A s^2 L_\odot}{16 \pi r^2 d^2} \tag{1}$$

   (d) [**5 points**] Consider a $s = 1$ km asteroid orbiting the sun with $d = 1$ AU, and at a distance $r = 0.1$ AU from the Earth. Asteroid albedos vary, but take an average value of $A = 0.1$. Find the brightness F of the asteroid, in ${\rm Watt/m^2}$. LSST is sensitive to fluxes as small as $F_{\rm min} \approx 10^{-17}\ {\rm Watt/m^2}$. Can LSST detect this asteroid?

   (e) [**5 bonus points**] We confirm that the object in part 1d is an asteroid by observing its motion relative to the Earth. Consider an asteroid with semimajor axis a and eccentricity e. Show that when the asteroid is at aphelion, it's speed is given by

   $$v_{\rm ap}^2 = \frac{1-e}{1+e} \frac{G M_\odot}{a} = \frac{1-e}{1+e}\, v_{\rm c}^2 \tag{2}$$

   where $v_{\rm c}$ is the speed the asteroid would have in a circular orbit of radius a.

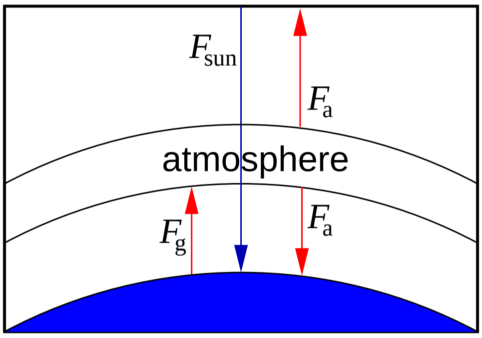
   (f) [**5 points**] A near-Earth asteroid with semi-major axis $a = 1$ AU and $e = 0.1$ is seen at opposition while it is at aphelion. Taking the Earth's orbit to be circular, find the *relative* speed δv between the asteroid and the Earth; express your answer in m/s. Then find the angular speed $\omega = \delta v / r$ of the asteroid as seen by Earth, where r the distance to the asteroid; convert your answer from radians/sec to degrees/day.

   (g) [**5 points**] LSST can easily measure changes in sky position ("proper motion") of 0.01 arcsec/year or larger. Can LSST detect the motion of asteroid in part 1f? Will it do this in days, months, years, or never? How well could it map the asteroid's orbit?

(h) **[5 points]** Given the results in part 1d, find the *maximum* size $s_{\rm min}$ of asteroids which are *not* detectable because they are at or below the flux limit $F_{\rm min}$ of LSST (again taking $d = 1$ AU and $r = 0.1$ AU). Find the energy released when such an undetectable object collides with the Earth, and comment on the danger posed by such an impactor.

2. *The Greenhouse Effect.* In class, we calculated planetary temperatures as a function of distance d from sun, albedo A, and whether the planet has a thick atmosphere or not. Our simple "first order" treatment was idealized, and for example did not allow for greenhouse effect. We can make a simple but instructive estimate of the greenhouse effect as follows.

Imagine the greenhouse gasses in the Earth's atmosphere to be a single layer that is completely transparent to the visible wavelengths of the light received from the Sun, but completely opaque to the infrared radiation emitted by the surface of the Earth. Assume (1) that the top and bottom surface areas of the atmosphere are each equal to the surface area of the planet, (2) that the entire layer is at the same temperature $T_{\rm a}$, and (3) that the layer radiates from both the top and the bottom.



(a) **[5 points]** In this model, the top of the atmosphere is responsible for radiating away all thermal energy from the Earth. Show that energy conservation demands that $T_{\rm a} = T_{\rm eq}$, where $T_{\rm eq}$ is the equilibrium temperature of the Earth as found in class.

(b) **[5 points]** For the moment set aside the result from part (a), and focus on the energy flows into and out of the atmosphere as well as those in and out of the ground. Show that energy conservation applied to the atmosphere gives

$$2T_{\rm a}^4 = T_{\rm g}^4 \tag{3}$$

and applied to the ground it gives

$$T_{\rm g}^4 = T_{\rm eq}^4 + T_{\rm eq}^4 \tag{4}$$

were $T_{\rm g}$ is the temperature of the ground, and again $T_{\rm eq}$ is the greenhouse-free equilibrium temperature found in class.

Using only these equations, solve for both $T_{\rm a}$ and $T_{\rm g}$ in terms of $T_{\rm eq}$. Verify that you recover the result in part (a). Also verify that $T_{\rm g} > T_{\rm eq}$ and thus the planet's surface is hotter than it would have been without an atmosphere.

(c) **[5 bonus points]** Now consider the case where the atmosphere consists of N layers of gas, each with its own temperature. If we call the top layer number 1, show that $2T_1^4 = T_2^4$. Layer N is then directly above the ground. Show that $T_{\rm g}^4 = T_N^4 + T_{\rm eq}^4$. Also show that for any other atmospheric layer $1 < n < N - 1$, we have $2T_n^4 = T_{n-1}^4 + T_{n+1}^4$ Finally, show that all of these equations are satisfied if we have, for all layers n,

$$T_n^4 = nT_{\rm eq}^4 \tag{5}$$

and the surface has

$$T_{\rm g}^4 = (N+1)T_{\rm eq}^4 \tag{6}$$

(d) **[5 points]** The true average temperature of the Earth ($A = 0.4$) is about 15°C. Convert this to Kelvin, and take this as $T_{\rm g}$. According to our model (eq. 6), how many layers N does the Earth's atmosphere have? This measure of atmospheric "effective thickness" to radiation need not be an integer.

Using the same albedo, compare the equilibrium temperature of Venus (orbital distance $d =$ 0.72 AU, albedo $A = 0.67$) with its surface temperature 460°C, and calculate the number N of layers in the Venus' atmosphere. Comment on your result.